Towards the continuum limit of the Hadronic Light-by-Light contribution to the muon anomalous magnetic moment

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Outline I

Muon g-2 experimental measurement [Bennett et al., 2006]

E821 at BNL measured relative precession of muon spin to it's momentum
$\omega_a = \frac{g-2}{2}\frac{eB}{m} = a_\mu \frac{eB}{m}$, the muon anomaly

The rate of detected electrons oscillates with ω_a, fit to
$N(t) = Be^{-\lambda t}(1 + A\cos\omega_a t + \phi)$

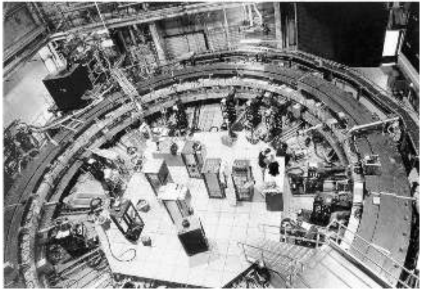


Figure 12. The storage-ring magnet. The cryostats for the inner-radius coils are clearly visible. The kickers have not yet been installed. The racks in the center are the quadrupole pulsers, and a few of the detector stations are installed, especially the quadrant of the ring closest to the person. The magnet power supply is in the upper left, above the plane of the ring. (Courtesy of Brookhaven National Laboratory)

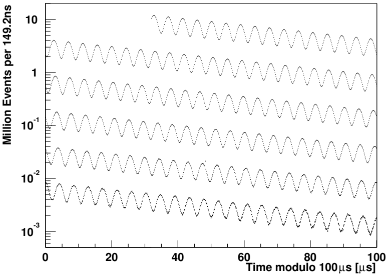
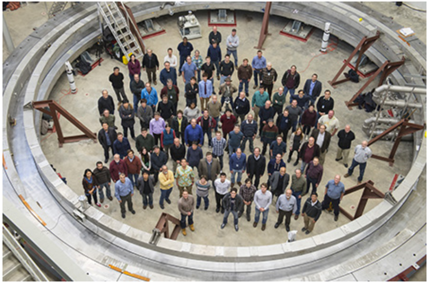


Figure 26. Histogram of the total number of electrons above 1.8 GeV versus time (modulo 100 μ s) from the 2001 μ^- data set. The bin size is the cyclotron period, $\approx$ 149.2 ns, and the total number of electrons is 3.6 billion.

New muon g-2 experiments

Storage ring moved to FNAL for E989, beginning in 2017



which is aiming for 0.14 ppm, $4\times$ improvement!

In Japan at J-PARC, the E34 experiment will measure a_μ using ultra-cold muons in a "table-top" experiment ($\sim$ 2020)

Experiment - Theory

SM Contribution	Value $\pm$ Error ($\times 10^{11}$)	Ref
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]
HVP LO	6923 ± 42	[Davier et al., 2011]
	6949 ± 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]
HLbL	105 ± 26	[Prades et al., 2009]
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014a]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592080 ± 63	[Bennett et al., 2006]
Diff (Exp − SM)	287 ± 80	[Davier et al., 2011]
	261 ± 78	[Hagiwara et al., 2011]
	249 ± 87	[Aoyama et al., 2012]

QCD errors largest, discrepancy large

New experiments+new theory=new physics?

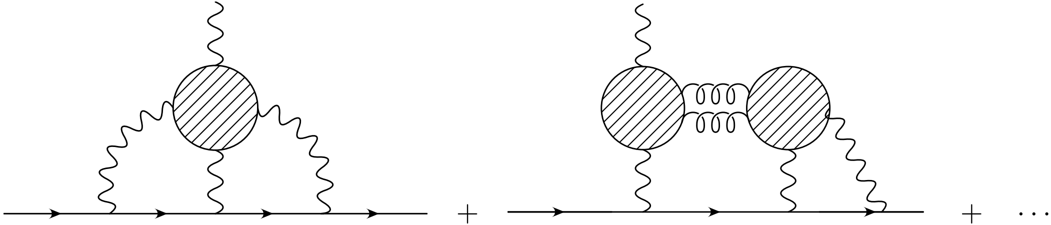
- Fermilab E989 begins 2017, aims for 0.14 ppm

  J-PARC E34 ∼2020, aims for 0.3-0.4 ppm

  Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,

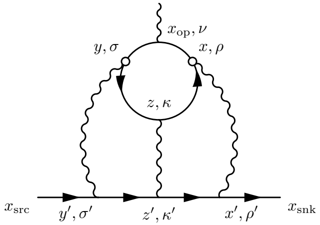
  E989 ($\sim 4\times$ smaller error) $\to \sim 5\sigma$
  E989+new HLBL theory (models+lattice, 10%) $\to \sim 6\sigma$
  E989+new HLBL +new HVP (50% reduction) $\to \sim 8\sigma$
- Good for discriminating models if discovery at LHC [Stckinger, 2013]
- Lattice calculations important to trust theory errors
  (see talks at Lattice 2016 (Southampton) for latest results by many groups)

Outline I



- Models: $(105 \pm 26) \times 10^{-11}$ [Prades et al., 2009, Benayoun et al., 2014]
  $(116 \pm 40) \times 10^{-11}$ [Jegerlehner and Nyffeler, 2009]
- Model errors difficult to quantify error now compatible with HVP error. see talk by A. Keshavarzi, 1st workshop muon g-2 theory initiative, June 2017, FNAL
- First lattice results [Blum et al., 2015, Blum et al., 2016, Blum et al., 2017] promise reliable errors (this talk and T. IZUBUCHI, next)
- Mainz group also calculating HLbL contribution (see talks in this session by N. ASMUSSEN and A. GERARDIN) [Green et al., 2015, Asmussen et al., 2016]
- Dispersive/data approach also systematic (see plenary talk tomorrow by G. COLANGELO) [Colangelo et al., 2014b, Pauk and Vanderhaeghen, 2014, Colangelo et al., 2015, Colangelo et al., 2017]

Point source method in pQED (L. Jin, Ph.D. thesis) [Blum et al., 2016]

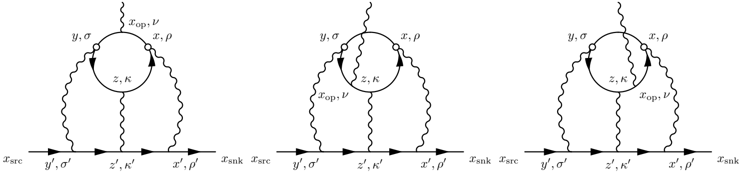


$$\mathcal{F}^C_\nu (\vec{q}; x, y, z, x_{\rm op}) = (-ie)^6 \mathcal{G}_{\rho,\sigma,\kappa}(\vec{q}; x, y, z)\mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op})$$

$$\begin{aligned} & i^4 \mathcal{H}^C_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op}) \\ = \quad & \sum_{q=u,d,s} \frac{(e_q/e)^4}{6} \left\langle \mathrm{tr}\left[-i \quad \gamma_\rho S_q(x,z) i\gamma_\kappa S_q(z,y) i\gamma_\sigma S_q\left(y, x_{\rm op}\right) i\gamma_\nu S_q\left(x_{\rm op}, x\right)\right]\right\rangle_{\rm QCD} + \text{other 5 permutations} \end{aligned}$$

$$\begin{aligned} & i^3 \mathcal{G}_{\rho,\sigma,\kappa}(\vec{q}; x, y, z) \\ = \quad & e^{\sqrt{m^2+\vec{q}^2/4}(t_{\rm snk}-t_{\rm src})} \sum_{x',y',z'} G_{\rho,\rho'}(x, x') G_{\sigma,\sigma'}(y, y') G_{\kappa,\kappa'}(z, z') \\ & \times \sum_{\vec{x}_{\rm snk},\vec{x}_{\rm src}} e^{-i\vec{q}/2\cdot(\vec{x}_{\rm snk}+\vec{x}_{\rm src})} S\left(x_{\rm snk}, x'\right) i\gamma_{\rho'} S(x', z') i\gamma_{\kappa'} S(z', y') i\gamma_{\sigma'} S\left(y', x_{\rm src}\right) + \text{other 5 permutations} \end{aligned}$$

Point source method in pQED (L. Jin, Ph.D. thesis) [Blum et al., 2016]



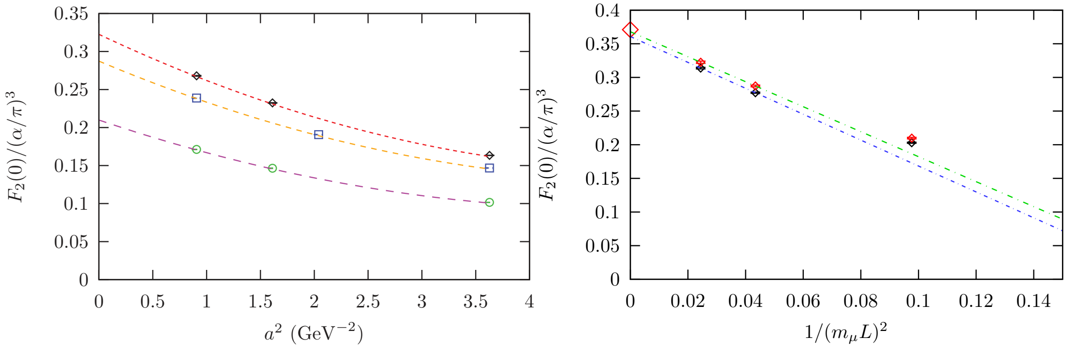
$$\frac{F_2(0)}{m} \bar{u}_{s'}(\vec{0}) \frac{\vec{\Sigma}}{2} u_s(\vec{0}) = \sum_r \left[\sum_{z, x_{\rm op}} \frac{1}{2} \vec{x}_{\rm op} \times \bar{u}_{s'}(\vec{0}) i \vec{\mathcal{F}}^C \left(\vec{0}; x = -\frac{r}{2}, y = +\frac{r}{2}, z, x_{\rm op} \right) u_s(\vec{0}) \right]$$

- Recall the definition for $\mathcal{F}$, we sum all the internal points over the entire space-time except we fix $x + y = 0$. (x, y) pairs stochastically sampled
- Key idea: contribution exponentially suppressed with $r = |x - y|$, so importance sample, concentrate on $r \lesssim \lambda_\pi$
- Three diagrams together insure Ward Identity on each configuration, so noise $\sim$ constant as $q \to 0$ (need seq. sources)
- $x_{\rm op}$ moment projects directly on to zero momentum transfer
- Photons: Feynman gauge, QED$_L$ [Hayakawa and Uno, 2008], muons: $L_s = \infty$ free DWF

Continuum and ∞ volume limits in QED [Blum et al., 2016]

Test method in pure QED

QED systematics large, $O(a^4)$, $O(1/L^2)$, but under control



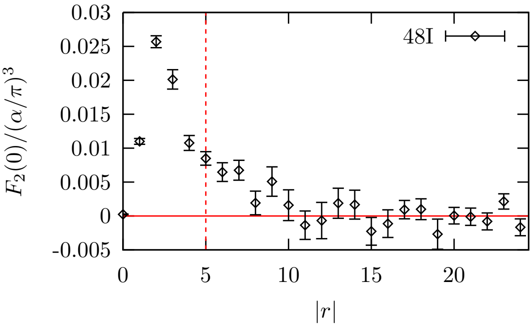
Limits quite consistent with well known PT result

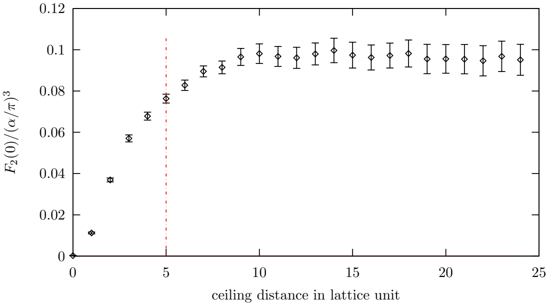
2+1f Möbius-DWF physical point ensembles (RBC/UKQCD) [Blum et al., 2014]

	$48^3 \times 96$	$64^3 \times 128$
a^{-1} (GeV)	1.73	2.36
a (fm)	0.114	0.084
L (fm)	5.47	5.38
m_π (MeV)	139	135
m_μ (MeV)	106	106

Physical point cHLbL contribution, 48^3, 1.73 GeV lattice [Blum et al., 2017]

- Uses AMA with 2000 low-modes of the Dirac operator and $\sim$ 2200 sloppy propagators per configuration (65 total)

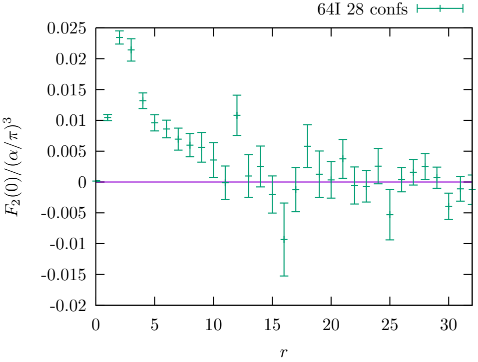


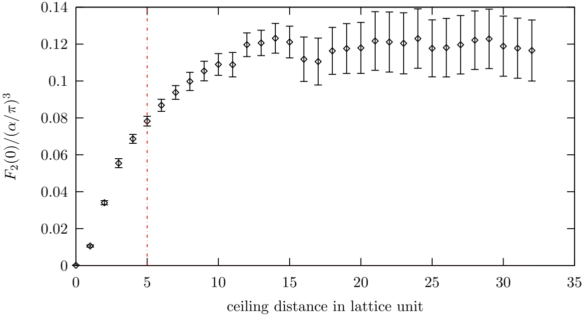


$$a_\mu^{\mathrm{cHLbL}} = 11.60 \pm 0.96 \times 10^{-10}$$

Physical point cHLbL contribution, 64^3, 2.36 GeV lattice (preliminary)

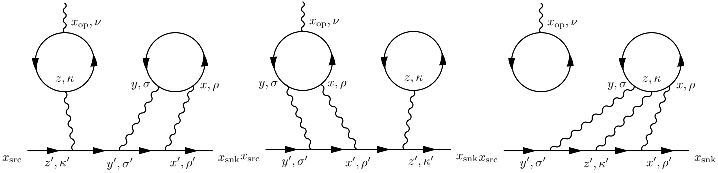
- Uses AMA with 2000 low-modes of the Dirac operator and $\sim$ 2200 sloppy propagators per configuration (28 total)



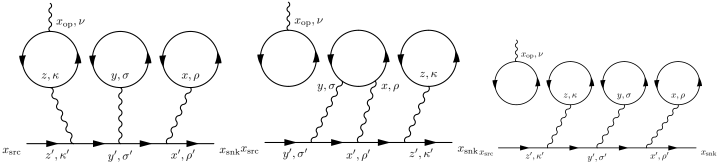


Disconnected contributions

SU(3) flavor:



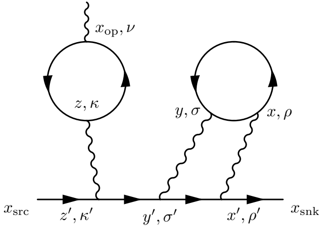
Leading $\qquad\qquad$ $O(m_s - m_{u,d})$



$O(m_s - m_{u,d})^2$ $\qquad\qquad$ and higher

- There should be gluons exchange between and within the quark loops, but are not drawn.
- We need to make sure that the loops are connected by gluons by "vacuum" subtraction. So the diagrams are 1-particle irreducible.

Leading disconnected contribution



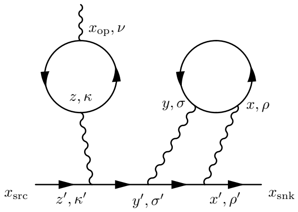
- We can use two point source photons at y and z, which are chosen randomly. The points $x_{\rm op}$ and x are summed over exactly on lattice.
- Only point source quark propagators are needed. We compute M point source propagators and all M^2 combinations are used to perform the stochastic sum over $r = z - y$.

$$\mathcal{F}^D_\nu(x, y, z, x_{\rm op}) = (-ie)^6 \mathcal{G}_{\rho,\sigma,\kappa}(x, y, z) \mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op})$$

$$\mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x, y, z, x_{\rm op}) = \left\langle \frac{1}{2} \Pi_{\nu,\kappa}(x_{\rm op}, z) \left[\Pi_{\rho,\sigma}(x, y) - \Pi^{\rm avg}_{\rho,\sigma}(x - y) \right] \right\rangle_{\rm QCD}$$

$$\Pi_{\rho,\sigma}(x, y) = -\sum_q (e_q/e)^2 \,\mathrm{Tr}[\gamma_\rho S_q(x, y) \gamma_\sigma S_q(y, x)].$$

Leading disconnected contribution



$$\frac{F_2^{\rm dHLbL}(0)}{m}\frac{(\sigma_{s',s})_i}{2} = \sum_{r,x}\sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(\tilde{x}_{\rm op}\right)_j \cdot i\bar{u}_{s'}(\vec{0})\mathcal{F}_k^D\left(x, y=r, z=0, x_{\rm op}\right)u_s(\vec{0})$$

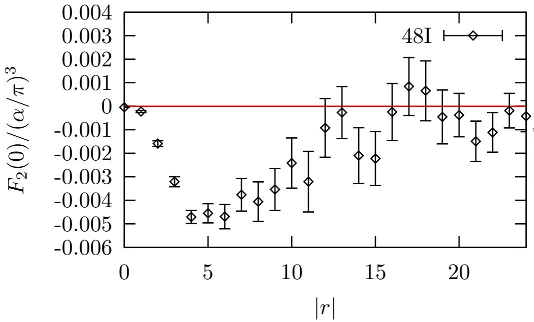
$$\mathcal{H}^D_{\rho,\sigma,\kappa,\nu}(x,y,z,x_{\rm op}) = \left\langle \frac{1}{2}\Pi_{\nu,\kappa}\left(x_{\rm op}, z\right)\left[\Pi_{\rho,\sigma}(x,y) - \Pi^{\rm avg}_{\rho,\sigma}(x-y)\right]\right\rangle_{\rm QCD}$$

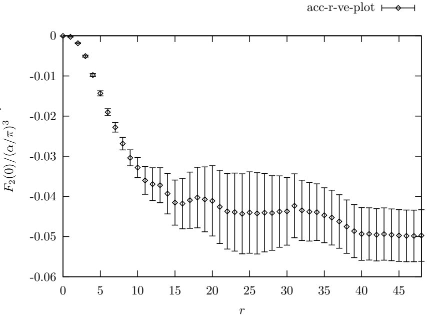
$$\sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(x_{\rm op}\right)_j\left\langle\Pi_{\rho,\sigma}\left(x_{\rm op},0\right)\right\rangle_{\rm QCD} = \sum_{x_{\rm op}}\frac{1}{2}\epsilon_{i,j,k}\left(-x_{\rm op}\right)_j\left\langle\Pi_{\rho,\sigma}\left(-x_{\rm op},0\right)\right\rangle_{\rm QCD} = 0$$

- Because of parity, the expectation value for the left loop averages to zero.
- $\left[\Pi_{\rho,\sigma}(x,y) - \Pi^{\rm avg}_{\rho,\sigma}(x-y)\right]$ is only a noise reduction technique. $\Pi^{\rm avg}_{\rho,\sigma}(x-y)$ should remain constant through out the entire calculation.

Physical point dHLbL contribution, 48^3, 1.73 GeV lattice [Blum et al., 2017]

- Uses AMA with 2000 low-modes of the Dirac operator and $(1024+512)^2$ measurements per configuration (65 total)



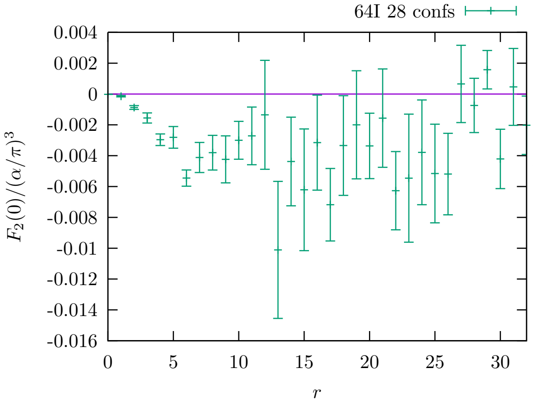


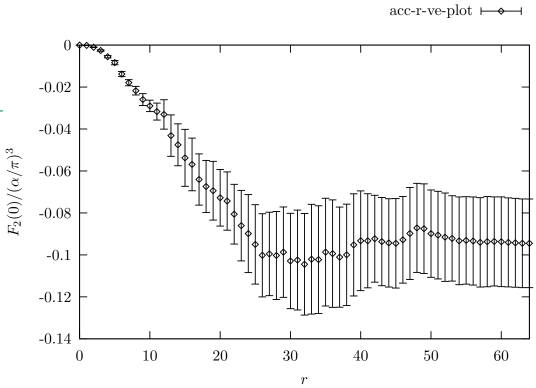
$$a_\mu^{\mathrm{dHLbL}} = -6.25 \pm 0.80 \times 10^{-10}$$

$$a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35 \pm 1.35 \times 10^{-10} \; (48^3, a^{-1} = 1.73\ \mathrm{GeV})$$

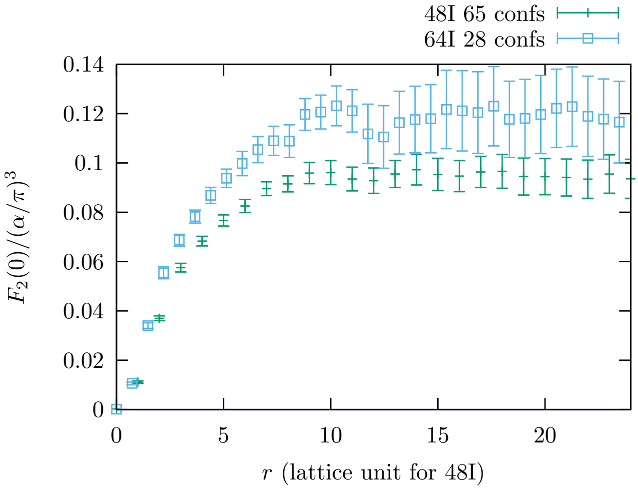
Physical point dHLbL contribution, 64^3, 2.36 GeV lattice (preliminary)

- Uses AMA with 2000 low-modes of the Dirac operator and $(1024 + 512)^2$ measurements per configuration (10 total)



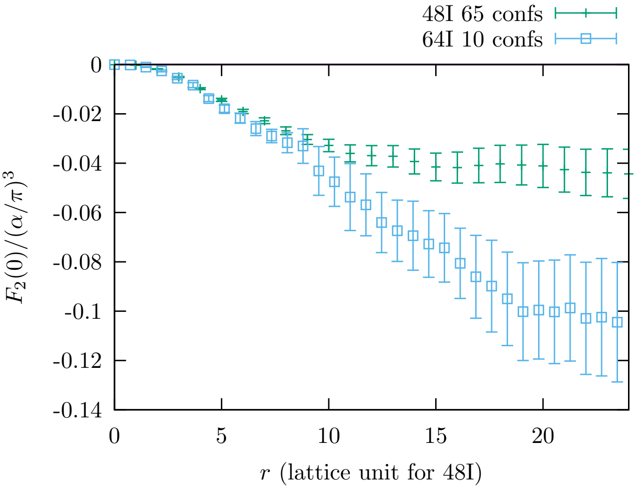


cHLbL contribution: lattice spacing effect (preliminary)



- Significant increase as $a \to 0$

dHLbL contribution: lattice spacing effect (preliminary)

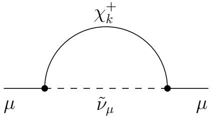


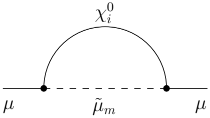
- Large negative increase tends to cancel connected one
- Collecting more statistics!

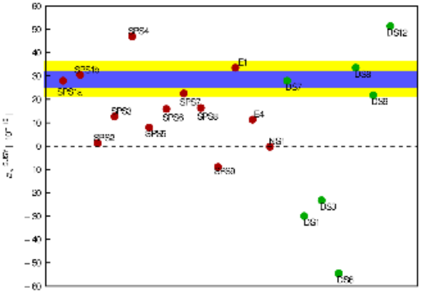
Physics beyond the SM

If there really is a discrepancy, where does it come from?

Most likely scenario is still SUSY (?) [Bach et al., 2015, Athron et al., 2016, Belyaev et al., 2016], . . .







SUSY signatures at LHC

But there are other models too: 2HDM[Crivellin et al., 2016, Cherchiglia et al., 2016], Dark Matter [Kobakhidze et al., 2016], . . . , LFV [Altmannshofer et al., 2016]

Outline I

Summary

- Lattice QCD(+QED) calculations with physical masses, large boxes + improved measurement algorithms are powerful
- Physical point calculations complete at $a = 0.114$ fm [Blum et al., 2017]
- Physical point nearly complete at $a = 0.084$ fm
- together, good control of non-zero a systematic error.
  Next: finite volume (next talk by T. Izubuchi)
- Need non-leading (SU(3) flavor) disconnected diagrams
- E989 measurement at Fermilab (goal 0.14 ppm) now underway (!), good opportunity to test the standard model
- Lattice results: seems unlikely that HLbL contribution will rescue standard model

Acknowledgments

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Outline I

Altmannshofer, W., Carena, M., and Crivellin, A. (2016).
A $L_\mu - L_\tau$ Theory of Higgs Flavor Violation and $(g-2)_\mu$.
Submitted to: Phys. Rev. Lett.

Aoyama, T., Hayakawa, M., Kinoshita, T., and Nio, M. (2012).
Complete Tenth-Order QED Contribution to the Muon g-2.
Phys.Rev.Lett., 109:111808.

Asmussen, N., Green, J., Meyer, H. B., and Nyffeler, A. (2016).
Position-space approach to hadronic light-by-light scattering in the muon $g-2$ on the lattice.
PoS, LATTICE2016:164.

Athron, P., Bach, M., Fargnoli, H. G., Gnendiger, C., Greifenhagen, R., Park, J.-h., Paehr, S., Stckinger, D., Stckinger-Kim, H., and Voigt, A. (2016).
GM2Calc: Precise MSSM prediction for $(g-2)$ of the muon.
Eur. Phys. J., C76(2):62.

Bach, M., Stckinger, D., Stckinger-Kim, H., and Park, J.-H. (2015).

News on Muon $(g-2)$.
Acta Phys. Polon., B46(11):2243.

Belyaev, A. S., Camargo-Molina, J. E., King, S. F., Miller, D. J., Morais, A. P., and Schaefers, P. B. (2016).
A to Z of the Muon Anomalous Magnetic Moment in the MSSM with Pati-Salam at the GUT scale.
JHEP, 06:142.

Benayoun, M., Bijnens, J., Blum, T., Caprini, I., Colangelo, G., et al. (2014).
Hadronic contributions to the muon anomalous magnetic moment Workshop. $(g-2)_\mu$: Quo vadis? Workshop. Mini proceedings.

Bennett, G. et al. (2006).
Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL.
Phys.Rev., D73:072003.

Blum, T., Chowdhury, S., Hayakawa, M., and Izubuchi, T. (2015).
Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment from lattice QCD.

Phys.Rev.Lett., 114(1):012001.

Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., Jin, L., Jung, C., and Lehner, C. (2017).

Connected and Leading Disconnected Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment with a Physical Pion Mass.

Phys. Rev. Lett., 118(2):022005.

Blum, T., Christ, N., Hayakawa, M., Izubuchi, T., Jin, L., and Lehner, C. (2016).

Lattice Calculation of Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment.

Phys. Rev., D93(1):014503.

Blum, T. et al. (2014).

Domain wall QCD with physical quark masses.

Cherchiglia, A., Kneschke, P., Stckinger, D., and Stckinger-Kim, H. (2016).

The muon magnetic moment in the $2\mathrm{HDM}$: complete two-loop result.

Colangelo, G., Hoferichter, M., Nyffeler, A., Passera, M., and Stoffer, P. (2014a).

Remarks on higher-order hadronic corrections to the muon g?2.
Phys. Lett., B735:90–91.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2014b).
Dispersive approach to hadronic light-by-light scattering.
JHEP, 1409:091.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2015).
Dispersion relation for hadronic light-by-light scattering: theoretical foundations.

Colangelo, G., Hoferichter, M., Procura, M., and Stoffer, P. (2017).
Dispersion relation for hadronic light-by-light scattering: two-pion contributions.
JHEP, 04:161.

Crivellin, A., Heeck, J., and Stoffer, P. (2016).
A two-Higgs-doublet model facing experimental hints.
EPJ Web Conf., 118:01014.

Davier, M., Hoecker, A., Malaescu, B., and Zhang, Z. (2011).
Reevaluation of the Hadronic Contributions to the Muon g-2 and to alpha(MZ).

Eur.Phys.J., C71:1515.

Gnendiger, C., Stckinger, D., and Stckinger-Kim, H. (2013).
The electroweak contributions to $(g-2)_\mu$ after the Higgs boson mass measurement.
Phys.Rev., D88:053005.

Green, J., Gryniuk, O., von Hippel, G., Meyer, H. B., and Pascalutsa, V. (2015).
Lattice QCD calculation of hadronic light-by-light scattering.

Hagiwara, K., Liao, R., Martin, A. D., Nomura, D., and Teubner, T. (2011).
$(g-2)_\mu$ and alpha(M_Z^2) re-evaluated using new precise data.
J.Phys., G38:085003.

Hayakawa, M. and Uno, S. (2008).
QED in finite volume and finite size scaling effect on electromagnetic properties of hadrons.
Prog.Theor.Phys., 120:413–441.

Jegerlehner, F. and Nyffeler, A. (2009).
The Muon g-2.

Phys. Rept., 477:1–110.

Kobakhidze, A., Talia, M., and Wu, L. (2016).
Probing the MSSM explanation of the muon g-2 anomaly in dark matter experiments and at a 100 TeV *pp* collider.

Kurz, A., Liu, T., Marquard, P., and Steinhauser, M. (2014).
Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order.
Phys.Lett., B734:144–147.

Pauk, V. and Vanderhaeghen, M. (2014).
Two-loop massive scalar three-point function in a dispersive approach.

Prades, J., de Rafael, E., and Vainshtein, A. (2009).
Hadronic Light-by-Light Scattering Contribution to the Muon Anomalous Magnetic Moment.

Stckinger, D. (2013).
The muon magnetic moment and new physics.

Hyperfine Interact., 214(1-3):13–19.