The spin content of the nucleon

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with

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Outline

- The proton spin
- Renormalization
- Calculation of disconnected quark line diagrams
- Fit strategy
- Results
- Summary

Spin of the nucleon

$$\frac{1}{2} = \overbrace{\frac{1}{2}\Delta\Sigma}^{\text{quark spin}} + \overbrace{\sum_q L_{q_+} + J_g}^{\text{dark spin}}$$

Ji decomposition into the contributions of the (longitudinal) quark spins

$$\Delta\Sigma = \Delta u_+ + \Delta d_+ + \Delta s_+ + \cdots , \quad (\Delta q_+ = \Delta q + \Delta\bar{q}),$$

the (longitudinal) quark orbital angular momenta $L_{q_+} = J_{q_+} - \frac{1}{2}\Delta q_+$ and the total (longitudinal) gluon angular momentum J_g.

Naïve non-relativistic SU(6) quark model: $\Delta\Sigma = 1$, $L_{q_+} = J_g = \Delta s_+ = 0$.
Relativistic quark models: $\Delta\Sigma \sim 0.6$, $L_{\text{quarks}} \sim 0.2$.

I will say nothing about the Jaffe and Manohar decomposition:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_{\text{quarks}} + \Delta G + \mathcal{L}_g \quad \left(J_g \neq \Delta G + \mathcal{L}_g, J_{q_+} \neq \frac{1}{2}\Delta q_+ + \mathcal{L}_{q_+}\right).$$

Individual quark spin contributions ($q \in \{u, d, s\}$)

$$\Delta q_+ \, s_\mu = \frac{1}{m_N} \langle N, s \,|\bar{q}\gamma_\mu\gamma_5 q|\, N, s\rangle = F_A^q(0) = \tilde{A}_{10}^q(0)$$

Axial charges:

$$a_3 = -s_\mu \frac{1}{m_N}\langle N, s|\bar{\psi}\gamma_\mu\gamma_5\lambda_3\psi|N, s\rangle = \Delta u_+ - \Delta d_+ = g_A$$

$$a_8 = -s_\mu \frac{1}{m_N}\langle N, s|\bar{\psi}\gamma_\mu\gamma_5\sqrt{3}\lambda_8\psi|N, s\rangle = \Delta u_+ + \Delta d_+ - 2\Delta s_+$$

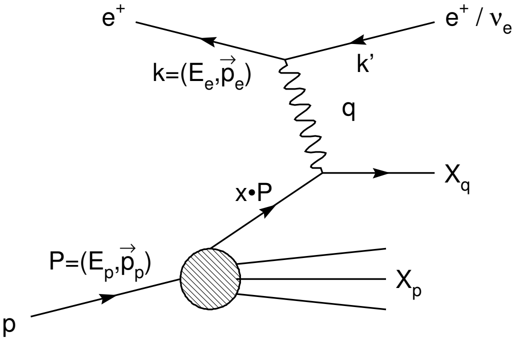
$$a_0(Q^2) = -s_\mu \frac{1}{m_N}\langle N, s|\bar{\psi}\gamma_\mu\gamma_5\mathbb{1}\psi|N, s\rangle = \Delta u_+ + \Delta d_+ + \Delta s_+ = \Delta\Sigma(Q^2)\,.$$

$\psi = (u, d, s)^t$, λ_j are Gell-Mann flavour matrices.

$a_3 = g_A$ known from neutron β decay, assuming isospin symmetry (and the standard model).

a_8 usually estimated from hyperon β decay, assuming $SU(3)_F$ symmetry.

Deep inelastic scattering (DIS)



x: longitudinal momentum fraction of proton, carried by scattered parton.
$Q^2 = -q^2$: "virtuality" of the exchanged γ, Z, $W^{\pm}$.

Extraction of the Δq's from experiment

DIS gives spin structure functions of proton and neutron $g_1^{p,n}(x, Q^2)$.

First moment related to a_i's via OPE (leading twist):

$$\Gamma_1^{p,n}(Q^2) = \int_0^1 \mathrm{d}x\, g_1^{p,n}(x, Q^2) \quad = \quad \frac{1}{36}\left[(a_8 \pm 3a_3)C_{NS} + 4a_0 C_S\right]$$

Use models to extrapolate g_1 from experimental x_{min} to $x = 0$!
$C_{S/NS} = C_{S/NS}(\alpha_s(Q^2))$.

Combinations of a_i give Δq's, e.g., $\Delta s_+(Q^2) = \frac{1}{3}[a_0(Q^2) - a_8]$

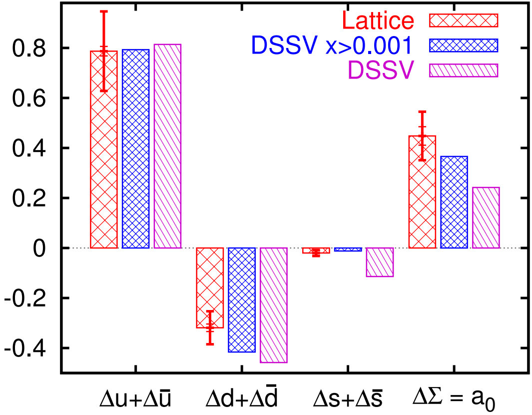
SIDIS in principle enables the direct determination of the $\Delta q(x)$, $\Delta\bar{q}(x)$ PDFs but requires fragmentation functions.
[COMPASS, arXiv 1001.4654]

	Naive Extrap.	combined with DSSV
$(\Delta s + \Delta\bar{s})(5\,\mathrm{GeV}^2)$	$-0.02 \pm 0.02 \pm 0.02$	$-0.10 \pm 0.02 \pm 0.02$

DSSV: [de Florian et al, arXiv:0904.3821]

Our 2011 paper

$N_f = 2$, no continuum limit, $M_\pi \approx 290$ MeV $\Rightarrow$ add 20 % systematic error.

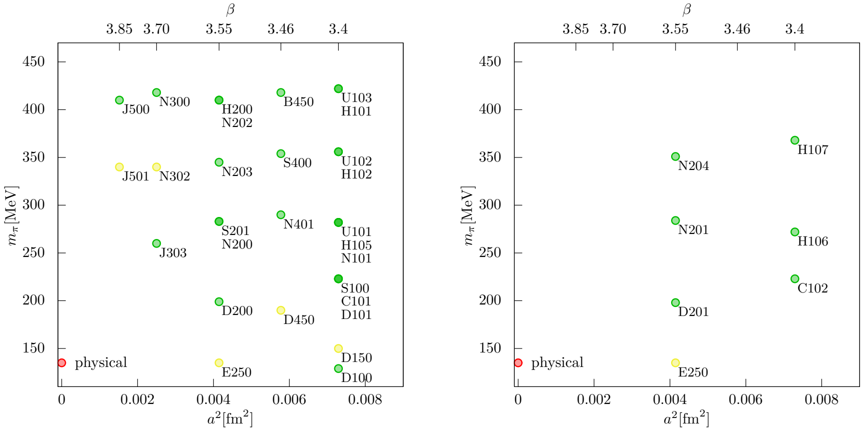


[QCDSF: GB et al, 1112.3354]

Result in the $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 7.4$ GeV^2:

$$\Delta\Sigma = \Delta u_+ + \Delta d_+ + \Delta s_+ = 0.45(4)(9)\,, \quad \Delta s_+ = -0.020(10)(4)$$

$N_f = 2 + 1$ CLS ensembles



E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,

S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$

$\rightarrow$ more detail in yesterday's talk by Daniel Mohler.

Renormalization

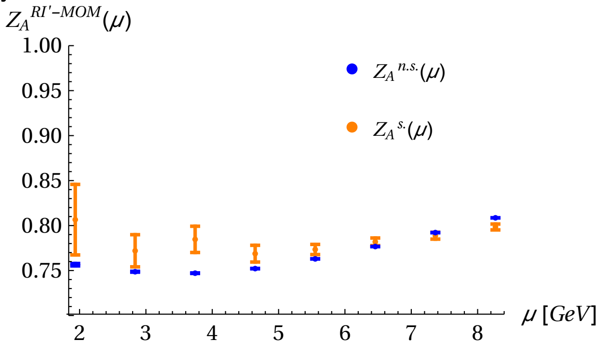
$$\Delta s_+ = \frac{1}{3}\left(a_0 - a_8\right) ,$$
$$\Delta u_+ = \frac{1}{6}\left(2a_0 + 3a_3 + a_8\right) ,$$
$$\Delta d_+ = \frac{1}{6}\left(2a_0 - 3a_3 + a_8\right) .$$

a_3 and a_8 renormalize with Z_A^{ns}, $a_0(Q^2)$ renormalizes with $Z_A^s(Q^2)$.
We can also write ($q \in \{u, d, s\}$):

$$\Delta q_+^{\overline{\mathrm{MS}}}(Q^2) = Z_A^{ns}\left(\Delta q_+^{\mathrm{lat}} - \frac{1}{3}a_0^{\mathrm{lat}}\right) + Z_A^s(Q^2)\frac{a_0^{\mathrm{lat}}}{3}$$
$$= Z_A^{ns}\Delta q_+^{\mathrm{lat}} + \frac{z(Q^2)}{3}\left(\Delta u_+^{\mathrm{lat}} + \Delta d_+^{\mathrm{lat}} + \Delta s_+^{\mathrm{lat}}\right) ,$$

where $z(Q^2) = Z_A^s(Q^2) - Z_A^{ns}$.
Next step: Order a improvement → talk by Piotr Korcyl.

Carried out with the Roma-Southampton method non-perturbatively to the RI'MOM scheme on $m_\ell = m_s$ ensembles and from MOM to $\overline{\mathrm{MS}}$ perturbatively.



$z(2\,\mathrm{GeV}) = Z_A^s(2\,\mathrm{GeV}) - Z_A^{ns} = 0.045(10)$, $Z_A^{ns} \approx 0.743$ (preliminary)
$z = 0.0065 + \mathcal{O}(\alpha^3)$ [M Constantinou et al, 1610.06744] ($c_{sw} = 1$)
$Z_A^{ns} = 0.722(11)$ (Schrödinger functional [J Bulava et al, 1604.05827])
$Z_A^{ns} = 0.728(4), 0.732(3)$ (X-space, [P Korcyl, 1705.06119])

Algorithmic choices

Quark line connected diagrams: sequential propagator.
Disconnected diagrams: stochastic quark loop.

- Dilution [S Bernardson et al, CPC 78 (93) 256; J Viehoff et al, NPPS 63 (98) 269; W Wilcox, arXiv:hep-lat/9911013]
  Partition into 4 sets, seeding only every 4th time slice.
- Hopping parameter expansion [C Thron et al, PRD 57 (98) 1642; C Michael et al, NPPS 83 (00) 185; GB et al, PRD 71 (05) 114513]:

$$M^{-1} = 2\kappa\left(\mathbb{1} - \kappa D\right)^{-1} = 2\kappa\sum_j(\kappa D)^j$$

$$= 2\kappa\sum_{j=0}^{n-1}(\kappa D)^j + (\kappa D)^n M^{-1}$$

  We use $n = 4$ as $\mathrm{Tr}(\gamma_\mu\gamma_5 D^j M^{-1}) = 0$ for $j < 4$.
  (more efficient than the clover HPE suggested in [V Gülpers, G von Hippel, H Wittig, 1309.2104])
- Truncated solver method [S Collins et al, 0709.3217; 0910.3970]

Truncated solver method

random noise vectors: $|\eta^\ell\rangle$, $\ell = 1, \ldots, n$, where

$$\frac{1}{n}\sum_\ell |\eta^\ell\rangle\langle\eta^\ell| = \overline{|\eta\rangle\langle\eta|}_n = \overline{|\eta\rangle\langle\eta|} = \mathbb{1} + \mathcal{O}(1/\sqrt{n}), \quad \overline{\langle\eta|} = \mathcal{O}(1/\sqrt{n}).$$

Here: $\eta^\ell(x)^\alpha_a \in \mathbb{Z}_2 \otimes i\,\mathbb{Z}_2/\sqrt{2}$ [S Dong, K-F Liu, PLB 328 (94) 130].
By solving $M|s^\ell\rangle = |\eta^\ell\rangle$ for the $|s^\ell\rangle$ one can construct an unbiased estimate:

$$M_E^{-1} = \overline{|s\rangle\langle\eta|} = M^{-1} + M^{-1}\underbrace{\left(\overline{|\eta\rangle\langle\eta|} - \mathbb{1}\right)}_{\mathcal{O}(1/\sqrt{n})}$$

TSM: approximate solutions $|s^\ell_{m_t}\rangle$ after m_t solver iterations (Here $m_t = 5$).

$$M_E^{-1} = \overline{|s_{m_t}\rangle\langle\eta|}_{n_{tr}} + \overline{(|s\rangle - |s_{m_t}\rangle)\langle\eta|}_{n_{ex}} \quad \text{with} \quad n_{ex} \ll n_{tr}\,.$$

n_{tr}/n_{ex} can be optimized to minimize the cost for a given error.

NB: Application of TSM to volume (rather than stochastic) averaging is discussed in [T Blum et al, 1208.4349].

Optimizing the TSM parameters

"Problem": multigrid solver set-up already does most of the job: small iteration counts $\rightarrow$ no computer time gain anymore for connected diagrams at $M_\pi > 200\,\text{MeV}$, due to the cost of contractions.

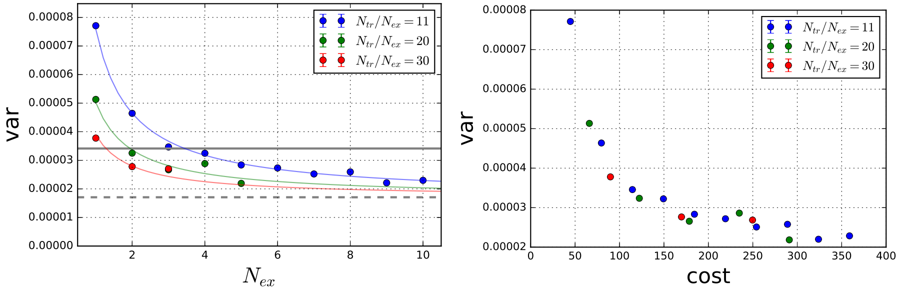
With modern solver still $\exists$ gain for disconnected loops.

Error on n_{conf} configurations for fixed n_{tr}/n_{ex}:

$$\sigma^2 = \sigma_0^2 + \sigma_{\text{stoch}}^2\,, \quad \sigma_0^2 \propto \frac{1}{n_{\text{conf}}}\,, \quad \sigma_{\text{stoch}}^2 \propto \frac{1}{n_{\text{conf}}\, n_{ex}}$$

Target: $\sigma^2 \lesssim 2\sigma_0^2$. Otherwise it is more cost-effective to increase the number of source positions (or gauge configurations).

Optimizing the TSM parameters II



We used $n_{tr}/n_{ex} = 11$, $n_{ex} = 10$ for U103 and $n_{ex} = 18$ for the other $L/a \leq 48$ ensembles at $a \approx 0.086\,\text{fm}$.

This is already overkill for Δq but cheap.

Fit function

$$C_{2\mathrm{pt}}(t_{\mathrm{sink}}) = e^{-m_N t_{\mathrm{sink}}}\left[A_0 + A_1 e^{-\Delta m_N t_{\mathrm{sink}}}\right] + \cdots$$

$$C_{3\mathrm{pt}}(t_{\mathrm{sink}}, t) = A_0 e^{-m_N t_{\mathrm{sink}}}\left\{B_0 + B_{01}\left[e^{-\Delta m_N (t_{\mathrm{sink}}-t)} + e^{-\Delta m_N t}\right] + B_1 e^{-\Delta m_N t_{\mathrm{sink}}}\right\} + \cdots ,$$

$B_0 = \langle N|J|N\rangle$, $B_{01} \propto \langle N'|J|N\rangle$, $B_1 \propto \langle N'|J|N'\rangle$, $\Delta m_N = m_{N'} - m_N$.

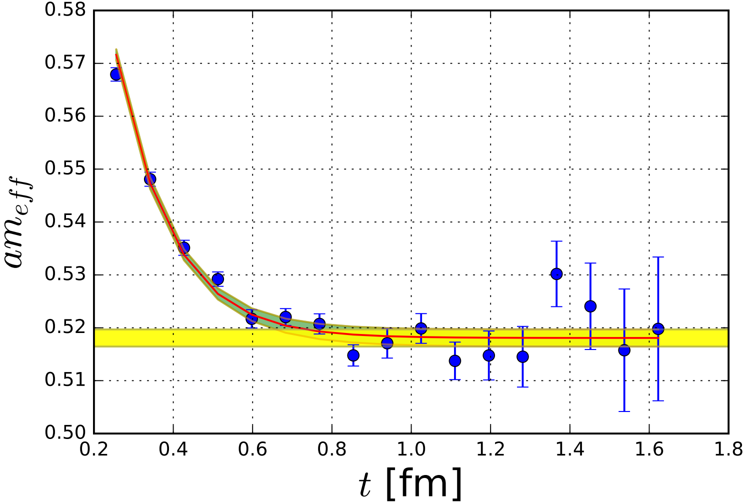
Fit $C_{2\mathrm{pt}}$ and $C_{3\mathrm{pt}}$ simultaneously for all t_{sink}, t with $t \in [\Delta t, t_{\mathrm{sink}} - \Delta t]$, varying Δt. Note that

$$\frac{C_{3\mathrm{pt}}(t_{\mathrm{sink}}, t)}{C_{2\mathrm{pt}}(t_{\mathrm{sink}})} = B_0 + \cdots = \frac{\langle N|J|N\rangle}{2m_N} + \cdots ,$$

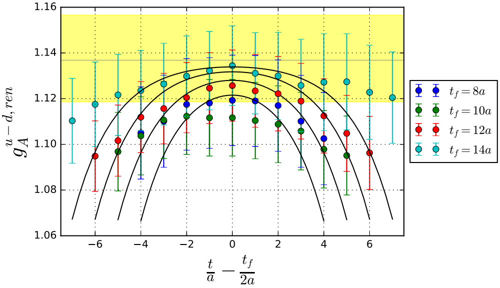
B_1 can only be identified, varying t_{sink}.

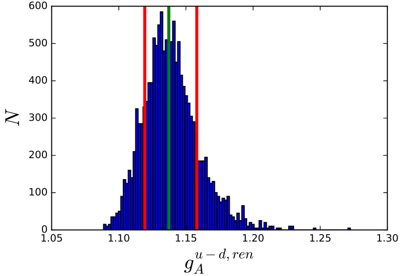
B_{01} corresponds to a transition to N' (which can be $N\pi\pi$ at large t).
$B_{01} = B_{10}$ since identical source and sink smearing.

Nucleon effective mass



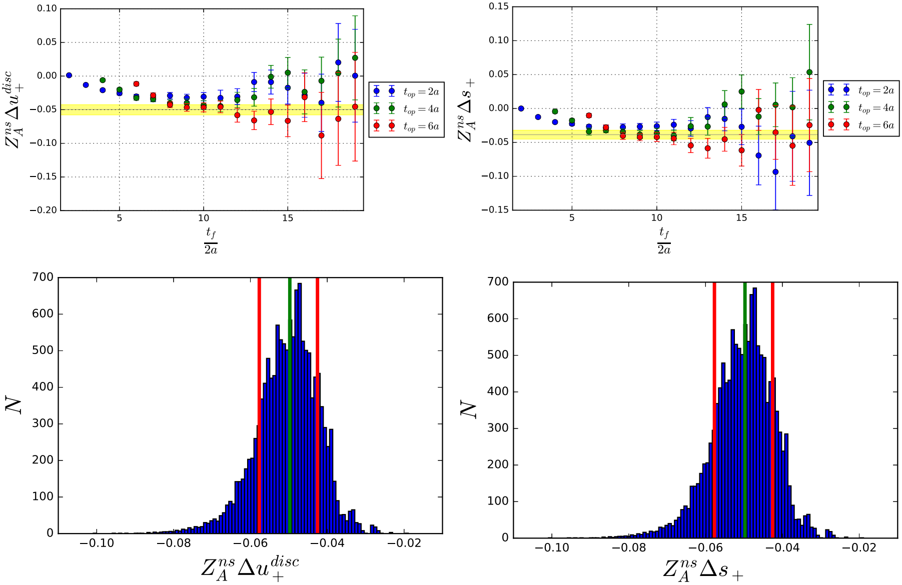
Quark line connected contributions (example: g_A)



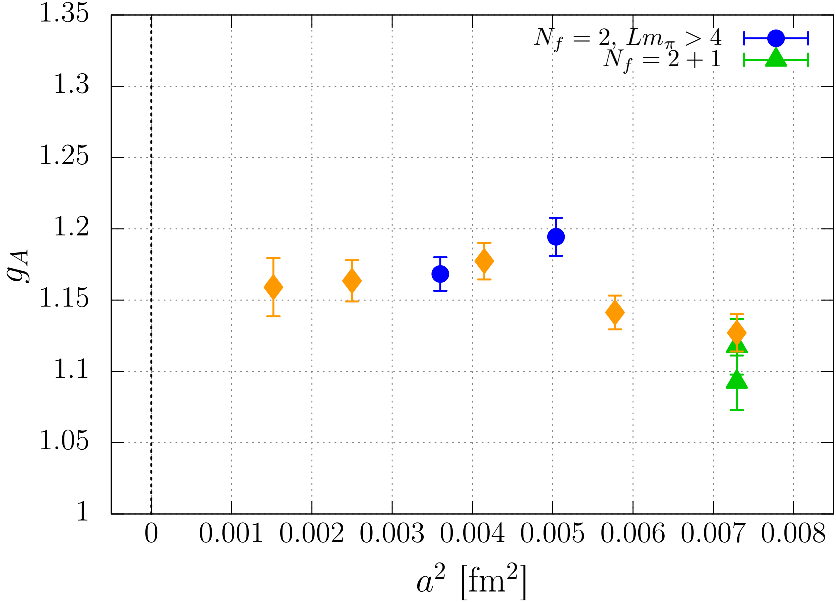


It is now considered cool to produce histograms of bootstraps of a range of valid fits a la
[GB et al, hep-lat/9308003;
S Dürr et al, 0906.3599]

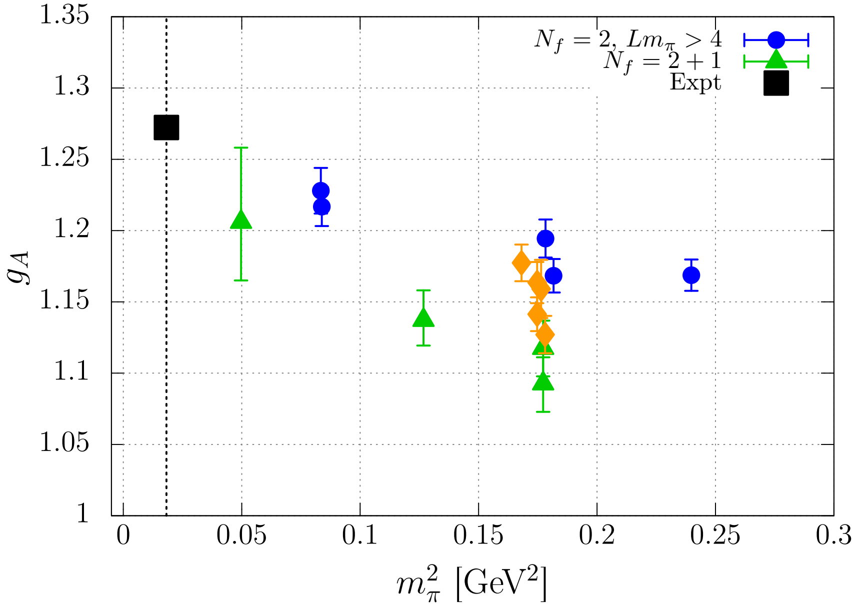
Disconnected contributions ($M_\pi \approx 350\,\mathrm{MeV}$, $a \approx 0.086\,\mathrm{fm}$)



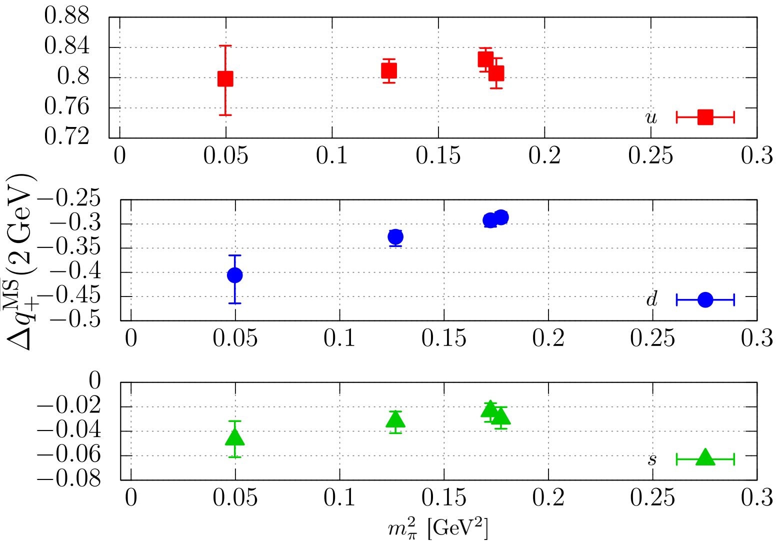
g_A: continuum limit at $M_\pi \approx 420\,\text{MeV}$



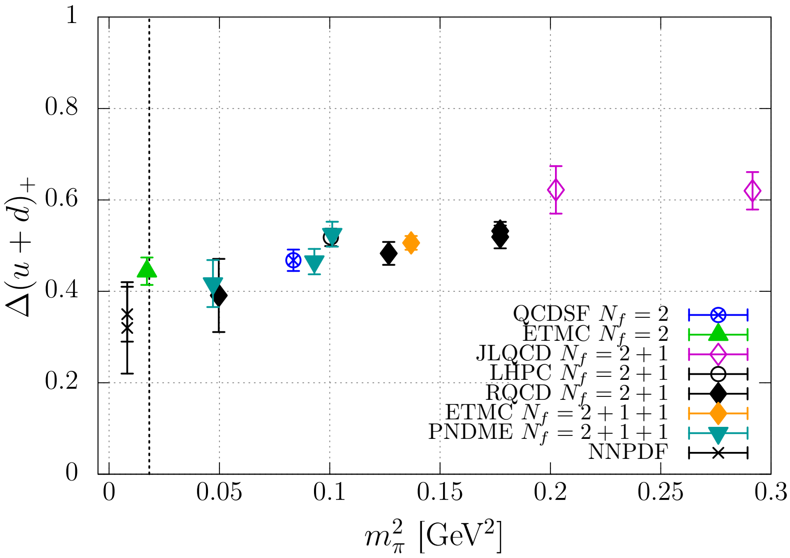
g_A: quark mass dependence



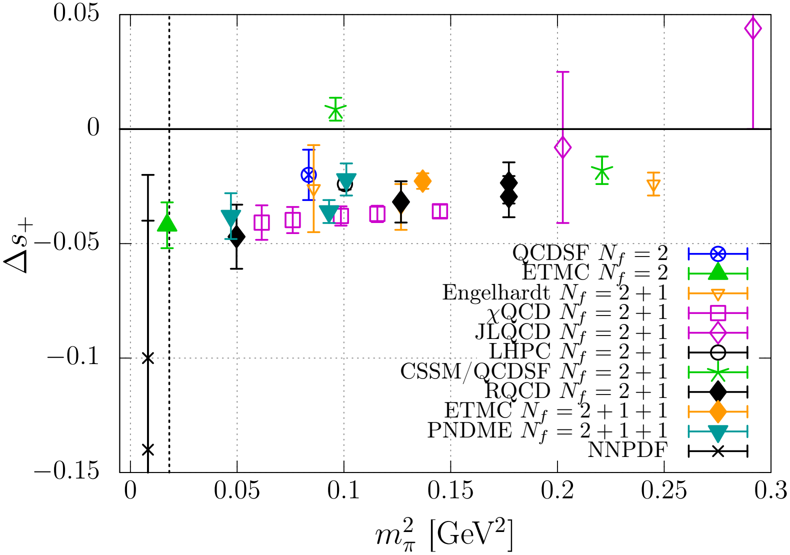
Flavour singlet combinations



Lightest point shown (C101) in progress. Everything is preliminary.



Different a, volumes etc., NNPDFpol1.1 is $x \in [0, 1]$, $x \in [0.001, 1]$.



Not all results are (non-perturbatively) renormalized (and some at $Q^2 > 4\,\text{GeV}^2$).

Summary

- The non-perturbative renormalization of flavour singlet quark bilinears is becoming a standard method.
- Δs is negative and small $\rightarrow$ will impact on polarized PDF parametrizations.
- Future plans: continuum and physical quark mass limits.
- The quark line connected contributions dominate in terms of computer time $\rightarrow$ next talk (Marius Löffler).