Fluctuations of conserved charges from imaginary chemical potential

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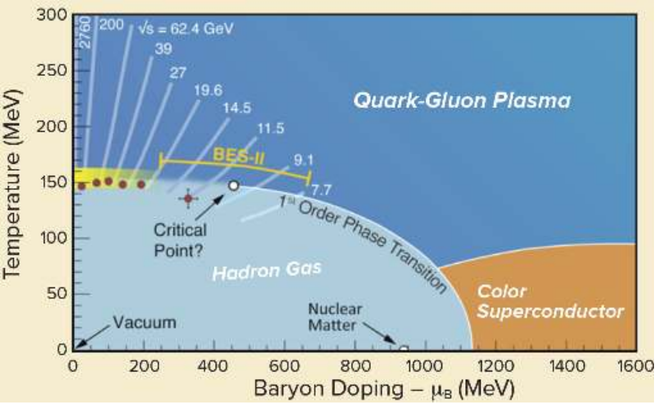
for the Wuppertal-Budapest-Collaboration

June 23rd 2017

SFB TR55

WB
collaboration

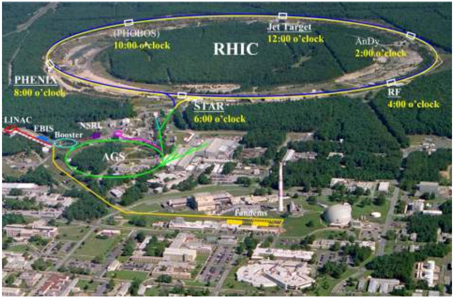
The (T, μ_B)-phase diagram of QCD



Our observables:
Last Years: T_c, Equation of state
This year: Fluctuations

Motivation

- Heavy ion colliders are used to investigate the QCD phase diagram experimentally
- Fluctuations of conserved charges are used as observables for the 2nd order critical end point



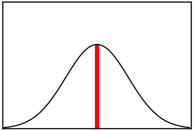
- An understanding of the relation between the freeze-out observed in experiment and the chiral transition temperature is needed

- Since heavy ion collisions happen at $\mu_B > 0$ we need theoretical values for the conserved charges at $\mu_B > 0$

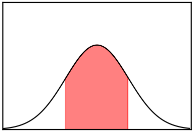
Observables

Cumulants of the net baryon number distributions:

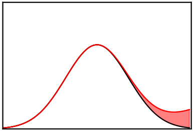
- mean M_B



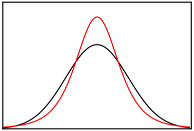
- variance σ_B^2



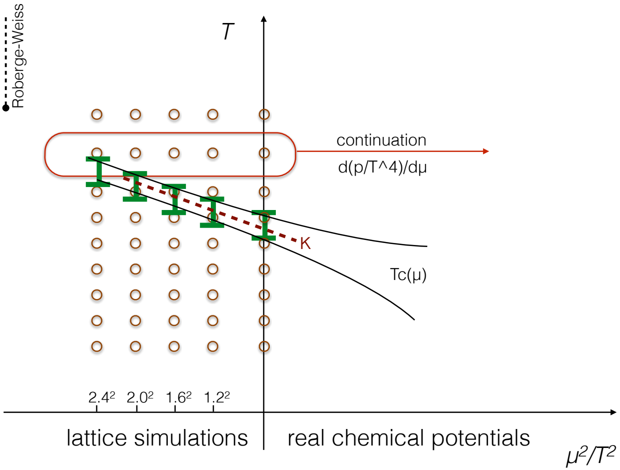
- skewness S_B: asymmetry of the distribution



- kurtosis κ_B: “tailedness” of the distribution



Analytic continuation



Common technique: [de Forcrand, Philipsen, deForcrand:2002hgr], [Bonati et al., Bonati:2015bha], [Cea et al., Cea:2015cya], [D'Elia et al., DElia:2016jqh] . . .

Overview over the Analysis

1. Do the simulations at $\mu_S = \mu_Q = 0$ and $\mu_B^2 \leq 0$

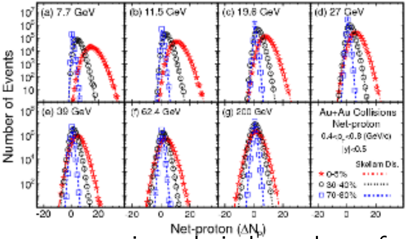
2. Determine different derivatives with respect to B, S, Q

3. Make a fit for each temperature in the μ_B direction to determine the derivatives precisely at $\mu_B = 0$

4. Calculate the correct combinations

Calculating observables

We have derivatives with respect to $\hat{\mu}_B$, $\hat{\mu}_Q$ and $\hat{\mu}_S$ of the pressure at $\mu_S = \mu_Q \mu_B = 0$. Notation:



$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k},$$

with $\hat{\mu}_i = \mu/T$.

We want ratios of the cumulants that are approximately independent of the volume at $\mu_B > 0$, $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4\langle n_B \rangle$:

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \dots$$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \dots$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \dots$$

[Bazavov et al., Bazavov:2017dus], [Karsch, Karsch:2017zzw]
plot: [STAR, Adamczyk:2013dal]

Calculating observables II

The μ_B dependence can be written in terms of the Taylor expansion:

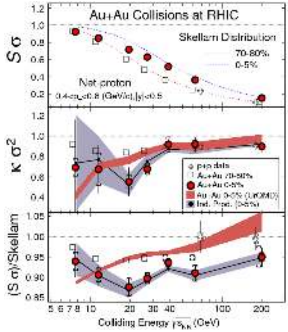
$$\chi^{BQS}_{i,j,k}(\hat{\mu}_B) = \chi^{BQS}_{i,j,k}(0) + \hat{\mu}_B \left[\chi^{BQS}_{i+1,j,k}(0) + q_1 \chi^{BQS}_{i,j+1,k}(0) + s_1 \chi^{BQS}_{i,j,k+1}(0)\right]$$
$$+ \frac{1}{2}\hat{\mu}_B^2 \left[\chi^{BQS}_{i+2,j,k}(0) + s_1^2 \chi^{BQS}_{i,j+2,k}(0) + q_1^2 \chi^{BQS}_{i,j,k+2}(0)\right.$$
$$\left. + 2q_1 s_1 \chi^{BQS}_{i,j+1,k+1}(0) + 2s_1 \chi^{BQS}_{i+1,j+1,k}(0) + 2q_1 \chi^{BQS}_{i+1,j,k+1}(0)\right] + \dots$$

with

$$q_j = \frac{1}{j!}\frac{d^j \hat{\mu}_Q}{(d\hat{\mu}_B)^j}(0) \qquad s_j = \frac{1}{j!}\frac{d^j \hat{\mu}_S}{(d\hat{\mu}_B)^j}(0)$$

From $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$ we get the conditions

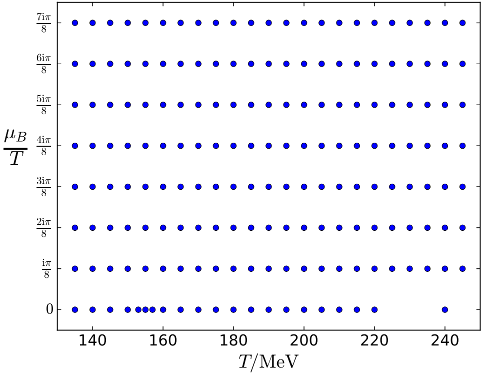
$$\chi_1^Q = 0.4 \chi_1^B\,, \qquad \chi_1^S = 0$$



[STAR, Adamczyk:2013dal]

After some calculations we arrive at formulas for $\frac{M_B}{\sigma_B^2}$, $\frac{S_B \sigma_B^3}{M_B}$ and $\kappa_B \sigma_B^2$.

Simulation details



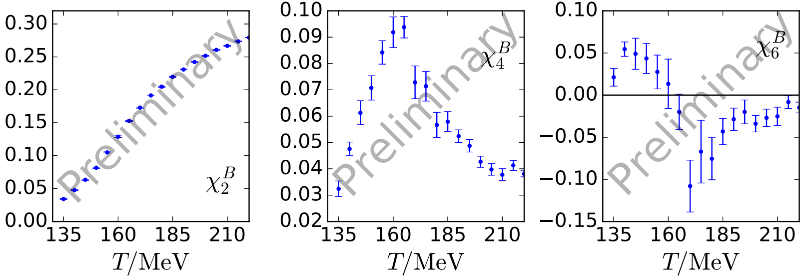
- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\mu_S = \mu_Q = 0$
- Lattice size: $48^3 \times 12$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 1, 2, 3, 4, 5, 6$ and 7

Measured observables

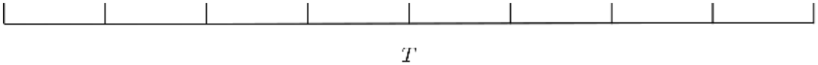
On each ensemble we measure the $\chi_{i,j,k}^{BQS}$ up to the fourth derivative:

- χ_1^B
- χ_2^B
- χ_3^B
- χ_4^B
- χ_1^Q
- $\chi_{1,1}^{BQ}$
- $\chi_{1,2}^{BQ}$
- $\chi_{1,3}^{BQ}$
- χ_1^S
- $\chi_{1,1}^{BS}$
- $\chi_{1,2}^{BS}$
- $\chi_{1,3}^{BS}$
- χ_2^Q
- $\chi_{2,1}^{BQ}$
- $\chi_{2,2}^{BQ}$
- χ_2^S
- $\chi_{2,1}^{BS}$
- $\chi_{2,2}^{BS}$
- $\chi_{1,1}^{QS}$
- $\chi_{1,1,1}^{BQS}$
- $\chi_{2,1,1}^{BQS}$
- χ_3^Q
- $\chi_{3,1}^{BQ}$
- χ_3^S
- $\chi_{3,1}^{BS}$
- $\chi_{2,1}^{QS}$
- $\chi_{1,2,1}^{BQS}$
- $\chi_{1,2}^{QS}$
- $\chi_{1,1,2}^{BQS}$
- χ_4^Q
- χ_4^S
- $\chi_{3,1}^{QS}$
- $\chi_{2,2}^{QS}$
- $\chi_{1,3}^{QS}$

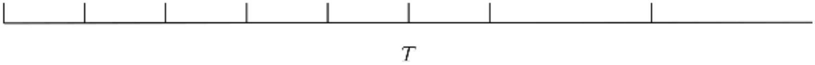
χ_2^B, χ_4^B and χ_6^B



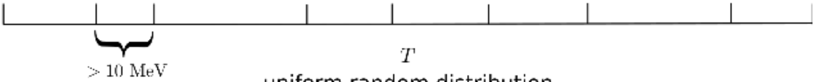
Spline node points



equidistant node points

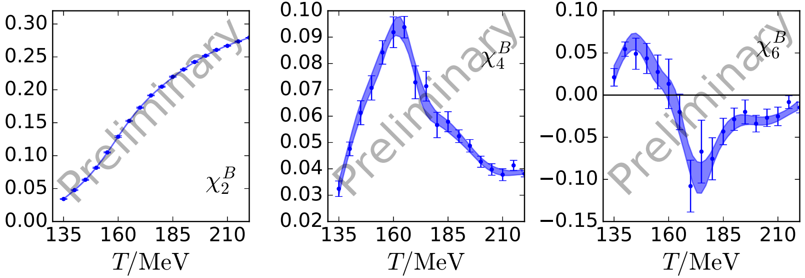


equidistant node points with gap at high temperatures



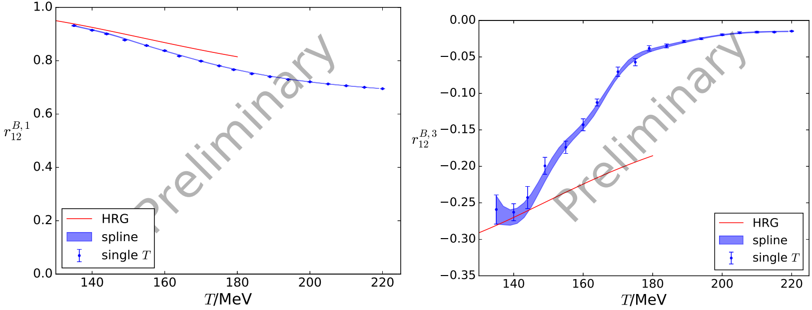
uniform random distribution

χ_2^B, χ_4^B and χ_6^B



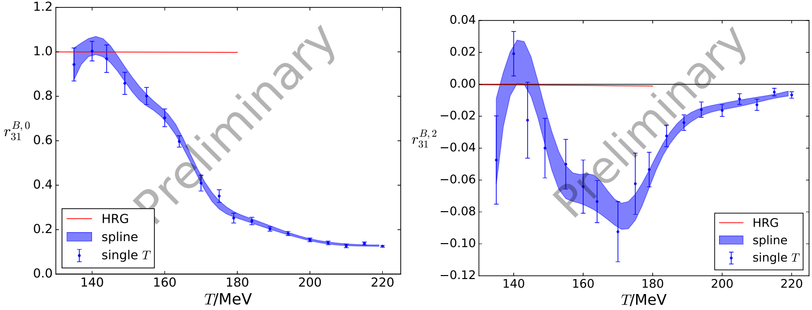
M_B/σ_B^2

$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \ldots$$



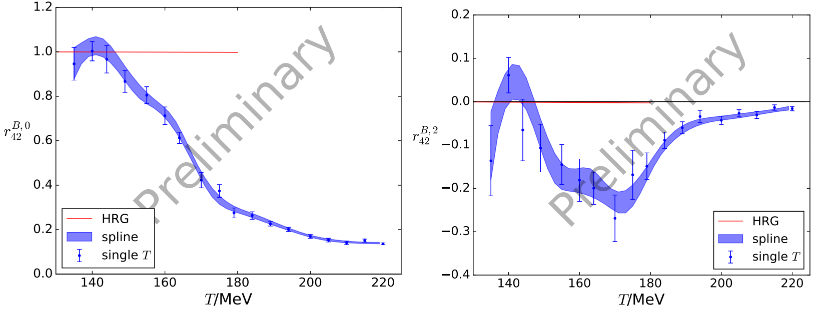
$S_B\sigma_B^3$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \dots$$

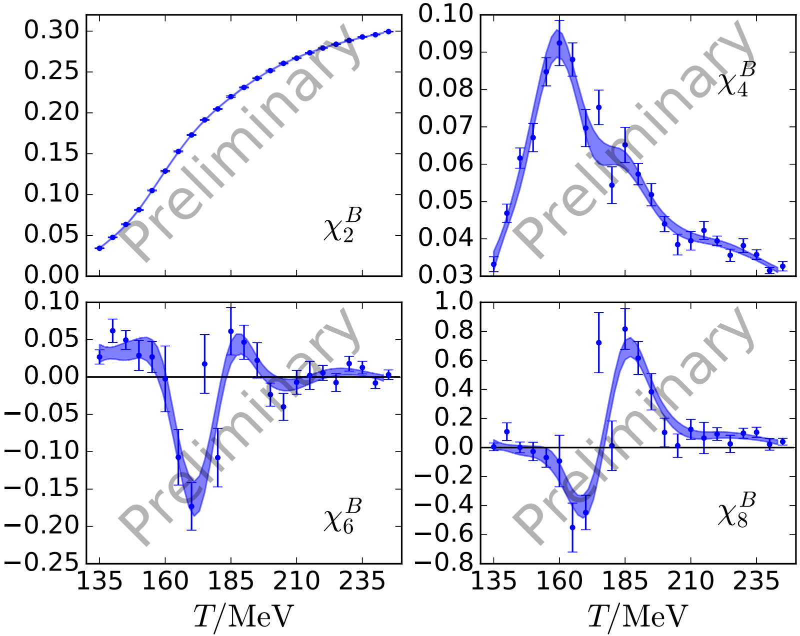


$\kappa_B \sigma_B^2$

$$\kappa_B \sigma_B^2 = \frac{\chi_4^B(T, \hat{\mu}_B)}{\chi_2^B(T, \hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \dots$$



χ_8^B



Preliminary
Preliminary
Preliminary
Preliminary
χ_2^B
χ_4^B
χ_6^B
χ_8^B
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
−0.25
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
135
160
185
210
235
T/MeV
T/MeV

Summary

The μ_B dependence can be written in terms of the Taylor expansion:

$$\begin{aligned}\chi^{BQS}_{i,j,k}(\hat{\mu}_B) = \chi^{BQS}_{i,j,k}(0) + \hat{\mu}_B \Big[\chi^{BQS}_{i+1,j,k}(0) \\ + q_1 \chi^{BQS}_{i,j+1,k}(0) + s_1 \chi^{BQS}_{i,j,k+1}(0)\Big] \\ + \frac{1}{2}\hat{\mu}_B^2 \Big[\chi^{BQS}_{i+2,j,k}(0) + s_1^2 \chi^{BQS}_{i,j+2,k}(0) + q_1^2 \chi^{BQS}_{i,j,k+2}(0) \\ + 2q_1 s_1 \chi^{BQS}_{i,j+1,k+1}(0) + 2s_1 \chi^{BQS}_{i+1,j+1,k}(0) \\ + 2q_1 \chi^{BQS}_{i+1,j,k+1}(0)\Big] + \ldots\end{aligned}$$