Updates on the Columbia plot and its extended/alternative versions

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Outline

1. **Standard QCD Columbia plot** $m_s - m_{u,d}$
   - Updates on $m_{heavy}^{Z_2}$ at $N_f = 2$ and $\mu = 0$

2. **Extended QCD Columbia plot** $\left(\frac{\mu}{T}\right)^2 - m_s - m_{u,d}$
   - Updates on $m_{light/heavy}^{tric}$ at $N_f = 2$ and $\mu = \mu_i^{RW}$

3. **Alternative QCD Columbia Plot** $m_{u,d} - N_f$
   - Updates on $m_{light}^{Z_2}(N_f)$ at $\mu = 0$

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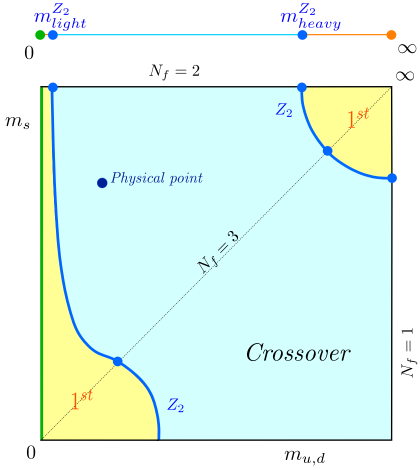
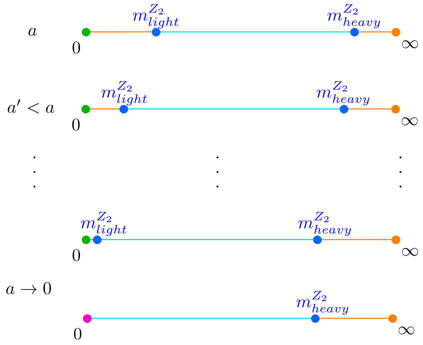
2D Columbia plot $m_s - m_{u,d}$ at $\mu = 0$

Deconfinement first order region towards the continuum limit





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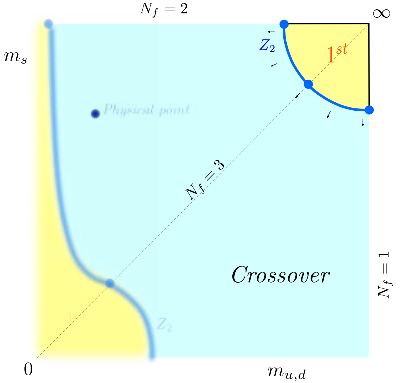
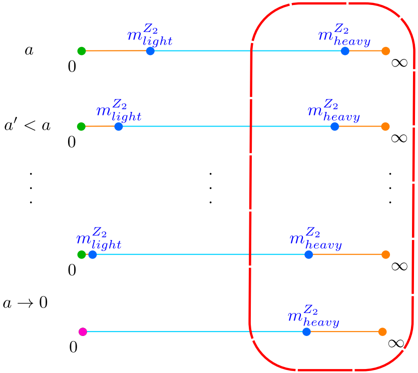




Saito et al. (2012) , doi.org/10.1103/PhysRevD.84.054502.
Fromm et al. (2012), 10.1007/JHEP01(2012)042.

m_{crit} KNOWING THE ORDER OF THE TRANSITION

- Analysis of the order parameter $\mathcal{O} = L_{\mathrm{Im}},\ \|L\|,\ \bar{\psi}\psi$ distribution and of its moments

$$B_n(\langle\mathcal{O}\rangle, \beta, m, \mu, V) \equiv B_n(\beta; m, \mu, N_\sigma) = \frac{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\rangle^{n/2}}$$

- $\forall m, \mu$ identify β_c by $\begin{cases} \text{the condition } B_3(\beta; m, \mu, N_\sigma) = 0\ \forall N_\sigma \\ \text{the crossing of } B_4(\beta; m, \mu, N_\sigma) vs \beta \text{ for different } N_\sigma \end{cases}$

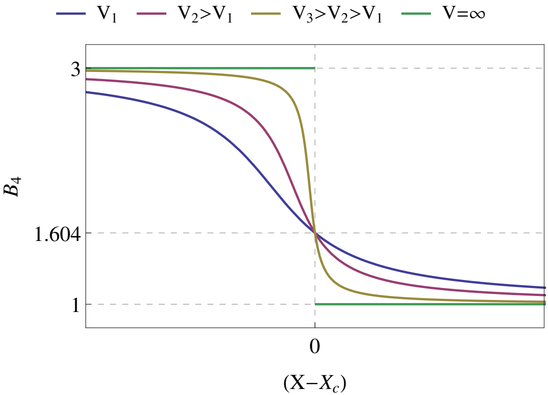
- $B_4(\beta_c, X)$, along with the critical exponent $\nu(\beta_c, X)$, depends on the order of the transition at that given X, with $X = \beta, m, \mu$

$$B_4(\beta_c, X, N_\sigma) = \left.\frac{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^4\rangle}{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\rangle^2}\right|_{\beta_c} \underset{N_\sigma\to\infty}{\sim} \begin{cases} 1.604, & 2^{nd}\text{order} Z_2 \\ 2, & \text{tricritical} \end{cases}$$

$$\nu(X, N_\sigma) \underset{N_\sigma\to\infty}{\sim} \begin{cases} 0.6301, & 2^{nd}\text{order} Z_2 \\ 0.5, & \text{tricritical} \end{cases}$$

Finite Size Scaling (FSS) analysis

Critical mass out of a multi-branch fit of the kurtosis at the transition



$$B_4(X, N_\sigma) = B_4(X_c, \infty) + c_1\, x + c_2\, x^3 + \mathcal{O}(x^4)$$

$$x \equiv (X - X_c) N_\sigma^{1/\nu}, \qquad X = \beta,\ m,\ \mu$$

Better method: quantitative collapse of the rescaled kurtosis around X_c

Features of the heavy mass region

- UV cutoff effects: $m_\pi \gtrsim \mathcal{O}(1)$ GeV with $am_\pi > 1$
  - location of Z_2 boundary not trustworthy in m_π at current lattice spacings
- Finite size effects
  - While $a \to 0$ we need $N_\sigma \to \infty$ to keep the size of the box fixed
  - Current strategy: locating the Z_2 boundary via FSS analysis
    - at least 3 different and large enough N_σ(per m)

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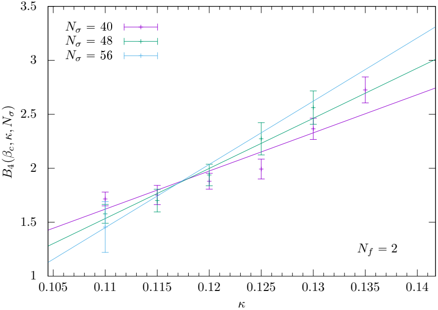
Devised alternative (cheaper) strategy: locate the Z_2 boundary in two steps

1. identify at some m the minimal physical volume V_{min}
   - reliable extraction of m_c out of a linear fit of the kurtosis
2. Locate the Z_2 boundary at a different m, N_τ
   - e.g. reweighting the effective potential V_{eff} at one fixed $V \gtrsim V_{min}$
     Saito et al. (2012), 10.1103/PhysRevD.84.054502

Minimal physical size of the box

$$B_4(\kappa, N_\sigma) = B_4(\kappa_c, \infty) + c\,(\kappa - \kappa_c) N_\sigma^{(1/\nu)} (1 + B N_\sigma^{y_t - y_h})$$



$$B_4(\kappa_c, \infty) = 1.604$$
$$\nu = 0.6301$$
$$y_t - y_h = -0.894$$

Jin et al. (2017), arXiv:1706.01178
Karsch et al. (2001),
10.1016/S0370-2693(01)01114-5

Smaller volumes gradually excluded from multi-branch fit till $B = 0$ within errors

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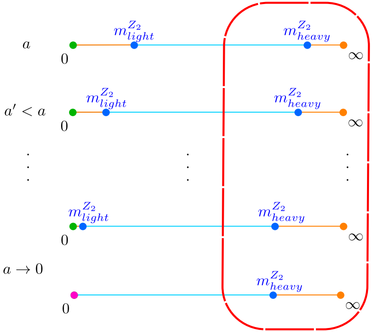

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PRELIMINARY RESULTS

N_τ	κ_c	a [fm]	am_π	m_π [MeV]	V_{min} [fm^3]	L_{min} [fm]
6	0.0890(27)	[0.118(1):0.123(1)]	[3.108:2.241]	[5198(35):3589(15)]	50	3.68
8	0.1128(29)	[0.088(1):0.092(1)]	[2.131(1):1.397(1)]	[4768(22):2995(33)]	45	3.56

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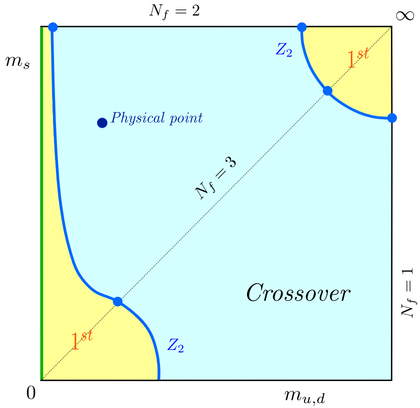
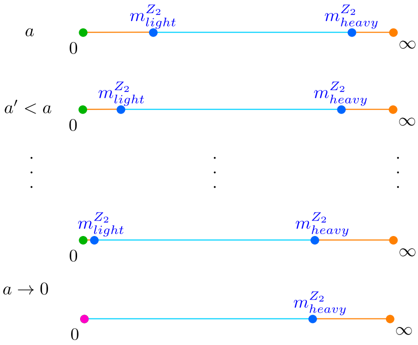
At least two possible scenarios on the nature of $N_f = 2$ chiral transition





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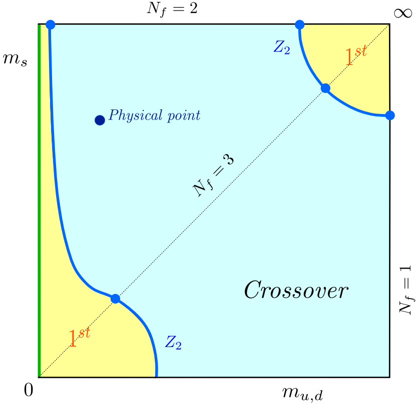
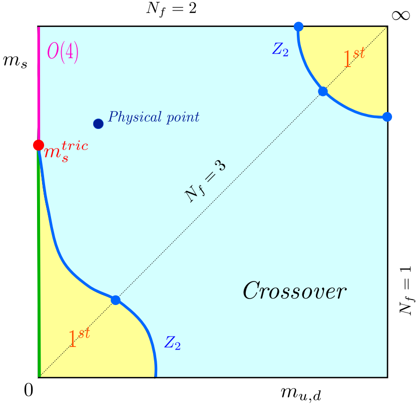
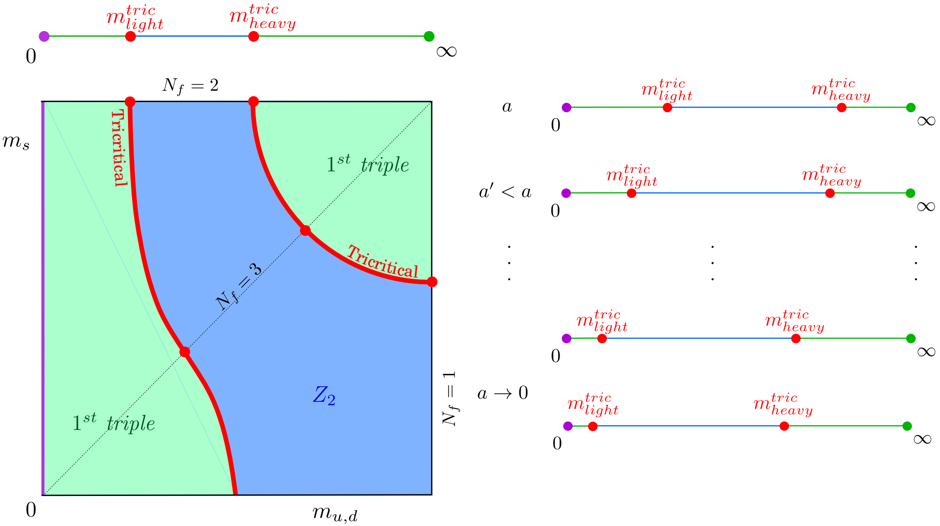




Direct (expensive) approach: $\mu = 0$ and $m_{u,d} \to 0$

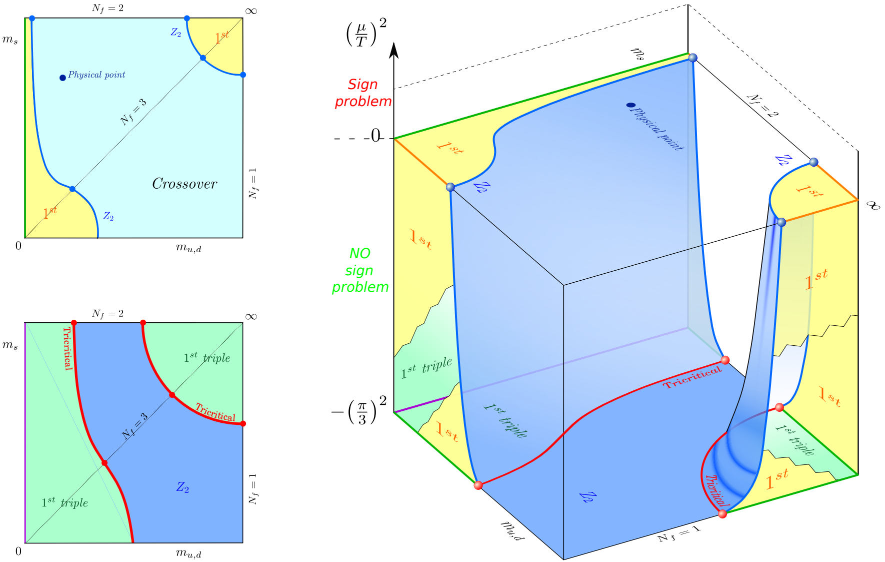
Svetitsky, Yaffe (1982), 10.1016/0550-3213(82)90172-9
Pisarski, Wilczek (1984), 10.1103/PhysRevD.29.338

2D Columbia plot $m_s - m_{u,d}$ at $\mu = \frac{i\pi T}{3}$

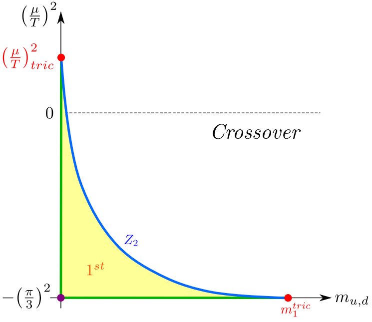


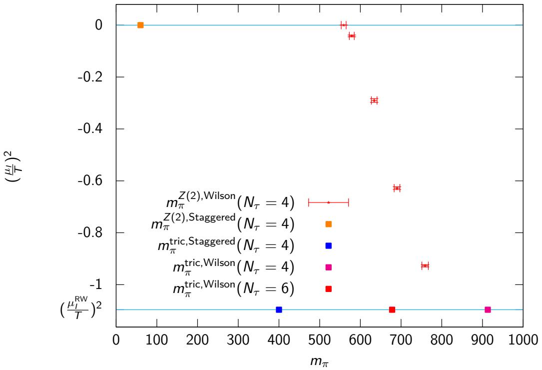
Indirect (cheaper) approach: $\mu = i\mu_i$, bigger $m_{u,d}$ and scaling laws

3D Columbia plot $\left(\frac{\mu}{T}\right)^2 - m_s - m_{u,d}$



FROM μ_i^{RW} TO $\mu = 0$ ALONG THE Z_2 LINE

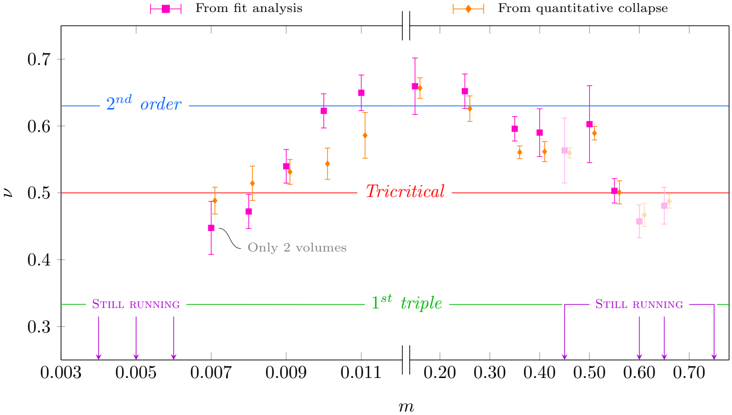




- Only qualitative agreement between different discretizations (cf. staggered Bonati et al. (2014), 10.1103/PhysRevD.90.074030)
- Unimproved Wilson: no need for extrapolation using tricritical scaling law
- Expected shift of $\mu_i^{Z_2}(m_{u,d})$ to smaller $m_{u,d}$ as $a \to 0$ (cf. F.C. et al. (2016), 10.1103/PhysRevD.93.054507)

Roberge-Weiss transition in $N_f = 2$ QCD

(Unimproved) staggered fermions on $N_\tau = 6$

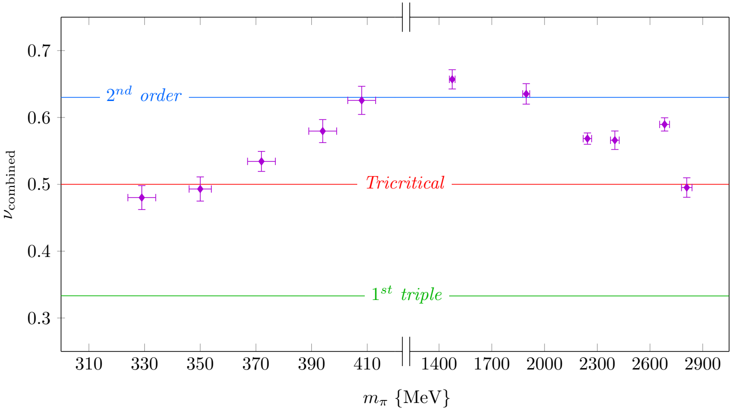


Two sets of data: quantitative collapse of the rescaled kurtosis as alternative to fit

(details in Sciarra (2017), github.com/AxelKrypton/PhD_Thesis)

Smallest/Highest masses still running

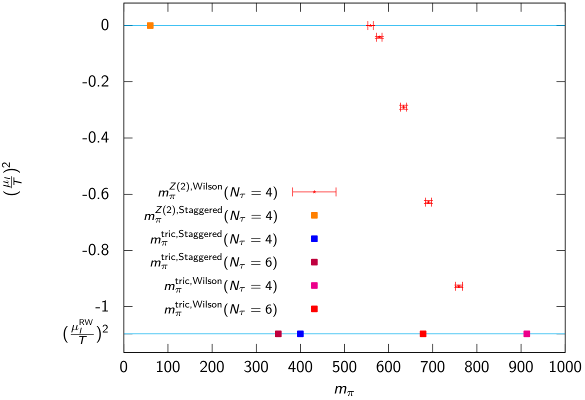
Roberge-Weiss transition in $N_f = 2$ QCD



0.12 fm $\lesssim a \lesssim 0.15$ fm $\qquad m_{\pi\ light}^{tric} = 350(20)$ MeV

PRELIMINARY RESULTS

Different discretizations compared



- on $N_\tau = 4$ and $a \sim 0.3$ fm (Bonati et al. (2011), 10.1103/PhysRevD.83.054505) it was found $m_{\pi\ light}^{tric} \sim 400$ MeV. Shift of 14% toward smaller masses for $N_\tau = 6$
- the analogous shift going from $N_\tau = 4$ to $N_\tau = 6$ with unimproved Wilson discretization was of about 35%

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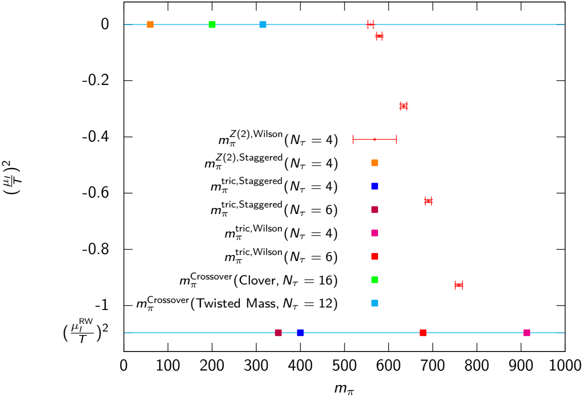

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- Wilson $\mathcal{O}(a)$ improved results from Burger et al. (2013), 10.1103/PhysRevD.87.074508 (Twisted Mass) and Brandt et al. (2014), arXiv:1310.8326 (clover)

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Continuous N_f in the path integral

- Partition function describing N_f flavors of degenerate mass m

$$Z_{N_f}(m, \mu, N_f) = \int \mathcal{D}A \left[\det M(A, m, \mu)\right]^{N_f} e^{-S_G}$$

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- Phase diagram in the N_f *vs* m plane
  - $m_{Z_2}(N_f)$ line constrained by tricritical scaling in the chiral limit

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- ∞ many interpolations such that $\lim_{N_f\to 2,3} \mathcal{Z}(N_f = 2.\#) = \mathcal{Z}(N_f = 2,3)$

$$\begin{aligned} Z_{N_f=2.\#} &= \int \mathcal{D}U\,[\det M(U,m,\mu)]^{2.\#}\, e^{-S_G} \\ Z_{N_f=2+1} &= \int \mathcal{D}U\,[\det M(U,m_1,\mu)]^{2}\,[\det M(U,m_2,\mu)]\, e^{-S_G} \end{aligned}$$

the relative position of N_f^{tric} w.r.t. $N_f = 2$ has to be uniquely determined by every interpolation

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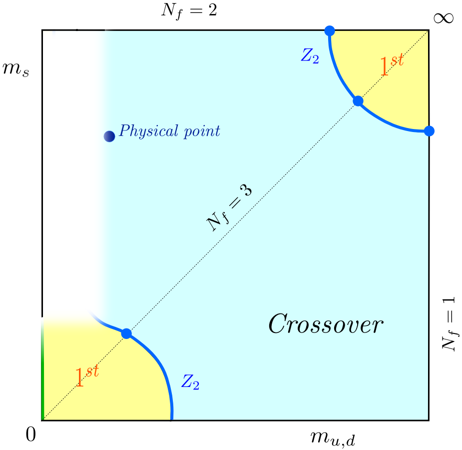
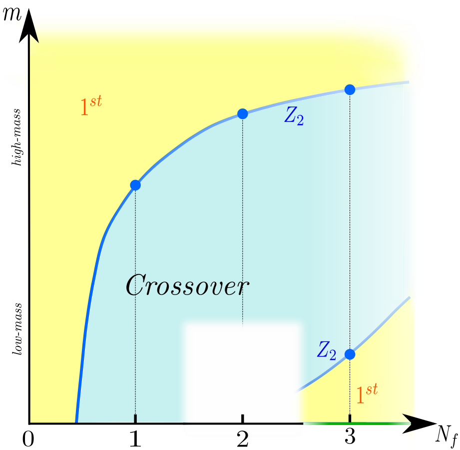
  the relative position of N_f^{tric} w.r.t. $N_f = 2$ has to be uniquely determined by every interpolation
- at non-integer N_f our theory lacks locality and does NOT correspond to a well defined QFT in the CL

Further insights into the phase diagram

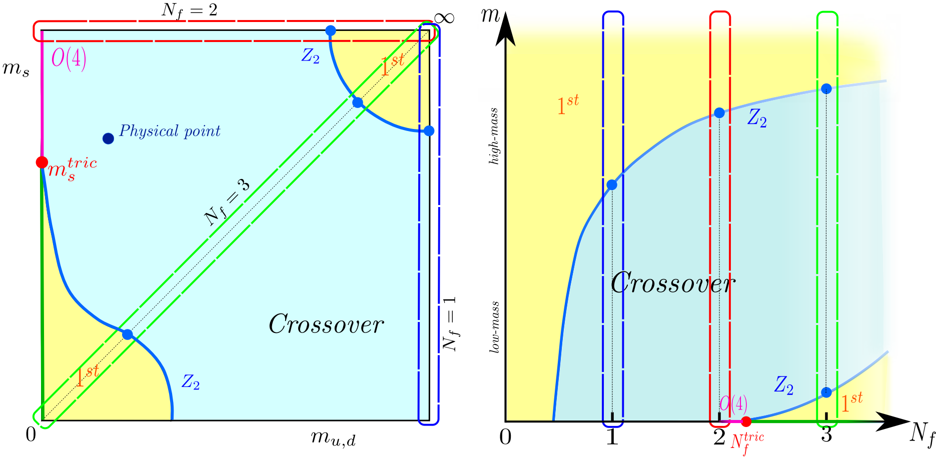
We still do not know the order of chiral transition for $N_f = 2$





Further insights into the phase diagram

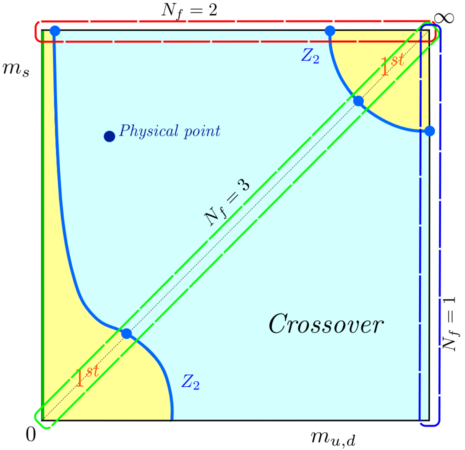
$m_{Z_2}(N_f)$ according to the second order scenario

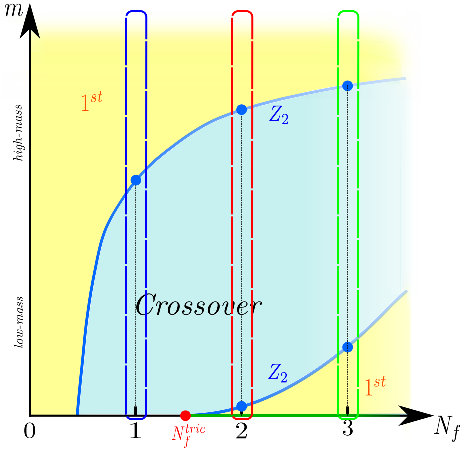


$$N_f^{tric} > 2$$

Further insights into the phase diagram

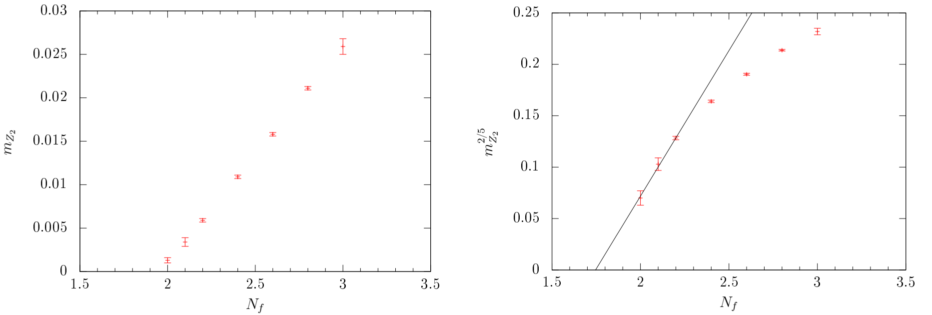
$m_{Z_2}(N_f)$ according to the first order scenario





$$N_f^{tric} < 2$$

Tricritical scaling region in $m_{Z_2}(N_f)$



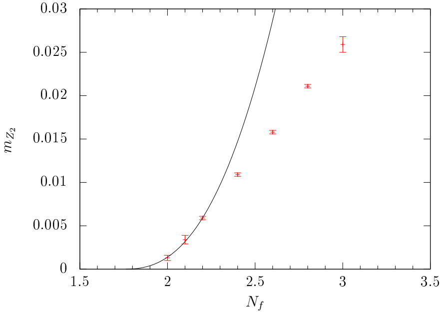
Bonati et al. (2014), 10.1103/PhysRevD.90.074030,

de Forcrand and Philipsen (2003), 10.1016/j.nuclphysb.2003.09.005

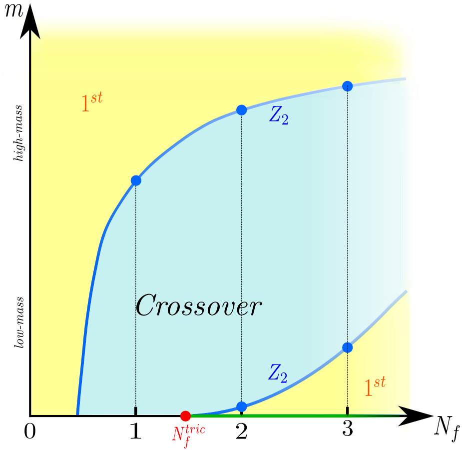
$$m_{Z_2}^{2/5}(N_f) = C\left(N_f - N_f^{tric}\right) \qquad \chi^2_{ndf=1} = 0.28$$

Tricritical scaling region in $m_{Z_2}(N_f)$

$N_f^{tric} = 1.75(32)$



PRELIMINARY RESULTS



- Width of the scaling window in m same as found in the extrapolation from μ_i
- Cheapest extrapolation while changing N_τ?

THANK YOU!

(...and A. Sciarra for plots from his PhD thesis)