Continuum extrapolation of charmonium correlators at non-zero temperature

Hauke Sandmeyer

in collaboration with
H.-T. Ding[1], O. Kaczmarek[1,2], A-L. Kruse[2], H. Ohno[3], M.Laine[4]

[1]Central China Normal University, [2]Bielefeld University, [3]Brookhaven National Laboratory/Tsukuba University, [4]University of Bern

Universität Bielefeld
Bielefeld University

19.06.2017

Motivation

Goal: Determination of spectral and transport properties of the Quark Gluon Plasma to understand:

- In-medium modification of charmonium and bottomonium states
- Heavy quark diffusion coefficients and their mass and temperature dependence

$\Rightarrow$ Based on continuum extrapolated correlation functions

For light quarks see

H-T.Ding, F.Meyer, O. Kaczmarek, PRD94 (2016) 034504,
J. Ghiglieri, M. Laine, F.Meyer, O. Kaczmarek, PRD94 (2016) 016005

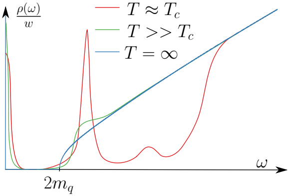
Properties related to charmonium correlators

A lot of information encoded in the spectral function $\rho(\omega)$.
Heavy quark diffusion coefficient D and bound state properties

$$D = \frac{\pi}{3\chi_q} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}(\omega, T)}{\omega}$$

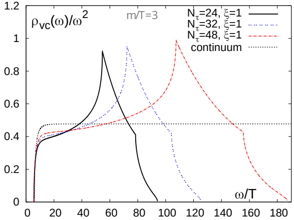
In general: ρ needs to be extracted from correlator

$$G(\tau) = \int_0^\infty \rho(\omega) K(\omega, \tau) d\omega, \quad K(\omega, \tau) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$



Extraction from lattice data is an ill-posed problem

Free Lattice spectral function



[H.T. Ding et al. PRD86 (2012) 014509]

Lattice setup

β	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	#confs.
7.192	0.018 (11.19)	96	48	0.75	237
			32	1.1	476
			28	1.3	336
			24	1.5	336
			16	2.25	237
7.394	0.014 (14.24)	120	60	0.75	171
			40	1.1	141
			30	1.5	247
			20	2.25	226
7.544	0.012 (17.01)	144	72	0.75	221
			48	1.1	462
			42	1.3	660
			36	1.5	288
			24	2.25	237
7.793	0.009 (22.78)	192	96	0.75	224
			64	1.1	291
			56	1.3	291
			48	1.5	348
			32	2.25	235

Sacale setting from r_0 scale [A. Francis, O. Kaczmarek et al. PRD91(2015)096002]

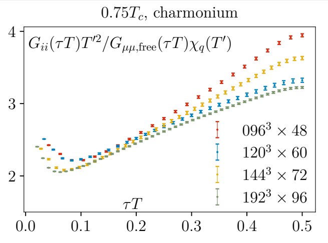
- **Quenched** gauge field configurations
- Wilson gauge action
- **500 sweeps** of 1 heatbath and 4 overrelaxation steps between each configuration
- 4 to 5 different temperatures
- fixed aspect ratios → **same phys. volume** $(8.43\mathrm{GeV}^{-1})^3$
- 4-5 sources on finest lattice

Computing resources

Bielefeld GPU Cluster, Paderborn PC^2(OCuLUS),
RWTH Compute Cluster, JUQUEEN - Jülich Blue Gene/Q

Overview of the method

Lattice correlators



↓

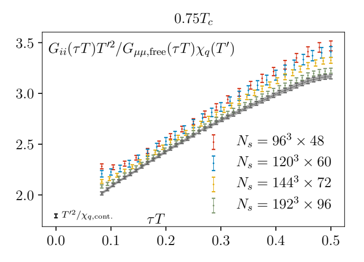
Interpolate correlators to same quark mass

→

Interpolate correlators between lattice points

↓

Continuum extrapolation



Meson-vector correlation function

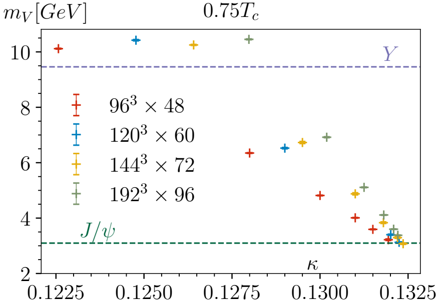
Meson vector correlation function:

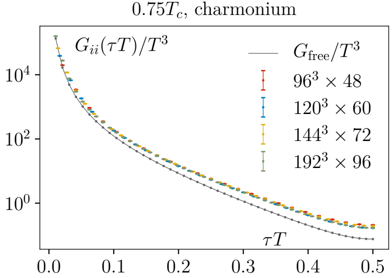
$$G_{\mu\nu} = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = \psi^\dagger(\tau,\vec{x})\gamma_\mu\psi(\tau,\vec{x})$$

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$

- Non-perturbatively improved **Clover Wilson fermions**
- 4-6 different kappa values distributed between bottom and charm quark mass





Hard to distinguish details on log scale
$\rightarrow$ normalize with G_{free}

Normalize with continuum non-interacting (free) vector correlator function

$$G_{\mu\mu,\mathrm{free}}(\tau)/T^3 = \int_{2m_q}^{\infty} \frac{\rho_{\mathrm{free}}(\omega, m_q)}{T^3} K(\omega,\tau) d\omega$$

$$\rho_{\mathrm{free}}(\omega, m_q) = \frac{3}{16\pi^2}\omega^2 \tanh\left(\frac{\omega}{4T}\right) \sqrt{1-\left(\frac{2m_q}{\omega}\right)^2} \left(4+2\left(\frac{2m_q}{\omega}\right)^2\right)$$

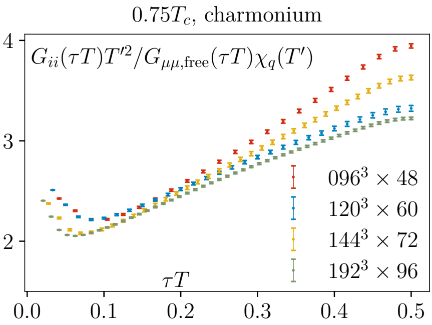
at fixed quark mass of $m_q = 1.5\mathrm{GeV}$

Use renormalization independent ratios $\rightarrow$ normalize with

$$\chi_q(T)/T^2 = G_{00}(\tau T = 0.5)/T^3 = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_{00}(\omega) K(\omega, \tau T)}{T^3},$$

$$\rho_{00} = 2\pi T^2 \omega\delta(\omega)$$

Choose $\chi_\mathbf{q}(\mathbf{T}' = \mathbf{2.25T_c})$



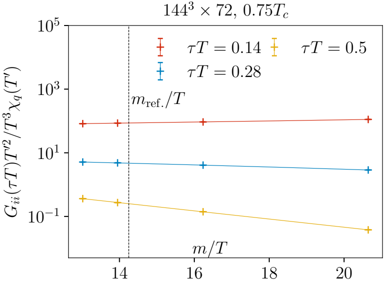
$\rightarrow$ Need interpolation of the correlators G_{ii} in the meson mass

Mass interpolation

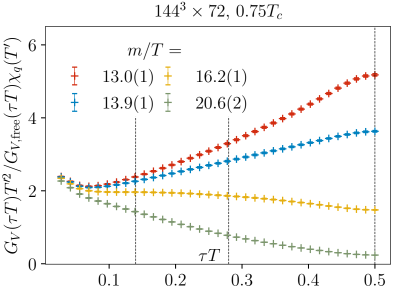
Interpolate the correlator at each τT separately using a fit with Ansatz

$$G(m/T, \tau T) = \exp(a(m/T)^2 + b(m/T) + c)$$

- Choose screening mass $\mathbf{m/T = 14.2}$ from the finest lattice as a reference
- Also at higher temperatures use $0.75\,T_C$ screening masses
- Interpolate on **bootstrap samples**



(Details of mass determination in next talk, by Hai-Tao Shu)

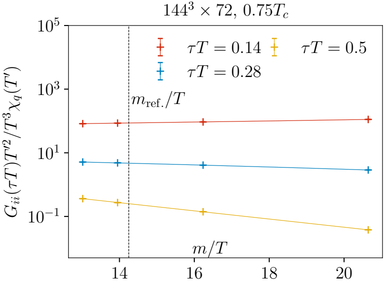


Mass interpolation

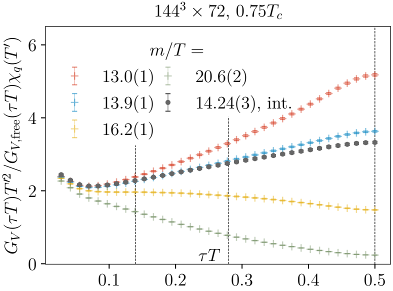
Interpolate the correlator at each τT separately using a fit with Ansatz

$$G(m/T, \tau T) = \exp(a(m/T)^2 + b(m/T) + c)$$

- Choose screening mass $\mathbf{m/T = 14.2}$ from the finest lattice as a reference
- Also at higher temperatures use $0.75\,T_C$ screening masses
- Interpolate on **bootstrap samples**

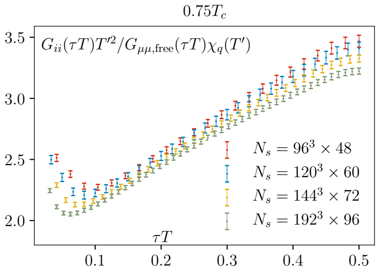


(Details of mass determination in next talk, by Hai-Tao Shu)

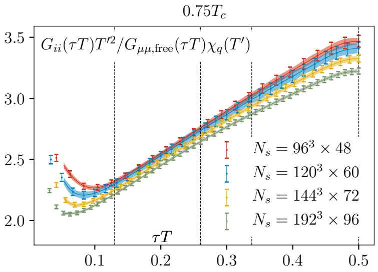


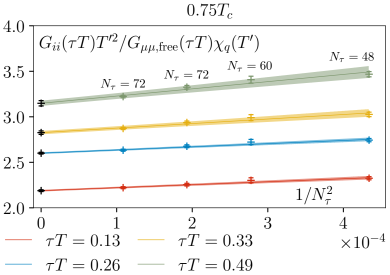
→ Correlators for all lattice spacings tuned to the same meson mass

Continuum limit



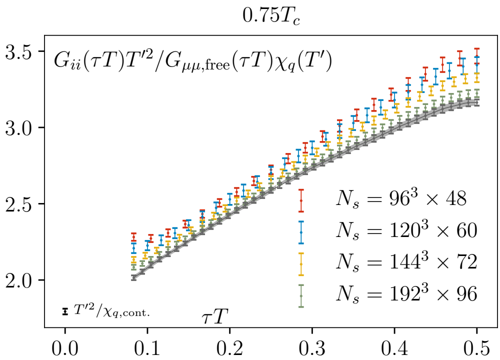
- Strong cut off effects at small distances
- Also at large distances continuum limit is needed





- Strong cut off effects at small distances
- Also at large distances continuum limit is needed
- interpolate correlator using b-splines
- Linear extrapolation **in $1/N_\tau^2$** on interpolated correlators

Continuum limit

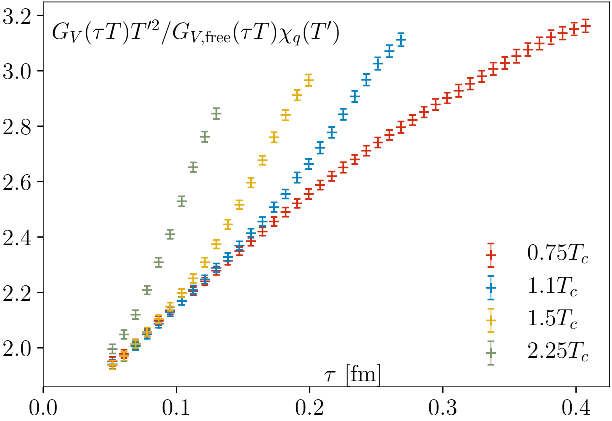


Well defined continuum extrapolated correlator for distances larger than $\tau T > 0.08$

- Approaching correct asymptotic limit for $\tau \to 0$
- χ_q-normalization can be removed easily

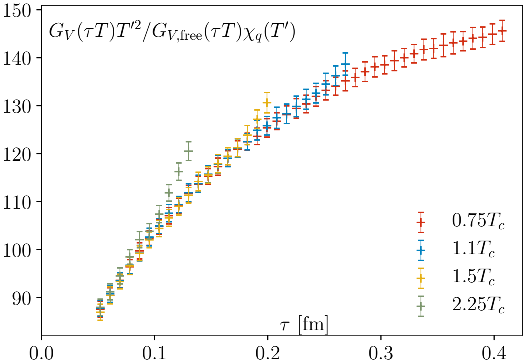
At different temperatures

Continuum correlators at four different temperatures



Observation of clear temperature dependence for charmonium.

At different mass (bottom)



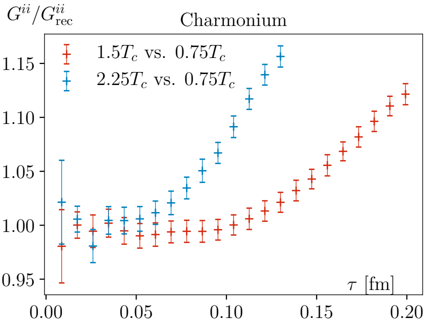
- We can scan for arbitrary quark masses between bottom and charm
- Smaller temperature dependence for bottomonium

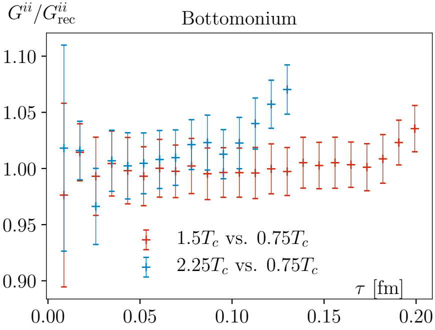
Reconstructed correlator

Remove trivial temperature dependence from the Kernel
[H. Meyer JHEP 1004 (2010) 099, H.T. Ding et al. PRD86 (2012) 014509]

$$G_{\mathrm{rec}}(\tau, T; T') = \int_0^\infty \rho(\omega, T') K(\omega, \tau, T) d\omega = \sum_{\tau'=\tau;\tau'+=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')$$

$$T' < T, \quad N'_\tau = m N_\tau, \quad m \in \mathbb{Z}$$





Indications for strong temperature effects for charmonium.

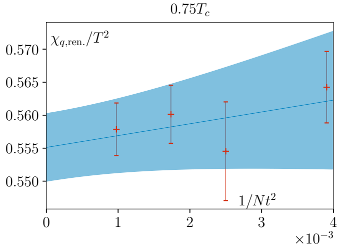
Conclusion and outlook

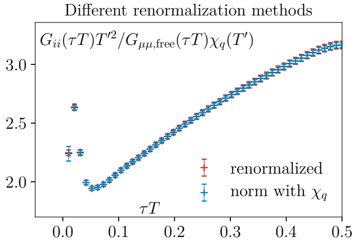
- Developed method to compute vector continuum correlators
- Interpolations for quark masses between m_c and m_b are possible
- Significant temperature effects for charmonium
- Smaller temperature dependence for bottomonium

- **Next steps**: pseudo-scalar, axial-vector, scalar, non-zero momentum
- Extraction of spectral and transport properties for continuum extrapolated correlators:
  - Screening masses at non-zero temperatures (Next talk, Hai-Tao Shu)
  - Extract spectral function via SAI or MEM
  - Heavy quark diffusion coefficient

Backup

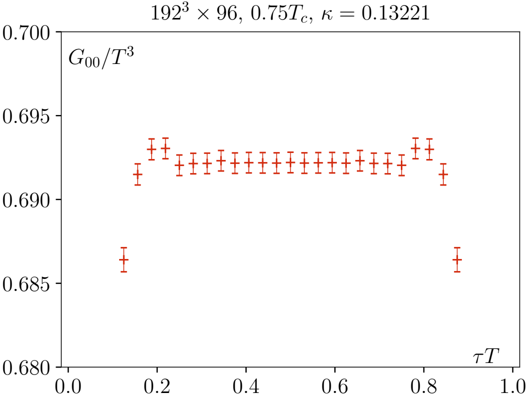
Renormalization methods





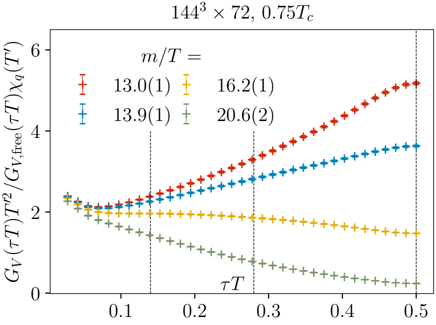
- Renormalize with non perturbative renormalization constants [Lüscher et al, Phys.Lett. B372(1996)275-282]
- Continuum limit of χ_q well defined
- Normalization with χ_q or using renormalization constants give same results

χ_q determination



$$\chi_q(T)/T^2 = G_{00}(\tau T = 0.5)/T^3$$
$$= \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{\rho_{00}(\omega) K(\omega, \tau T)}{T^3},$$
$$\rho_{00} = 2\pi T^2 \omega \delta(\omega)$$

Mass dependence



- Different fitting methods
- **Correlated two-state** fit gives best mass plateau
- Mass plateau for each lattice selected separately based on mass plateau and $\chi^2/\mathrm{d.o.f}$

- Two-state Ansatz:

$$G(n_\tau) = A_1 \cosh(m_1(n_\tau - N_\tau/2)) + A_2 \cosh(m_2(n_\tau - N_\tau/2))$$