Topology in the chiral symmetry restored phase of unquenched QCD and axion cosmology.

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Outline

- Brief historical summary
- σ and η susceptibilities
- Phase diagram of QCD in the $Q = 0$ topological sector
- Conclusions

Phys. Rev. D 94, 094505 (2016)
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- K. Suzuki, in this Conference

σ and η susceptibilities

$$L_{QCD} = \sum_f L_F^f + \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + i\theta \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

$$Q = \frac{g^2}{64\pi^2} \int d^4x \; \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x)$$

Ginsparg-Wilson fermions

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Ginsparg-Wilson fermions

- $U(1)_A$ anomalous symmetry
- Good chiral properties
- Quantized topological charge
- Exact index theorem on the lattice

σ and η susceptibilities

- The topological effects of the $U(1)_A$ anomaly survive in the high temperature chiral symmetric phase of QCD

$$Z(\theta) = \sum_Q Z_Q e^{i\theta Q}$$

$$Z(\theta) = e^{-V_x L_t E(\beta, m, \theta)}$$

$$\langle O \rangle_{Q=0} = \frac{\int d\theta \, \langle O \rangle_\theta \, Z(\theta, m)}{\int d\theta Z(\theta, m)}$$

σ and η susceptibilities

The free energy density, as a function of θ, has its absolute minimum at $\theta = 0$ for non-vanishing quark masses.

$$\langle O \rangle_{Q=0} = \langle O \rangle_{\theta=0}$$

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$$\langle O \rangle_{Q=0} = \langle O \rangle_{\theta=0}$$

In spite of the fact that the $Q = 0$ topological sector is free from the $U(1)_A$ global anomaly, and spontaneously breaks the $U(N_f)_A$ axial symmetry at $T = 0$, this equation is compatible with a massive flavor-singlet pseudoscalar meson in the chiral limit (Phys. Rev. D 94, 094505 (2016))

σ and η susceptibilities

Two-flavour model

- $T > T_c$ the $SU(2)_A$ symmetry is fulfilled in the ground state for massless quarks.
- $\langle S \rangle$ and any order parameter vanishes in the chiral limit.
- The infinite lattice volume limit and the chiral limit should commute.

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- $T > T_c$ the $SU(2)_A$ symmetry is fulfilled in the ground state for massless quarks.
- $\langle S \rangle$ and any order parameter vanishes in the chiral limit.
- The infinite lattice volume limit and the chiral limit should commute.
- $\langle S \rangle_{\theta=0} = \langle S \rangle_{Q=0} \Longrightarrow SU(2)_A$ symmetry is also fulfilled in the $Q = 0$ topological sector.

σ and η susceptibilities

Assume $\chi_\sigma(m)$ is finite

$$\langle S \rangle_{\theta=0} = \langle S \rangle_{Q=0} \underset{m\to 0}{\approx} \chi_\sigma(0)\, m$$

The $Q=0$ sector is free from the $U(1)_A$ global anomaly

$$\chi_{\bar{\pi}}(m)_{Q=0} = \chi_\eta(m)_{Q=0} = \frac{\langle S \rangle_{Q=0}}{m} \to \chi_\sigma(0)$$

Anomalous $U(1)_A$ transformation

$$\langle S(x)\, S(0) \rangle_\theta^{m=0} = \cos^2\left(\frac{\theta}{2}\right) \langle S(x)\, S(0) \rangle_{\theta=0}^{m=0} + \sin^2\left(\frac{\theta}{2}\right) \langle P(x)\, P(0) \rangle_{\theta=0}^{m=0}$$

$$\langle S(x)\, S(0) \rangle_{Q=0}^{m=0} = \frac{1}{2} \langle S(x)\, S(0) \rangle_{\theta=0}^{m=0} + \frac{1}{2} \langle P(x)\, P(0) \rangle_{\theta=0}^{m=0}$$

σ and η susceptibilities

$$\chi_\sigma(0) = \chi_\sigma(0)_{Q=0} = \frac{\chi_\sigma(0) + \chi_\eta(0)}{2}$$

A finite value of $\chi_\sigma(m)$ in the chiral limit is not compatible with the survival of the topological effects of $U(1)_A$ at high T.

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A finite value of $\chi_\sigma(m)$ in the chiral limit is not compatible with the survival of the topological effects of $U(1)_A$ at high T.
Critical behaviour

$$\langle S \rangle_{\theta=0\ m\to 0} \approx C(T)\, m^{\frac{1}{\delta}}$$

$$\chi_\sigma(m) \approx C(T)\, \frac{1}{\delta} m^{\frac{1-\delta}{\delta}}$$

$SU(2)_A$ symmetry is not anomalous

$$\chi_{\bar{\pi}}(m) = \frac{\langle S \rangle}{m}$$

σ and η susceptibilities

$$\chi_{\bar{\pi}}(m) \approx C(T)\, m^{\frac{1-\delta}{\delta}}$$

σ and η susceptibilities

$$\chi_{\bar{\pi}}(m) \approx C(T)\, m^{\frac{1-\delta}{\delta}}$$

The vector meson $\bar{\delta}$ susceptibility, $\chi_{\bar{\delta}}$, is bounded by the scalar susceptibility, χ_{σ}

$$\chi_{\bar{\pi}}(m) - \chi_{\bar{\delta}}(m) \geq \chi_{\bar{\pi}}(m) - \chi_{\sigma}(m) \approx C(T)\, \frac{\delta - 1}{\delta} m^{\frac{1-\delta}{\delta}}$$

$\chi_{\bar{\pi}}(m) - \chi_{\bar{\delta}}(m)$, which is an order parameter for the $U(1)_A$ symmetry, diverges in the chiral limit

Phase diagram of QCD in the $Q = 0$ topological sector

$SU(2)_A$ symmetry is fulfilled $T > T_c \Longrightarrow \langle S_u \rangle = \langle S_d \rangle = 0$ in the chiral limit

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QCD with two non degenerate quark flavors: m_u, m_d

- $m_u = 0 \; m_d \neq 0 \; U(1)_u$ anomalous symmetry $\Longrightarrow \langle S_u \rangle \neq 0$
- $m_u \neq 0 \; m_d = 0 \; U(1)_d$ anomalous symmetry $\Longrightarrow \langle S_d \rangle \neq 0$

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QCD with two non degenerate quark flavors: m_u, m_d

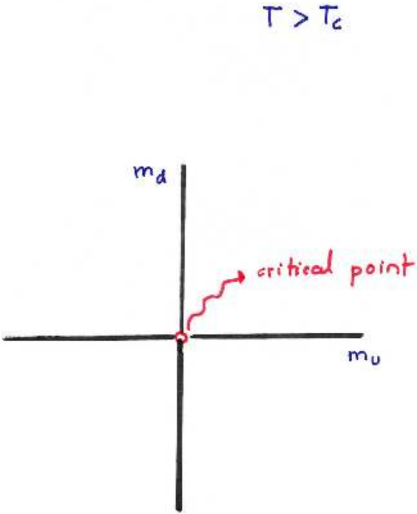
- $m_u = 0$ $m_d \neq 0$ $U(1)_u$ anomalous symmetry $\Longrightarrow \langle S_u \rangle \neq 0$
- $m_u \neq 0$ $m_d = 0$ $U(1)_d$ anomalous symmetry $\Longrightarrow \langle S_d \rangle \neq 0$

For non-vanishing quark masses

$$\langle S_{u,d} \rangle_{Q=0} = \langle S_{u,d} \rangle_{\theta=0}$$

Conclusion: the $Q = 0$ topological sector, which is not anomalous, spontaneously breaks the $U(1)_u$ axial symmetry at $m_u = 0, m_d \neq 0$, and the $U(1)_d$ symmetry at $m_d = 0, m_u \neq 0$.

Phase diagram of QCD in the $Q = 0$ topological sector



Phase diagram of QCD in the $Q = 0$ topological sector

- The critical chiral equation of state of QCD in the $Q = 0$ topological sector, which should show a divergent correlation length at any $T > T_c$, should be the same as in QCD at $\theta = 0$. We expect therefore a continuous finite temperature chiral transition, and a divergent correlation length for any $T \geq T_c$ in the chiral limit.

Phase diagram of QCD in the $Q = 0$ topological sector

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- Because the symmetry breaking pattern is, in the two flavor model, $SU(2)_L \times SU(2)_R \to SU(2)_V$, the critical equation of state should be that of the three-dimensional $O(4)$ vector universality class, which shows a critical exponent $\delta = 4.789(6)$ (M. Hasenbusch, J. Phys. A 34 8221 (2001)) ($\delta = 3$ in the mean field or Landau approach).

Phase diagram of QCD in the $Q = 0$ topological sector

For $N_f \geq 3$ a similar argument on the phase diagram of the $Q = 0$ sector applies, but the scenario that emerges in this case seems not plausible because no stable fixed points are expected in the corresponding Landau-Ginzburg-Wilson Φ^4 theory compatible with the given symmetry-breaking pattern (E. Vicari, Proc.Sci., LATTICE2007 (2007) 023).

Conclusions

- We have analyzed the physical consequences of assuming that the topological effects of the $U(1)_A$ anomaly persist in the chiral symmetric phase of QCD at high temperature.
- We have shown, in the two-flavour model, that the free energy density is a singular function of the quark mass m, in the chiral limit, and that the σ and $\bar{\pi}$ susceptibilities diverge in this limit at any $T \geq T_c$.
- We have also shown that the difference between the $\bar{\pi}$ and $\bar{\delta}$ susceptibilities diverges in the chiral limit at any $T \geq T_c$.
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Conclusions

- These results can also be obtained from an analysis of the phase diagram of the $Q = 0$ sector.
  - The results for the two-flavor model apply also to $N_f \geq 3$
  - Universality and renormalization-group arguments suggest that this scenario is not plausible for $N_f \geq 3$ because no stable fixed points are expected in the corresponding Landau-Ginzburg-Wilson Φ^4 theory.

A. Tomiya, G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko and J. Noaki, arXiv:1612.01908 [hep-lat].

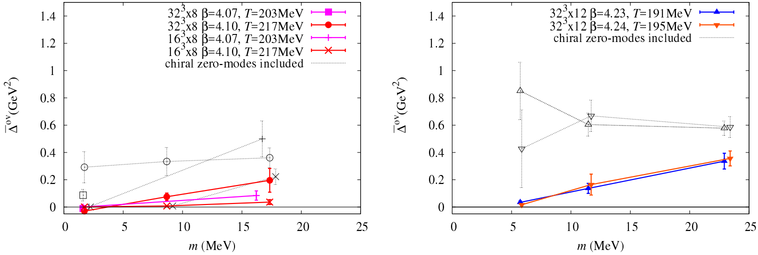

FIG. 14: The quark mass dependence of $\bar{\Delta}^{\rm ov}_{\pi-\delta}$ (solid symbols) and $\Delta^{\rm ov}_{\pi-\delta}$ (dashed). Data for coarse (left panel) and fine (right) lattices are shown.

the chiral phase transition ($T \sim$ 190–220 MeV) on different physical volume sizes (L = 2–4 fm), where frequent topology tunnelings occur.

Our results for the histograms of the Möbius domain-wall and (reweighted) overlap Dirac operators both show a strong suppression of the near zero modes as decreasing the quark mass. This behavior is stable against the changes in the lattice volume and lattice spacing.

If we do not perform the reweighting of their determinants, the overlap Dirac spectrum shows unphysical peaks near zero. We have identified them as partially quenched lattice

V. Dick, F. Karsch, E. Laermann, S. Mukherjee, and S. Sharma, Phys. Rev. D 91, 094504 (2015).

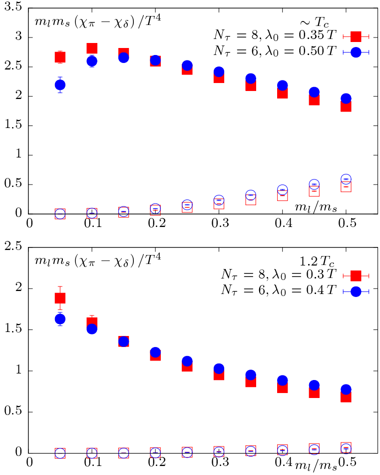

Fig. 6: A renormalized measure of $U_A(1)$ breaking for a range of valence light quark masses, $m_s/20 \leq m_l \leq m_s/2$ for ensembles with different N_τ at $T \sim T_c$ and $T \sim 1.2\,T_c$. The filled points denote the contribution from near-zero modes

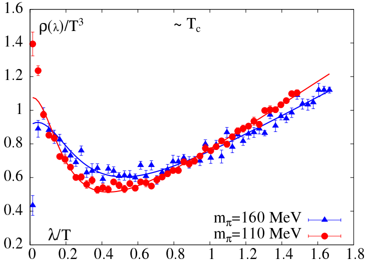

Fig. 7: Eigenvalue distribution at $T \sim T_c$ for $N_\tau = 6$ and two different sea quark masses compared to fits with Eq. (9).

unchanged under renormalization. We therefore take the same ansatz as in Eq. (9) with $\lambda \to \lambda/m_s$ and the density replaced by its renormalized definition $m_s\rho(\lambda)/T^4$. Fits to the renormalized spectrum in dimensionless units at two different temperatures, T_c and 1.2 T_c, are shown in Fig. 8. Taking a closer look at the near-zero mode peak, it is evident that the accumulation of near-zero modes is almost independent of the lattice spacing and is unlikely to be just a lattice artifact.