Towards extracting the timelike pion form factor on CLS 2-flavour ensembles

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Motivation

- The ρ resonance on the lattice, whose principal decay is $\rho \to \pi\pi$ is intersting to study on the lattice.
- It has been shown by Meyer that this resonance gives access to the pion form factor in the timelike region $2m_\pi \leq \sqrt{s} \leq 4m_\pi$. [Meyer '11]
- Knowing F_π is crucial to reduce the uncertainty in theoretical calculations of the muon $g-2$. [Bernecker, Meyer '11]
- Challenge with $\pi\pi$-interpolators on the lattice: Also sink-to-sink quark propagators have to be calculated.
- One approach which is able to handle those is (stochastic) distillation using Laplacian-Heavyside (LapH) smearing, which is used in this work. [Peardon et al. '09; Morningstar et al. '11]

(Stochastic) Distillation // LapH

- The inversion of the Dirac matrix K is computationally very expensive.
- Distillation uses a special kind of hermitian smearing matrix $\mathcal{S} = V_S V_S^\dagger$. $\Rightarrow$ Just $V_S^\dagger K^{-1} V_S$, a much smaller matrix has to be calculated and stored on disk.
- Quark lines $\mathcal{Q}$ (a smeared-to-smeared quark propagator) can be expressed as an expectation value of Distillation-sink vectors φ and Distillation-source vectors ϱ:

$$\mathcal{Q} = \sum_b E(\varphi^{[b]}(\rho)(\varrho^{[b]}(\rho))^\dagger).$$

$$\varphi^{[b]}(\rho) = \mathcal{S}\,\mathrm{K}^{-1}\, V_S P^{(b)} \rho \qquad \varrho^{[b]}(\rho) = V_S P^{(b)} \rho.$$

ρ: noise vectors. $P^{(b)}$: dilution projectors.

Interpolator Setup

$$\rho^0(\mathbf{P}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{P}\cdot\mathbf{x}} \Big(\bar{u}\Gamma u - \bar{d}\Gamma d \Big)(t) \;\; , \;\; \Gamma \in \; \{\gamma_i, \gamma_0\gamma_i\}$$

$$(\pi\pi)(\mathbf{P}, t) = \pi^+(\mathbf{p}_1, t)\pi^-(\mathbf{p}_2, t) - \pi^-(\mathbf{p}_1, t)\pi^+(\mathbf{p}_2, t),$$

$$\pi^+(\mathbf{q}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}} \big(\bar{u}\gamma_5 d\big)(\mathbf{x}, t)$$

$$\pi^-(\mathbf{q}, t) = \frac{1}{2L^{3/2}} \sum_{\mathbf{x}} e^{-i\mathbf{q}\cdot\mathbf{x}} \big(\bar{d}\gamma_5 u\big)(\mathbf{x}, t)$$

Frames and irreps

- We analyse 4 different frames:
- Centre-of-mass frame (CMF), $\mathbf{P} = \frac{2\pi}{L}\mathbf{d} = 0$
- Moving frames: $\mathbf{d}^2 \in 1, 2, 3$, averaged over all possible directions on the lattice.
- We analyse different lattice irreps in the moving frames.

Variational method

Correlator matrix:

$$C(t) = \begin{pmatrix} \langle \rho(t)\rho^\dagger(0)\rangle & \langle \rho(t)(\pi\pi)^\dagger(0)\rangle \\ \langle (\pi\pi)(t)\rho^\dagger(0)\rangle & \langle (\pi\pi)(t)(\pi\pi)^\dagger(0)\rangle \end{pmatrix}.$$

Generalized eigenvalue problem (GEVP) of this matrix:

[Michael '85; Lüscher, Wolff '90]

$$C(t)\mathbf{v} = \lambda(t)C(t_0)\mathbf{v}$$

window method: $t_0 = t - t_w; \quad t_w = const.$ [Blossier et al. '09]
fixed-t0 method: $t_0 = const.$
extracted eigenvalues $\lambda^{(k)}(t)$ lead to effective masses E_k

Phase shift

Lüscher condition: [Lüscher '86]

$$\delta_1(k) + \phi(q) = n\pi$$

$k = \frac{2\pi}{L} q$ and $E_{cm} = E_{cm}(k) = 2\sqrt{k^2 + m_\pi^2}$.
$\Rightarrow$ for every energy level extracted we get a value for the phase shift δ.

Timelike pion form factor

We are interested in $|A_\Psi| = |\langle\Omega|J(t)|n\rangle|$, because it gives us access to the timelike pion form factor:

$$|F_\pi(E)|^2 = G(\gamma)\left(q\frac{d\phi(q^2)}{dq} + k\frac{\partial\delta_1(k)}{\partial k}\right)\frac{3\pi E^2}{2k^5}|A_\Psi|^2$$

- In addition to the correlator matrix $C(t)$ we have calculated the matrix elements $\langle J_\mu(t)O_i^\dagger(0)\rangle$.
- The GEVP eigenvectors $v_n(t)$ can be used to form some optimal operators $X_n(t)$ which couple very well to the energy state E_n
- We can use these operators X_n to form a two-point function with the current insertions at the sink:

$$\langle J(t)X_n^\dagger(0)\rangle \rightarrow \langle\Omega|J(t)|n\rangle Z_n^* e^{-E_n t}$$

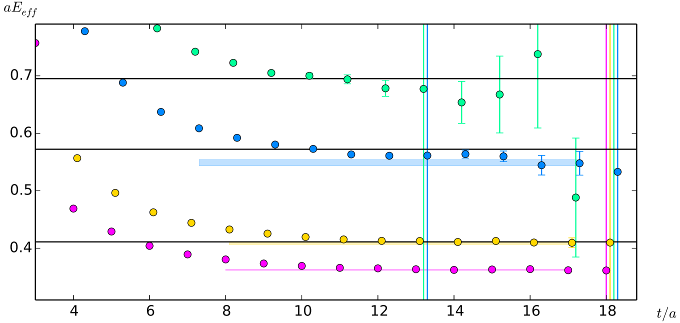
CLS Lattices

We use three different CLS 2-flavor lattices with $\beta = 5.3$ and a lattice spacing of $a = 0.0658(7)(7)\mathrm{fm}$

	T/a	L/a	m_π [MeV]	κ	$m_\pi L$	N_{conf}	N_{meas}
E5	64	32	437	0.13625	4.7	500	2000
F6	96	48	311	0.13635	5.0	300	300 (900)
F7	96	48	265	0.13638	4.2	350	350 (1050)

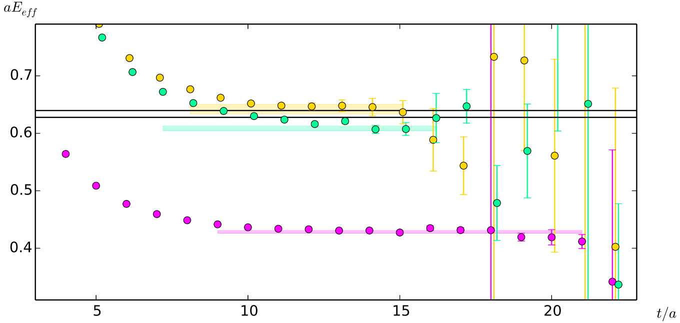
E5: full distillation for the quark lines connected to the source timeslice, stochastic distillation for the sink-to-sink lines.
F6/F7: stochastic distillation for all lines.

E5 lattice, moving frame with $d^2 = 1$, A_1 irrep



Window method, $t_W = 3$. Fit to ground state plus one excited state.

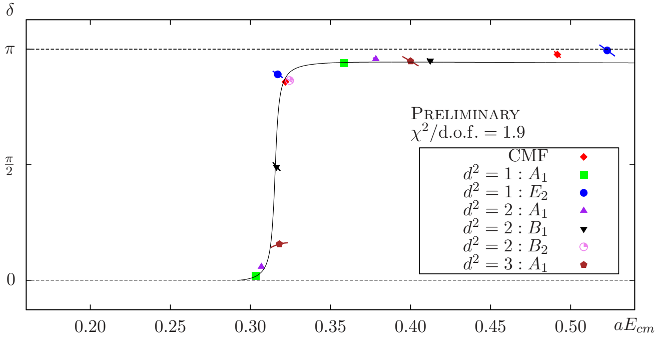
E5 lattice, moving frame with $d^2 = 2$, B_2 irrep



Window method, $t_W = 3$. Fit to ground state plus one excited state.

Lüscher's method

We can match the finite-box lattice data to the infinite volume:



Phase shift δ on the E5 lattice. (window method)

Fit procedure

Assumed resonance: Breit-Wigner curve (effective-range formula):

$$\cot\delta_1(k) = \frac{6\pi}{g^2_{\rho\pi\pi}} \frac{(m_\rho^2 - E_{cm}^2)E_{cm}}{k^3}$$

Lüscher condition:

$$\delta_1(k) + \phi(q) = n\pi \Rightarrow \cot\delta_1(k)\big|_{\text{Luscher}}$$

$$f(E_{cm}, g_{\rho\pi\pi}, m_\rho) = \cot\delta_1(k)\big|_{\text{Luscher}} - \cot\delta_1(k, g_{\rho\pi\pi}, m_\rho)$$

Given $g_{\rho\pi\pi}$ and m_ρ, the zeros of $f(E_{cm}, g_{\rho\pi\pi}, m_\rho)$ correspond to energy levels $E_{cm,i}(g_{\rho\pi\pi}, m_\rho)$ We can define a χ^2-function:

$$\chi^2(g_{\rho\pi\pi}, m_\rho) = \sum_{i,j} (E_{cm,i}(g_{\rho\pi\pi}, m_\rho) - E_{\text{lat},i}) C^{-1}_{i,j} (E_{cm,j}(g_{\rho\pi\pi}, m_\rho) - E_{\text{lat},j})$$

Fit procedure

The results of this fit are: window method:

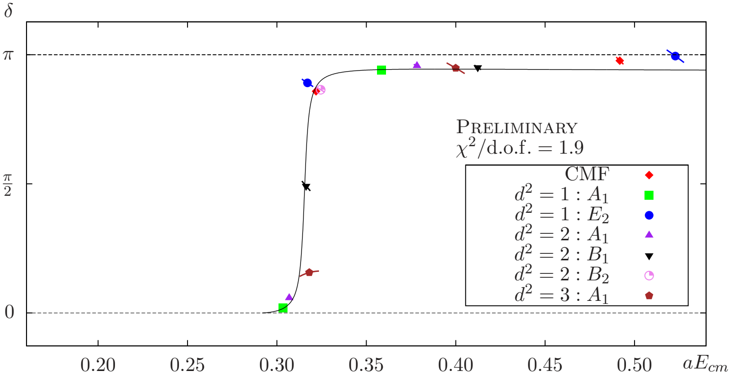
$$am_\rho = 0.3163(11) \qquad g_{\rho\pi\pi} = 5.86(24) \qquad \chi^2/d.o.f. = 1.9$$

fixed-t0 method:

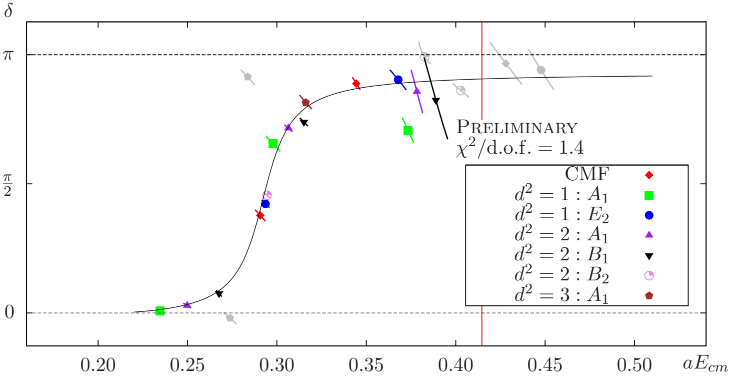
$$am_\rho = 0.3145(17) \qquad g_{\rho\pi\pi} = 6.26(51) \qquad \chi^2/d.o.f. = 0.7$$

The naive rho mass in comparison: $am_{\rho,\mathrm{naive}} = 0.3208(29)$

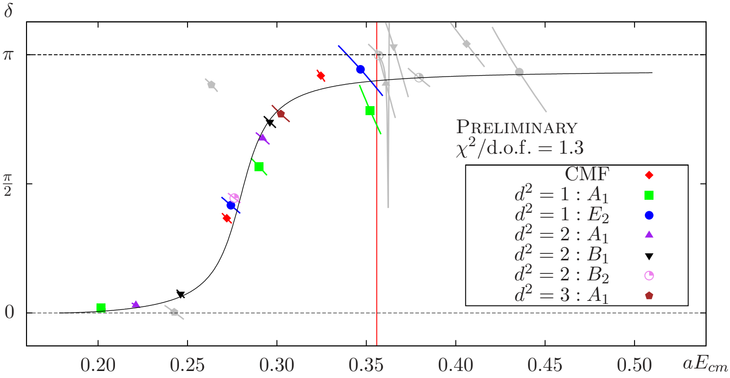
Phase shift δ on E5 ($m_\pi = 437$ MeV)



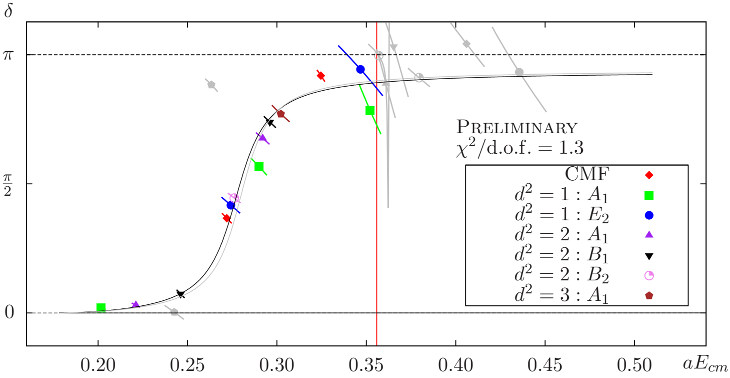
Phase shift δ on F6 ($m_\pi = 311$ MeV)



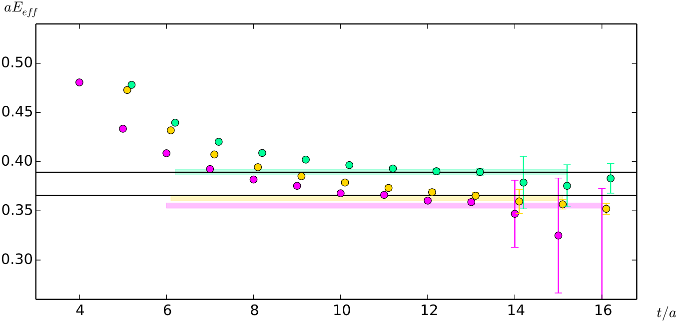
Phase shift δ on F7 ($m_\pi = 265$ MeV)



Phase shift δ on F7, uncorrelated fit

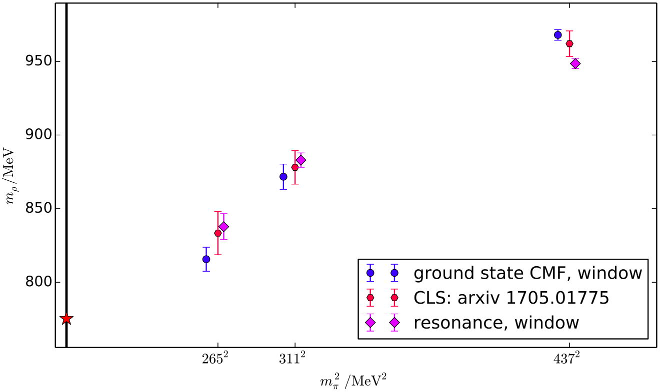


F6 lattice, moving frame with $d^2 = 3$, A_1 irrep



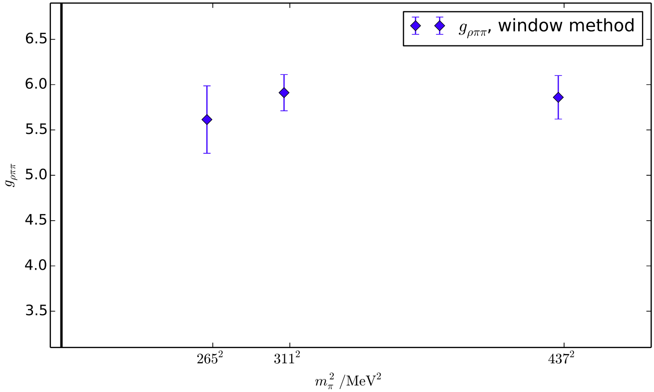
Effective energies of the 3 states in the A_1 irrep in the $\mathbf{d}^2 = 3$ moving frame. The lowest two levels are not resolvable at our level of statistics.

ρ mass



Fit results for m_ρ in F7, F6 and E5.

coupling



Fit results for $g_{\rho\pi\pi}$ in F7, F6 and E5.

Conclusion

- Using disillation we can precisely compute energy levels in different (moving) frames on the lattice, which we can use to map out the phase shift of the ρ resonance.
- We can perform a global fit to those phase shift points, which takes into account the error of the datapoints along the Lüscher curves.
- We are planning to use our data to map out the timelike $|F_\pi|$ in the ρ-resonance region.

Thank you for your attention!