

Charmed states and $SU(3)$ flavour symmetry breaking

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– QCDSF-UKQCD-CSSM Collaborations –

Edinburgh – Adelaide – RIKEN (Kobe) – Leipzig – Liverpool – DESY – Hamburg

Lattice 2017, Granada, Spain

[Wednesday 21/6/17 10:40 (Andalucia III)]



QCDSF related talks with 2 + 1 flavours:

- **James Zanotti** Monday 19/6/17 17:20 (Andalucia III)
Isospin breaking effects on nucleon structure from fully dynamical QCD+QED
- **Ross Young** Tuesday 20/6/17 17:50 (Andalucia III)
Partonic structure from the Compton amplitude
- **Holger Perlt** Tuesday 20/6/17 19:30 (Palacio de Congresos) [Poster]
The IMOM renormalization scheme on the lattice
- **Hinnerk Stüben** Tuesday 20/6/17 19:30 (Palacio de Congresos) [Poster]
An update on the BQCD Hybrid Monte Carlo program
- **Gerrit Schierholz** Friday 23/6/17 15:00 (Seminarios 6+7)
Dynamical QCD+Axion simulation: First results
- **Paul Rakow** Friday 23/6/17 17:50 (Seminarios 1+2)
Charge creation, finite size effects, and infra-red photons in simulations of QCD plus QED

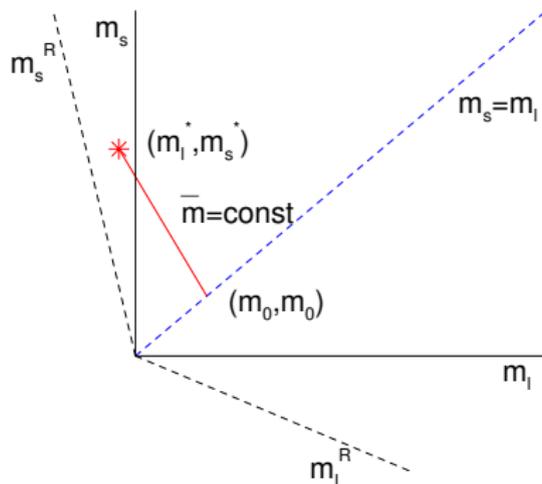
Introduction

- Background:
 - Given $2 + 1$ simulations (at quark masses larger than physical quark masses), how can we usefully approach the physical point?
 - Possibility: $SU(3)$ flavour expansion about flavour symmetric line
- Extend expansion to PQ quark masses (ie valence quarks \neq sea quarks)
- Determine (quenched) charm quark
- Open charm masses
- Conclusions

QCDSF strategy:

[arXiv:1102.5300]

2 + 1 simulations: many paths to approach the physical point

[$m_u = m_d \equiv m_l$ case]

QCDSF: extrapolate from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_0, m_0) \longrightarrow (m_l^*, m_s^*)$$

Choice here: keep the singlet quark mass \bar{m} constant

$$\bar{m} = m_0 = \frac{1}{3} (2m_l + m_s)$$

QCDSF strategy

[arXiv:1102.5300]

- develop $SU(3)$ flavour symmetry breaking expansion for hadron masses
- expansion in:

$SU(3)$ flavour symmetric point $\delta m_q = 0$

$$\delta m_q = m_q - \bar{m}, \quad \bar{m} = \frac{1}{3}(m_u + m_d + m_s) = m_0$$

- expansion coefficients are functions of \bar{m}
- trivial constraint

$$\delta m_u + \delta m_d + \delta m_s = 0$$

- path called 'unitary line' as expand in both sea and valence quarks

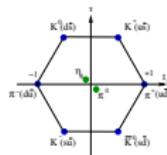
$SU(3)$ flavour symmetry breaking expansions

[known to $O(3)$ in δm , ie NNLO]

- octet pseudoscalar mesons:

$$\begin{aligned}
 M^2(a\bar{b}) &= M_{0\pi}^2 + \alpha(\delta m_a + \delta m_b) \\
 &+ \beta_0 \frac{1}{6} (\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
 &+ \beta_1 (\delta m_a^2 + \delta m_b^2) + \beta_2 (\delta m_a - \delta m_b)^2 \\
 &+ \dots
 \end{aligned}$$

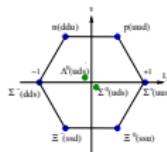
[$a, b = u, d, s$ (outer ring)]



- octet baryons:

$$\begin{aligned}
 M^{N2}(aab) &= M_{0N}^2 + A_1(2\delta m_a + \delta m_b) + A_2(\delta m_b - \delta m_a) \\
 &+ B_0 \frac{1}{6} (\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
 &+ B_1(2\delta m_a^2 + \delta m_b^2) + B_2(\delta m_b^2 - \delta m_a^2) + B_3(\delta m_b - \delta m_a)^2 \\
 &+ \dots
 \end{aligned}$$

[$a, b = u, d, s$ (outer ring)]



stable under strong ints.

$$\begin{aligned}
 M^{\Lambda 2}(aab) &= M_{0\Lambda}^2 + A_1(2\delta m_a + \delta m_b) - A_2(\delta m_b - \delta m_a) \\
 &+ B_0 \frac{1}{6} (\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
 &+ B_1(2\delta m_a^2 + \delta m_b^2) - B_2(\delta m_b^2 - \delta m_a^2) + B_4(\delta m_b - \delta m_a)^2 \\
 &+ \dots
 \end{aligned}$$

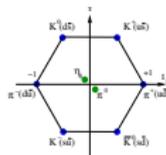
[$a, b = l, s$ (when no $\Lambda^0 - \Sigma^0$ mixing)]

Singlet quantities

- Pseudoscalar mesons (centre of mass):

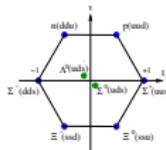
$$\begin{aligned} X_\pi^2 &= \frac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^0}^2 + M_{K^0}^2 + M_{K^-}^2) = (0.4116 \text{ GeV})^2 \\ &= M_{0\pi}^2 + \left(\frac{1}{6}\beta_0 + \frac{2}{3}\beta_1 + \beta_2\right)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_{0\pi}^2 + \mathcal{O}(\delta m_q^2) \end{aligned}$$

[no QED, $m_U = m_D$, FLAG3]



- Octet baryons (centre of mass):

$$\begin{aligned} X_N^2 &= \frac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^0}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160 \text{ GeV})^2 \\ &= M_{0N}^2 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_{0N}^2 + \mathcal{O}(\delta m_q^2) \end{aligned}$$



- Many other possibilities

$$X_S^2 = \begin{cases} \frac{1}{2}(M_\Sigma^2 + M_\Lambda^2) & S = \Lambda & \text{baryon octet} \\ M_{\Sigma^*}^2, \frac{1}{2}(M_\Delta^2 + M_{\Xi^*}^2) & S = \Sigma^*, \Delta & \text{baryon decuplet, unstable under QCD} \\ \frac{1}{6}(M_{K^*}^2 + M_{K^*0}^2 + M_\rho^2 + M_{\rho^-}^2 + M_{K^*0}^2 + M_{K^*}^2) & S = \rho & \text{vector octet} \\ 1/t_0^2, 1/t_0, 1/w_0^2 & S = r_0, t_0, w_0 & \text{force, Wilson flow scales} \\ \frac{1}{6}(f_{K^+} + f_{K^0} + f_{\pi^+} + f_{\pi^0} + f_{\pi^-} + f_{K^0} + f_{K^-}) & S = f & \text{decay constants} \end{cases}$$

stable under strong ints.

- Form dimensionless ratios (within a multiplet):

$$\tilde{M}^2 \equiv \frac{M^2}{X_S^2}, \quad S = \pi, N, \dots, \quad \tilde{A}_i \equiv \frac{A_i}{M_{0N}^2}, \dots \quad \text{in expansions}$$

Lattice

- $O(a)$ NP improved clover action
 - tree level Symanzik glue
 - mildly stout smeared 2 + 1 clover fermion
 - $\beta = [5.40], 5.50, 5.65, 5.80, 5.95$ [$24^3 \times 48, 32^3 \times 64, 48^3 \times 96$],
unitary $M_\pi \sim 220 - 410$ MeV [$M_\pi L \geq 4$]

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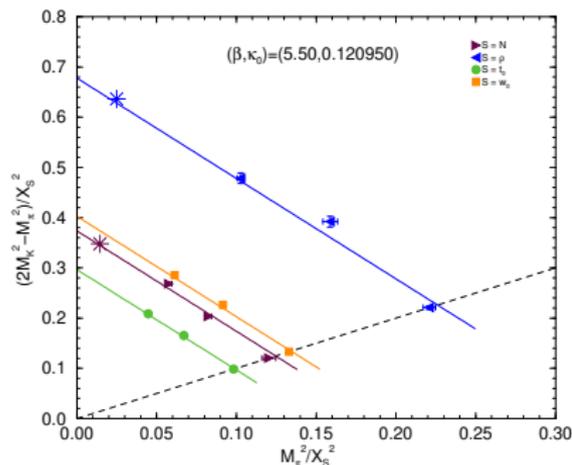
$$m_q = \frac{1}{2} \left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}} \right)$$

κ_{0c} is chiral limit along symmetric line

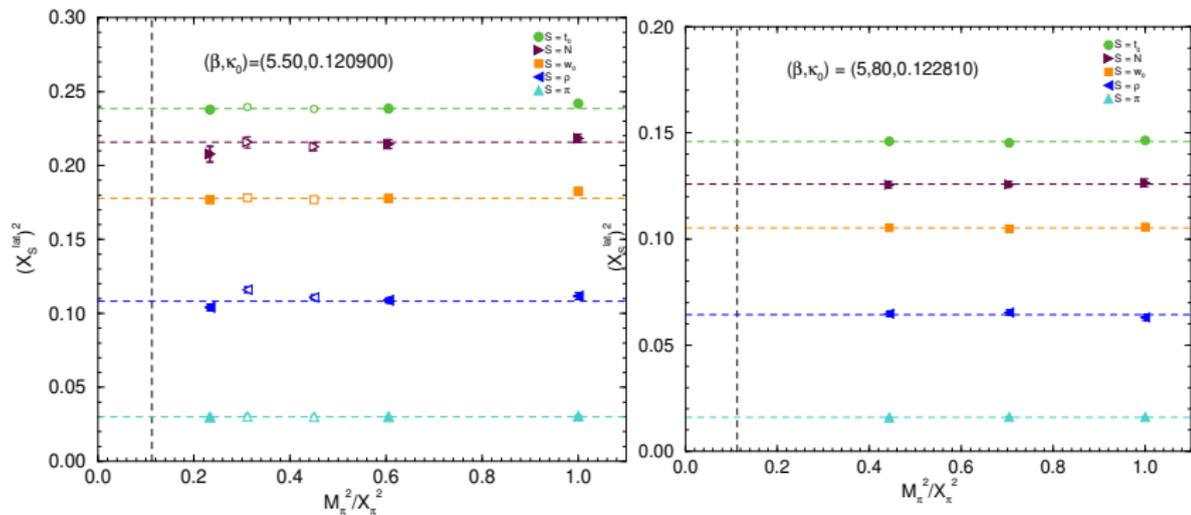
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$$\delta m_q = m_q - \bar{m} = \frac{1}{2} \left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0} \right)$$

- Path in quark mass plane

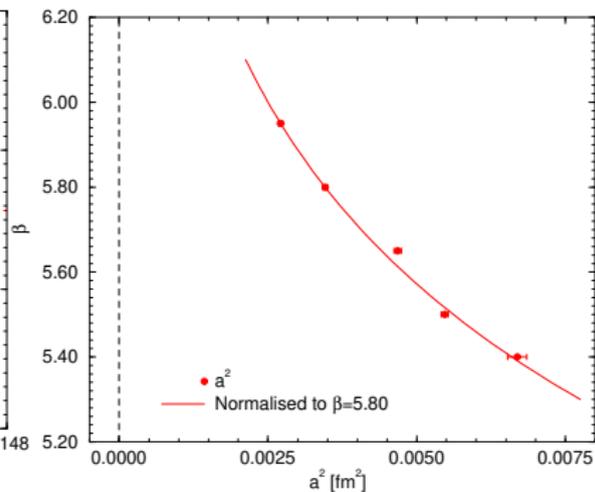
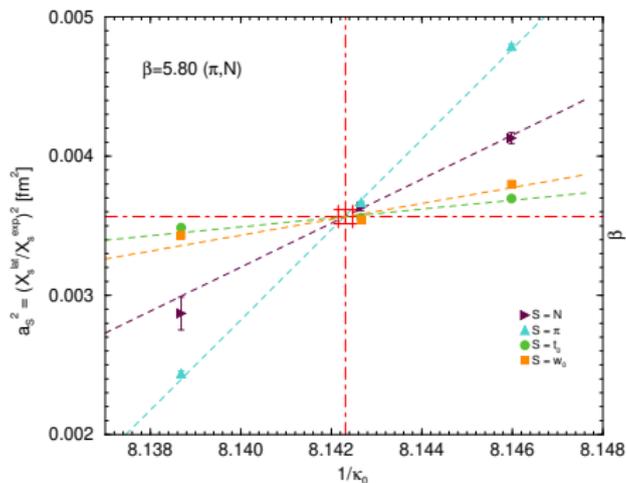


Constancy of X_5^2



Use this constancy to determine the scale

Scale



- Crossing of

$$a_S^2 = \frac{X_S^{\text{LAT}}}{X_S^{\text{exp}}}^2$$

for (π, N) , ... determines the $SU(3)$ flavour symmetric starting point κ_0 and the scale a^2

-

$$\beta_0 = 5.80$$

$$\frac{a^2(\beta)}{a^2(\beta_0)} = \left(\frac{\beta_0}{\beta}\right)^{-\frac{b_1}{b_0}} \exp\left(-\frac{1}{10b_0}(\beta - \beta_0)\right)$$

- $a \sim 0.052, 0.059, 0.068, 0.074,$
[0.082] fm

Reaching the charm quark mass range

- Unitary range rather small so introduce **PQ** partially quenching (ie valence quark masses \neq sea quark masses) and **NNLO**
- Furthermore can generalise to different valence quark masses, μ_q to sea quark masses m_q without increasing number of expansion coefficients

$$\delta\mu_q = \mu_q - \bar{m}$$

- Meson octet

$$\text{Useful: } \tilde{M}^2 = M^2/X_\pi^2; \tilde{\alpha}(\bar{m}) = \alpha/M_{0\pi}^2, \dots$$

$$\begin{aligned} \tilde{M}^2(a\bar{b}) &= 1 + \tilde{\alpha}(\delta\mu_a + \delta\mu_b) \\ &\quad - \left(\frac{2}{3}\tilde{\beta}_1 + \tilde{\beta}_2\right)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{\beta}_1(\delta\mu_a^2 + \delta\mu_b^2) + \tilde{\beta}_2(\delta\mu_a - \delta\mu_b)^2 + \dots \end{aligned}$$

- Baryon octet

$$\text{Useful: } \tilde{M}^2 = M^2/X_N^2; \tilde{A}_i = A_i/M_{0N}^2, \dots$$

$$\begin{aligned} \tilde{M}_{\Sigma}^2(aab) &= 1 + \tilde{A}_1(2\delta\mu_a + \delta\mu_b) + \tilde{A}_2(\delta\mu_b - \delta\mu_a) \\ &\quad - (\tilde{B}_1 + \tilde{B}_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{B}_1(2\delta\mu_a^2 + \delta\mu_b^2) + \tilde{B}_2(\delta\mu_b^2 - \delta\mu_a^2) + \tilde{B}_3(\delta\mu_b - \delta\mu_a)^2 + \dots \\ \tilde{M}_{\Lambda}^2(aa'b) &= 1 + \tilde{A}_1(2\delta\mu_a + \delta\mu_b) - \tilde{A}_2(\delta\mu_b - \delta\mu_a) \\ &\quad - (\tilde{B}_1 + \tilde{B}_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{B}_1(2\delta\mu_a^2 + \delta\mu_b^2) - \tilde{B}_2(\delta\mu_b^2 - \delta\mu_a^2) + \tilde{B}_4(\delta\mu_b - \delta\mu_a)^2 + \dots \end{aligned}$$

- mixed sea/valence mass terms
- unitary limit: $\delta\mu_q \rightarrow \delta m_q$

Method

- Use PQ data to determine expansion coefficients
 - $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ – meson octet
 - $\tilde{A}, \tilde{B}, \tilde{C}$ – baryon octet
- Determine ‘physical’ quark masses

$$\underbrace{\delta m_u^*, \delta m_d^*}_{\delta m_l^*}, \quad \delta m_s^*, \quad \delta \mu_c^*$$

by fitting to (eg)

[FLAG3 where possible, minimise isospin effects]

$$M_\pi^{\text{exp}}, \quad M_K^{\text{exp}}, \quad M_{\eta_c}^{\text{exp}}(c\bar{c})$$

- Can then determine open charmed states

κ_0 (fine) tuning

- If miss (slightly) starting point on $SU(3)$ flavour symmetric line
- Tune using PQ results so that get physical values of (say)

$$M_{\pi}^{\text{exp}} \quad X_N^{\text{exp}} \quad M_K^{\text{exp}}$$

correct

- Gives

$$\delta\mu_l^* \quad \delta\mu_s^*$$

- Philosophy: most change is due to change in valence quark mass, rather than sea quark mass

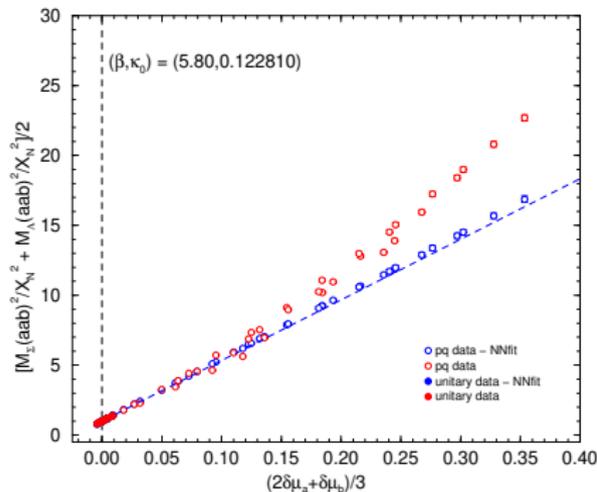
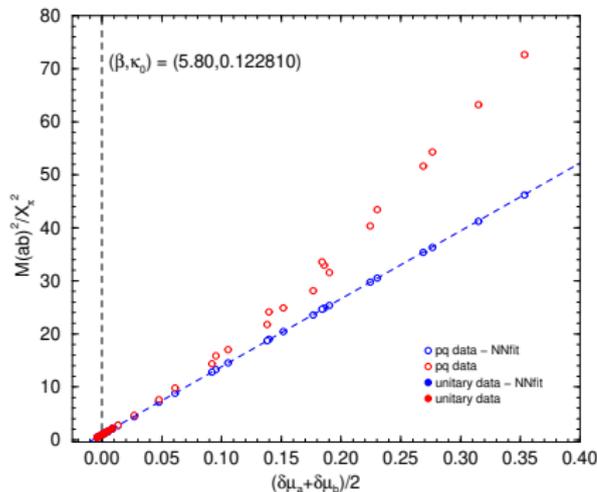
$$\delta\bar{\mu} \equiv (2\delta\mu_l^* + \delta\mu_s^*)/3 \neq 0 \text{ necessarily}$$

- Modifying dimensionless appropriately gives

$$\tilde{M}^2(a\bar{b}) \rightarrow \tilde{M}^2(a\bar{b})/c \quad c = \frac{X_{\pi}^{\text{LAT}^2}}{X_N^{\text{LAT}^2}} / \frac{X_{\pi}^{\text{exp}^2}}{X_N^{\text{exp}^2}}$$

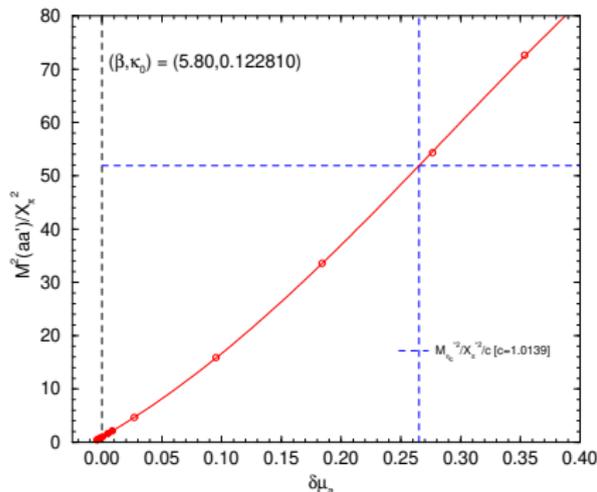
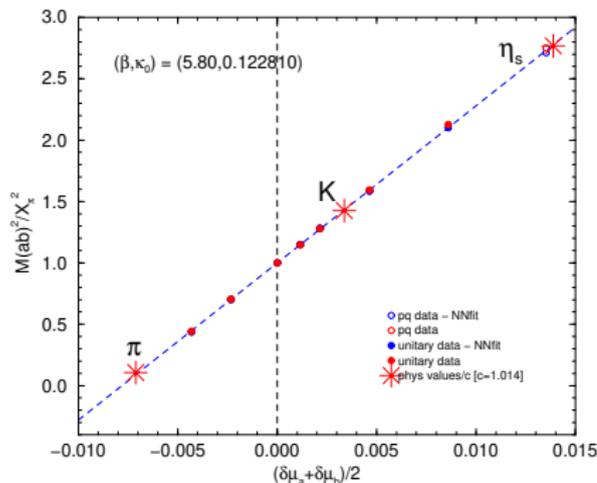
- $c = 1 + O(1\%)$, ie typically a $\sim 1\%$ effect

Determining the expansion coefficients



- PQ+unitary data
 - At LO, $\tilde{M}(a\bar{b})^2$ only function of $\delta\mu_a + \delta\mu_b$
 - To compare to fit, subtract NN terms, LO remains
 - Very different x-regions (for unitary/PQ data)
- PQ+unitary data
 - At LO, $(\tilde{M}_\Sigma(aab)^2 + \tilde{M}_\Lambda(aab)^2)/2$ only function of $2\delta\mu_a + \delta\mu_b$

Determining 'physical' quark masses



- blow-up of previous graph
- no visible curvature
- π , K to determine $\delta\mu_l^*$, $\delta\mu_s^*$
- Test: compare determined $\delta\mu_s^*$ with fictional $M_{\eta_s}(s\bar{s}) = 0.6885 \text{ GeV}$ [HPQCD] – little/no $O(a^2)$ effects

- Diagonal terms, with fit
- $\tilde{M}_{\eta_c}^2 \implies \delta\mu_c^*$

Open Charm masses

Can describe states with same wavefunction (and hence expansion) as previously used

- pseudoscalar mesons

$$\underbrace{D^0(c\bar{u}), D^+(c\bar{d})}_{D(c\bar{1})}, \quad D_s^+(c\bar{s})$$

which all have the wavefunction $\mathcal{M} = \bar{q}\gamma_5 c$ $q = u, d, s$

- baryons

- single open charm ($C = 1$) states

$$\underbrace{\Sigma_c^{++}(uuc), \Sigma_c^0(ddc)}_{\Sigma_c(lc)}, \quad \Omega_c^0(ssc)$$

which all have the wavefunction $\mathcal{B} = \epsilon(q^T C \gamma_5 c) q$ $q = u, d, s$

- double open charm ($C = 2$) states

$$\underbrace{\Xi_{cc}^{++}(ccu), \Xi_{cc}^+(ccd)}_{\Xi_{cc}(ccl)}, \quad \Omega_{cc}^+(ccs)$$

which all have the wavefunction $\mathcal{B} = \epsilon(c^T C \gamma_5 q) c$ $q = u, d, s$

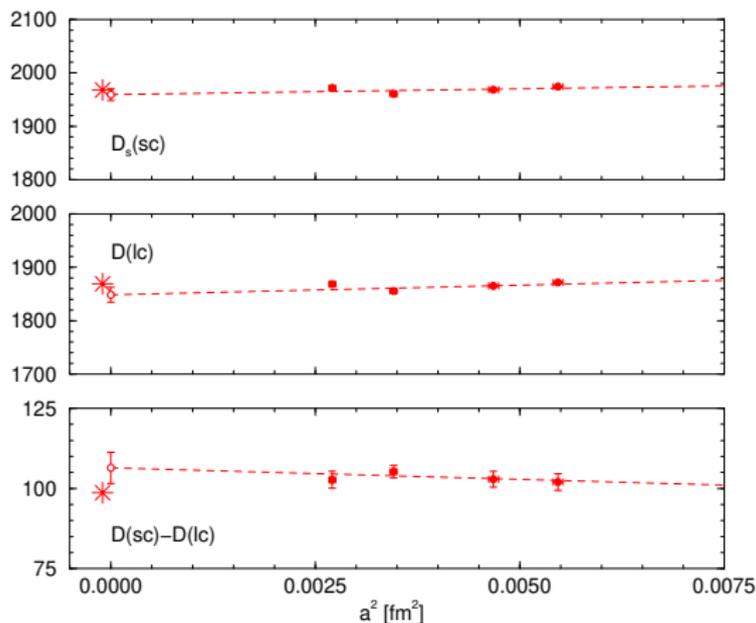
SU(4) 20-plet

Present status of results

[we presently only have $m_u = m_d = m_l$ results]

C	S	I	I_3	baryon	wavefunction
0	0	$\frac{1}{2}$	$+\frac{1}{2}$	p	$\epsilon(u^T C \gamma_5 d)u$
0	0	$\frac{1}{2}$	$-\frac{1}{2}$	n	$\epsilon(d^T C \gamma_5 u)d$
0	1	1	+1	Σ^+	$\epsilon(u^T C \gamma_5 s)u$
0	1	1	0	Σ^0	$\frac{1}{\sqrt{2}}\epsilon[(u^T C \gamma_5 s)d + (d^T C \gamma_5 s)u]$
0	1	1	-1	Σ^-	$\epsilon(d^T C \gamma_5 s)d$
0	2	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ^0	$\epsilon(s^T C \gamma_5 u)s$
0	2	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ^-	$\epsilon(s^T C \gamma_5 d)s$
0	1	0	0	Λ^0	$\frac{1}{\sqrt{6}}\epsilon[2(u^T C \gamma_5 d)s + (u^T C \gamma_5 s)d - (d^T C \gamma_5 s)u]$
1	0	1	+1	Σ_c^{++}	$\epsilon(u^T C \gamma_5 c)u$
1	0	1	0	Σ_c^+	$\frac{1}{\sqrt{2}}\epsilon[(u^T C \gamma_5 c)d + (d^T C \gamma_5 c)u]$
1	0	1	-1	Σ_c^0	$\epsilon(d^T C \gamma_5 c)d$
1	1	$\frac{1}{2}$	$+\frac{1}{2}$	$\Xi_c^{'+}$	$\frac{1}{\sqrt{2}}\epsilon[(s^T C \gamma_5 c)u + (u^T C \gamma_5 c)s]$
1	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\Xi_c'^0$	$\frac{1}{\sqrt{2}}\epsilon[(s^T C \gamma_5 c)d + (d^T C \gamma_5 c)s]$
1	2	0	0	Ω_c^0	$\epsilon(s^T C \gamma_5 c)s$
1	0	0	0	Λ_c^+	$\frac{1}{\sqrt{6}}\epsilon[2(u^T C \gamma_5 d)c + (u^T C \gamma_5 c)d - (d^T C \gamma_5 c)u]$
1	1	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ_c^+	$\frac{1}{\sqrt{6}}\epsilon[2(s^T C \gamma_5 u)c + (s^T C \gamma_5 c)u - (u^T C \gamma_5 c)s]$
1	1	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ_c^0	$\frac{1}{\sqrt{6}}\epsilon[2(s^T C \gamma_5 d)c + (s^T C \gamma_5 c)d - (d^T C \gamma_5 c)s]$
2	0	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ_{cc}^{++}	$\epsilon(c^T C \gamma_5 u)c$
2	0	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ_{cc}^+	$\epsilon(c^T C \gamma_5 d)c$
2	1	0	0	Ω_{cc}^+	$\epsilon(c^T C \gamma_5 s)c$

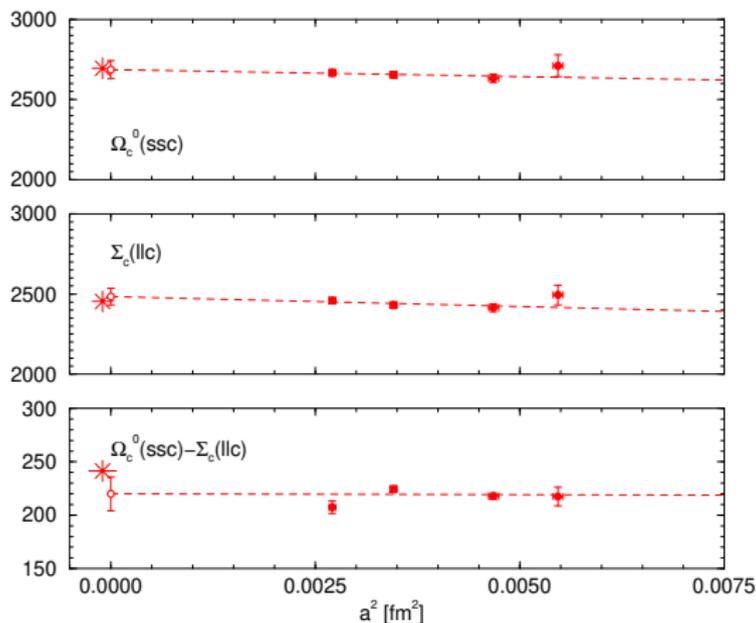
Charmed pseudoscalar mesons



- $D_s(c\bar{s})$ [ie $D_s^+(c\bar{s})$], $D(c\bar{l})$ [ie $D^0(c\bar{u})$, $D^+(c\bar{d})$]
- $D(c\bar{s}) - D(c\bar{l})$, ie $SU(3)$ flavour splitting effects
- **very** small lattice artifacts

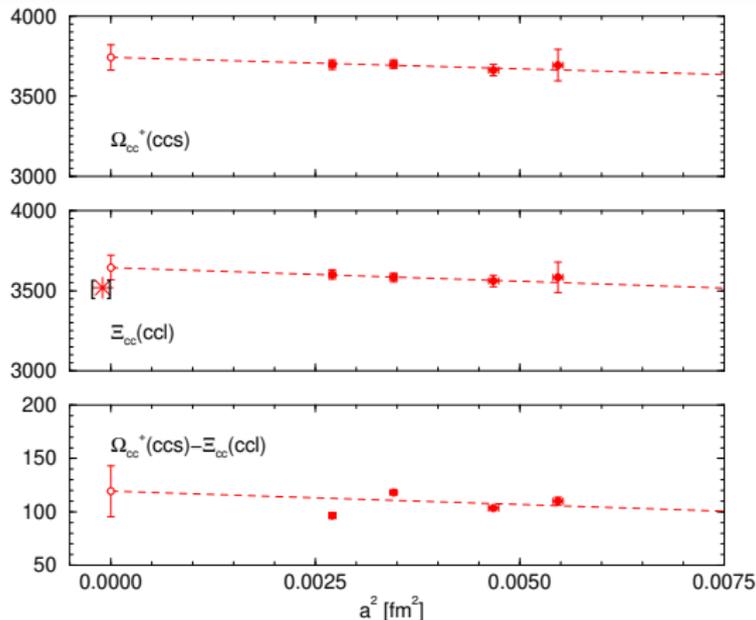
[+Dashen to minimise QED effects]

Charmed $C = 1$ baryons



- $\Omega_c(ssc)$, [ie $\Omega_c^0(ssc)$], $\Sigma_c(llc)$ [ie $\Sigma_c^{++}(uuc)$, $\Sigma_c^0(ddc)$]
- $\Omega_c(ssc) - \Sigma_c(llc)$, ie $SU(3)$ flavour splitting effects
- **very** small lattice artifacts

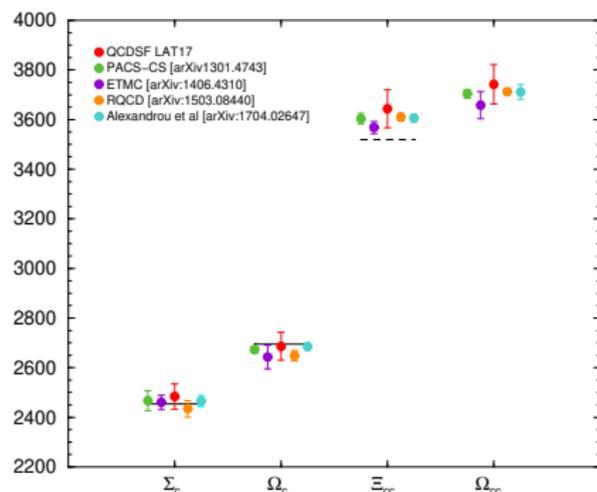
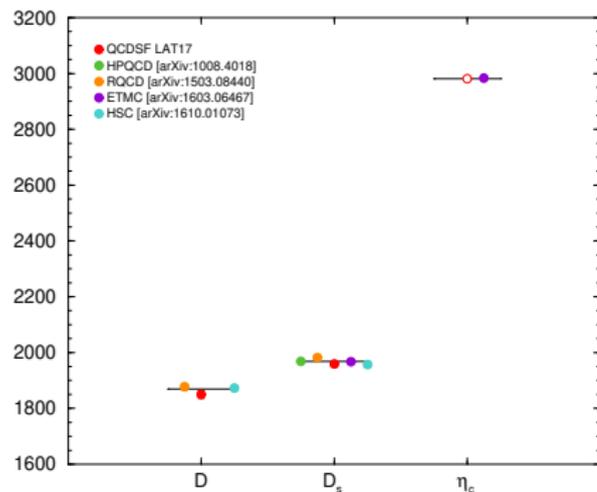
Charmed $C = 2$ baryons



- $\Omega_{cc}(ccs)$, [ie $\Omega_{cc}^+(ccs)$], $\Xi_{cc}(ccl)$ [ie $\Xi_{cc}^{++}(ccu)$, $\Xi_{cc}^+(ccd)$]
- $\Omega_{cc}(ccs) - \Xi_{cc}(ccl)$, ie $SU(3)$ flavour splitting effects
- [\star] SELEX
- **very** small lattice artifacts

[Inflate final error bar]

Comparison with (some) other results



Conclusions

- For u, d, s quarks, have developed a method to approach the physical point
- Precise $SU(3)$ flavour symmetry breaking expansions – nothing ad-hoc
- Extend expansions – PQ (mass valence quarks \neq mass sea quarks) to
 - better determine expansion coefficients
 - determine c quark mass
- Applied method to determine some open charm states.
 - $O(a^2)$ extrapolations rather delicate
- Future:
 - Immediate: Λ_c, N -octet
 - Extend to $m_u \neq m_d$
 - mixing:
 - in a $2 + 1$ world no $\Sigma^0 - \Lambda^0$ mixing, but determined coefficients can be used to determine $\Sigma^0(uds) - \Lambda^0(uds)$ mixing
 - generalise to eg $\Sigma_c^+ - \Lambda_c^+, \Xi_c^0 - \Xi_c^{\prime 0}$ mixing
 - baryon decuplet
 - QED effects