

# Progress in the improved lattice calculation of direct CP-violation in the Standard Model

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IN THE CITY OF NEW YORK

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# Motivation for studying $K \rightarrow \pi\pi$ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in  $K^0 \rightarrow \pi\pi$ :

$$\eta_{00} = \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)}, \quad \eta_{+-} = \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)}.$$

$$\text{Re}(\epsilon'/\epsilon) \approx \frac{1}{6} \left( 1 - \left| \frac{\eta_{00}}{\eta_{\pm}} \right|^2 \right) = 16.6(2.3) \times 10^{-4} \quad (\text{experiment})$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states:  $\Delta I=3/2$  decay to  $I=2$  final state, amplitude  $A_2$   
 $\Delta I=1/2$  decay to  $I=0$  final state, amplitude  $A_0$

$$A(K^0 \rightarrow \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2e^{i\delta_2},$$

$$A(K^0 \rightarrow \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2e^{i\delta_2}.$$



$$\epsilon' = \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \left( \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right)$$

( $\delta_i$  are strong scattering phase shifts.)

$$\omega = \text{Re}A_2/\text{Re}A_0$$

- Small size of  $\epsilon'$  makes it particularly sensitive to new direct-CPV introduced by many BSM models.

# Lattice Calculation

- First calculation performed using 216 configurations of  $32^3 \times 64$  Mobius DWF ensemble ( $L_s=12$ ) with physical pions  $m_\pi=143.1(2.0)$  MeV.
- Iwasaki+DSDR gauge action ( $\beta=1.75$ )  $\Rightarrow$  coarse lattice spacing  $a^{-1}=1.378(7)$  GeV but large, 4.6fm box - vital for controlling FV effects.
- Use G-parity boundary conditions (GPBC) in 3 directions to match  $\pi\pi$  and kaon energies, ensuring physical interaction without multi-state fits.
- Use Trinity-style all-to-all propagators : 900 exact low-modes (Lanczos) and stochastic approximation to high mode component.
  - Allows usage of “physical” sources - specifically 1s hydrogen wavefunction with radius 2 - to minimize overlap with vacuum
  - Maximal sampling of source/sink and operator locations.
- Operators non-perturbatively renormalized using RI-SMOM scheme and matched to  $\overline{\text{MS}}$  via 1-loop PT.
- Basis includes:
  - 7 independent dim-6 four-quark operators  $Q_1'-Q_7'$
  - All relevant dim-3 and dim-4 operators.
  - **The single two-quark dim-6 operator  $G_1$ , which mixes at  $O(\alpha)$ . [NEW]**
- Neglecting two other dim-6 operators that mix at  $O(\alpha^2)$ .
- Neglecting dim-5 chirality-breaking operators that come with coefficient of quark mass and thus expected to be small.

## Results for $\varepsilon'$

- $\text{Re}(A_0)$  and  $\text{Re}(A_2)$  from expt.
- Lattice values for  $\text{Im}(A_0)$ ,  $\text{Im}(A_2)$  and the phase shifts,

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \text{Re} \left\{ \frac{i\omega e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\varepsilon} \left[ \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right] \right\}$$

$= 1.38(5.15)(4.43) \times 10^{-4}$	(this work)
$16.6(2.3) \times 10^{-4}$	(experiment)

- Total error on  $\text{Re}(\varepsilon'/\varepsilon)$  is  $\sim 3x$  the experimental error.
- Find reasonable consistency with Standard Model (at  $2.1\sigma$  level).
- Tantalizing hint of discrepancy strong motivation for continued study!
- Error is dominated by that on  $A_0$ .

Our main focuses are therefore to:

- Increase statistics on  $A_0$  calculation, enabling improved precision and better systematic error estimation.
- Reduce dominant systematic errors, particularly NPR and finite lattice spacing.

# Statistics increase

- Aim for 4x increase in # of measurements from original 216 by the end of year.

## Configuration generation

- Achieve goal by generating multiple independent streams originating from widely separated configurations of original ensemble.
- On 512-nodes of BG/Q using the highly-tuned CPS/BFM/Bagel libraries we could formerly generate 1 trajectory every 7.7 hours.
- Time is large due to 2x from GPBC Dirac op being explicitly 2-flavor, plus additional slowdown caused by necessity to use RHMC to take root of light-quark  $M^\dagger M$  determinant to obtain 2f action.
- We have implemented and improved TWQCD's "exact one-flavor action", a formulation of DWF that reproduces 1f determinant (2f with GPBC) without rooting. [Y-C Chen, T-W Chiu, arXiv:1403.1683] [D.J. Murphy arXiv:1611.00298]
- Achieved 4.2x speed-up of  $32^3$  evolution by combining EOFA with mixed-precision CG and multiple Hasenbusch splittings now possible due to decreased linalg overheads  $\Rightarrow$  1.8 hours per trajectory on 512-nodes BG/Q!

[cf. D.Murphy's talk, Tuesday]

- Autocorrelation length  $\lesssim 4$  MD time units, choose 4 as separation between indep. configs.
- Allow 20 trajectories ( $\sim 5x$  autocorrelation length) to ensure streams independent from origin configuration.

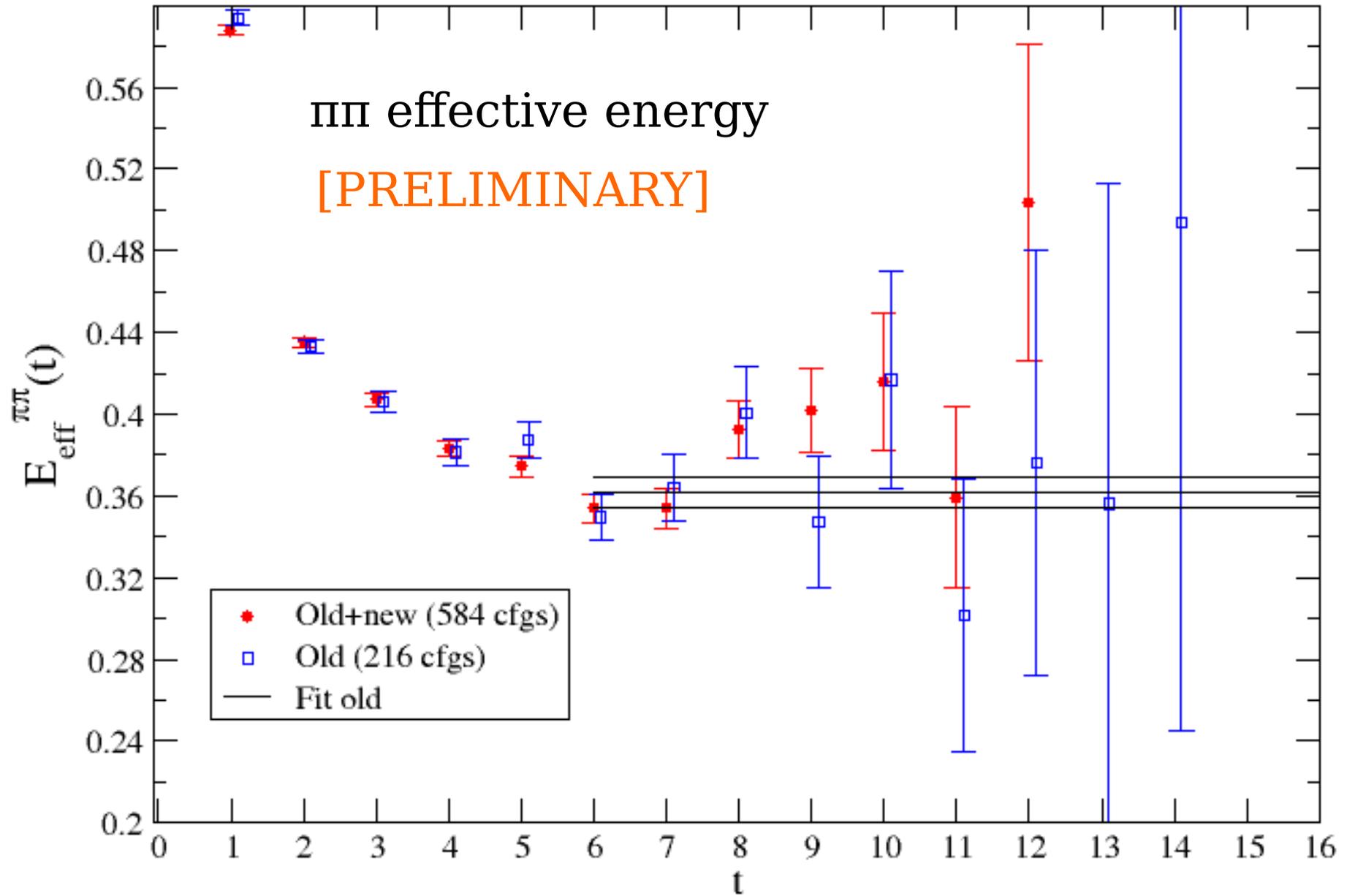
Source	Determinant computation	Trajectories	Independent configs.
Blue Waters	RHMC	116+69+37+32	24+12+4+3
KEKSC	RHMC	441	105
BNL	RHMC	854	208
DiRAC	RHMC	309	72
KEKSC	EOFA	557+344	134+81
BNL	EOFA	871	212
Mira	EOFA	157	34
			889 total

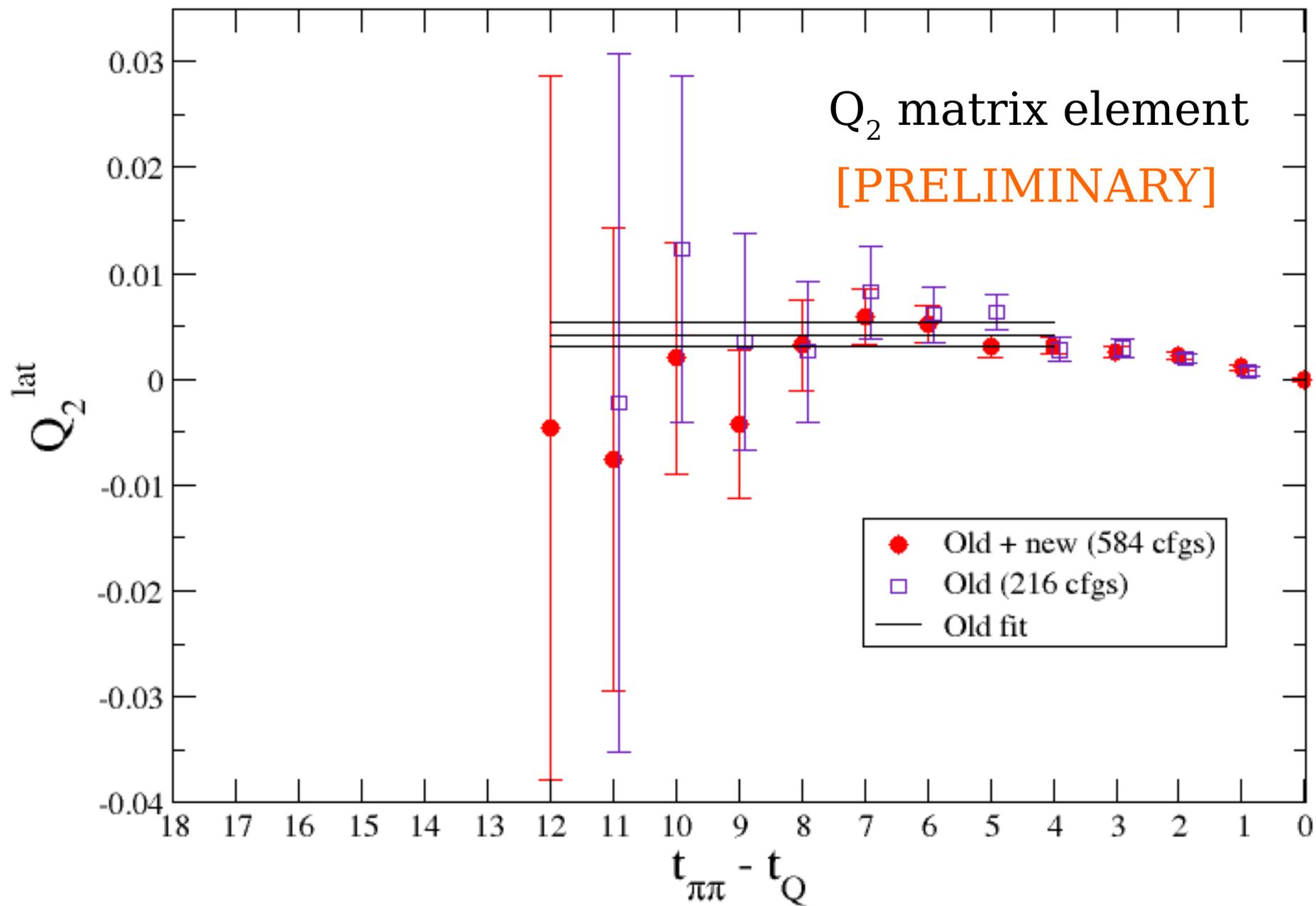
- **Very close to achieving goal of 4x increase (surpassed if include original data).**

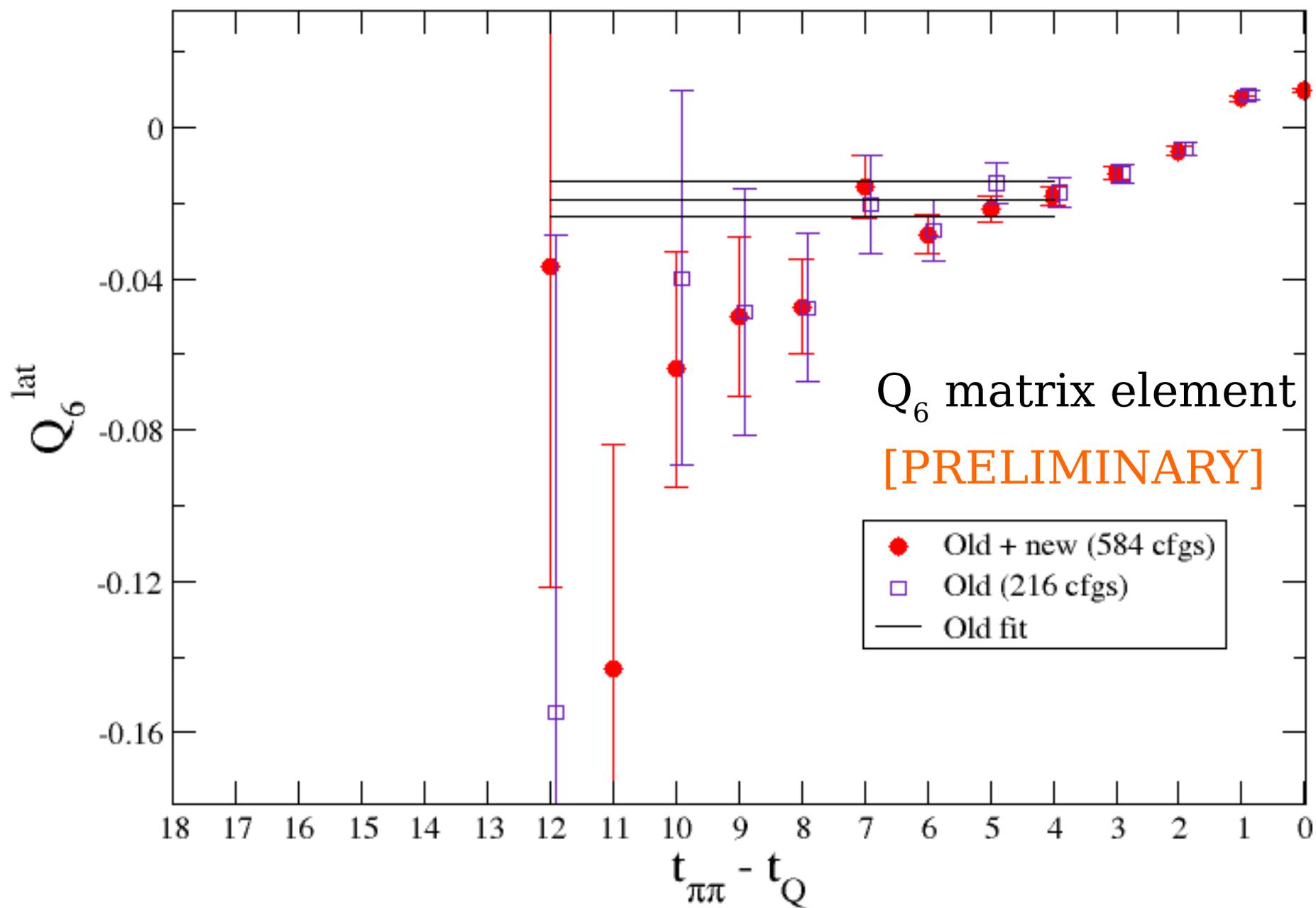
## Measurements

- Since publication we have completely re-implemented and substantially improved performance of CPS measurement code.
- Can run with BFM/Bagel for optimal BG/Q performance and now also with Grid for high performance on modern SIMD machines (Intel KNL [AVX512], Cray XC40 [AVX2]) for Lanczos and CG.
- Contraction code performance improvements include:
  - SIMD lattice decomposition of A2A vectors, particularly important for AVX512.
  - Hand-coded AVX512 assembly kernel for “meson field” contraction, achieving 400+ Gflops/node on KNL.
  - Optimized work distribution over nodes and threads of matrix operations, hooks for performant matrix algebra libraries (MKL, ATLAS).
  - Optimized parallel FFT.
  - Flexible structure allowing easy adaptation to memory constraints (v. important for BG/Q).
  - Large programme of general optimization.
- Measurements being performed on  
Cori II (Intel KNL + Cray Aries “Dragonfly” network),  
Cori I (Cray XC40 + Aries),  
BNL SDCC (Intel KNL + dual-rail Omnipath)  
IBM BG/Q
- 352 new measurements produced thus far! (06/07)

216 (old) + 368 (new) = 584 cfgs







# Systematic error improvements

Description	Error	Description	Error
Finite lattice spacing	12%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics $\leq 3\%$		Lellouch-Lüscher factor	11%
Total (added in quadrature)			27%

[RBC&UKQCD PRL 115 (2015) 21, 212001]

## NPR+Wilson Coefficients

- NPR error large due to use of 1-loop PT to match to  $\overline{\text{MS}}$  at low, 1.53 GeV renormalization scale.
- Previously reported NPR error 12%  $\rightarrow$  8% (preliminary) by increasing scale to 2.29 GeV using step-scaling procedure. [CK, Lattice 2016]
- Inclusion of  $G_1$  operator which mixes with  $Q_i$ , effects demonstrated to be percent-scale as expected. [G. McGlynn arxiv:1605.08807] [CK, Lattice 2016]
- Do not expect significant improvement in Wilson coeffs. error as dominated by use of PT to cross the charm threshold (1.29 GeV).
- Working on circumventing this by computing 3 $\rightarrow$ 4 flavor matching non-perturbatively. [M. Tomii @ Columbia]
- Requires  $\mu \ll m_c$ . At these low energies, MOM-scheme NPR severely hampered by increased mixing with tower of gauge-noninvariant operators.
- Circumvent using position-space NPR which does not require gauge fixing.

## Finite lattice-spacing errors

- Currently have results only on single lattice with coarse lattice spacing  $a^{-1}=1.38(1)$  GeV.
- Finite-lattice spacing errors estimated by comparing  $A_2$  results on near-identical (non-Gparity) lattice to our continuum results.
- Fine lattice spacing repeat of  $A_0$  calculation unfeasible with current gen. hardware. Instead consider a **coarser** lattice.
- For other applications we have generated an ensemble with the following parameters (without Gparity) that is an ideal candidate:

[cf. R. Mawhinney's talk, Tuesday]

  - $24^3 \times 64$  Ls=24 Mobius DWF + IDSDR ensemble
  - $\beta=1.633 \Rightarrow a^{-1} \sim 1.0$  GeV
  - 4.7fm box almost identical to size of  $32^3$  lattice : GPBC in 3 dirs (may need slight tuning in  $\beta$ ).
  - Physical pion mass  $\sim 140$  MeV
- Strong evidence from large number of observables and ensembles that DSDR action scales extremely well - observed scaling errors  $O(2\%)$  for  $f_\pi$ ,  $w_0$  etc on 1 GeV ensemble.
- Will begin tuning a corresponding ensemble with G-parity BCs shortly.

# Resolving the pi-pi puzzle

- Our result for I=0 pi-pi scattering phase shift  $\delta_0=23.8(4.9)(1.2)^\circ$  lower than the value of  $38.3(1.3)^\circ$  obtained by combining expt. data with Roy Equations  
[RBC&UKQCD PRL 115 (2015) 21, 212001] [G.Colangelo, priv. comm.]
- Matching pheno. requires  $E_{\pi\pi} \sim 470$  MeV vs. our 498(11) MeV.
- Possibility of nearby excited state unresolvable from ground-state pi-pi. (The presence of such a pair of states would also be inconsistent with pheno.)
- To address and to improve fitting in general, have expanded measurement programme, introducing new operators that also couple to I=0 pi-pi:
  - Added scalar bilinear ( $\sigma$ -meson) operator. In continuum  $\sigma$  is resonant state of the I=0 pi-pi system. In finite-volume is a stable state that mixes with pi-pi.
  - $\sigma$ - $\sigma$ ,  $\sigma$ - $\pi\pi$  matrix elements with a number of operator choices soon to be included in fits.
  - Alongside 1s hydrogen-wavefunction pion source smearing, have added 2s form. Alternative coupling to radial excited states of pion and sigma.
- Combining these new handles on the pi-pi state and modern multi-operator methods such as GEVP will allow us to obtain the finite-volume spectrum in more detail.

[T. Wang @ Columbia]

## Related projects on the horizon:

- Performing calculation taking advantage of modern multi-operator techniques to fit excited-state  $\pi\pi$  contributions directly, without G-parity BCs.  
[Blum, Hoying @ UConn]
- Laying the groundwork for non-perturbatively computing the effects of isospin breaking and electromagnetism.  
[cf. N. Christ's talk, earlier today]
- Study of complete, non-perturbative calculation of Wilson coefficients  
[cf. M. Bruno, Friday 17:30]

# Conclusions and outlook

- Substantial progress made towards 4x increase in statistics combining large-scale running with algorithmic and computational improvements.
- A number of complementary projects intending to reduce systematic errors and/or support this work:
  - › Many new pi-pi operators to improve energy and phase-shift determination and resolve apparent discrepancy with phenomenological result.
  - › Step-scaling from 1.53 GeV to 2.29 GeV decreases renormalization systematic by  $\sim 35\%$ . Considering taking this further.
  - › Inclusion of  $G_1$  operator in NPR.
  - › Attempt calculation of  $3f \rightarrow 4f$  matching using position-space NPR.
  - › Second lattice spacing with coarser,  $a^{-1}=1$  GeV to enable continuum limit.
  - › Independent calculation using periodic BCs and high-statistics alongside modern multi-operator methods.
  - › Investigation into future determination of EM and isospin-breaking effects.
- We aim to publish new results by the end of 2017.

Thank you