High precision determination of w_0

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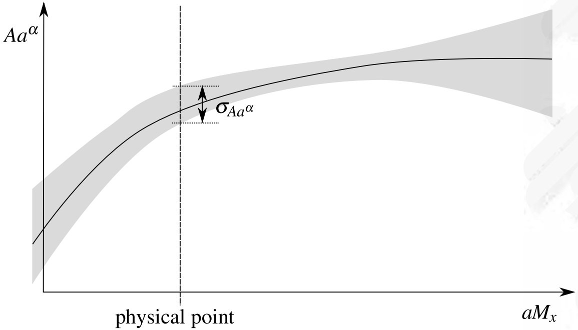
Introduction

High precision scale setting is important for any high precision calculation on the lattice, especially for dimensionfull quantities.

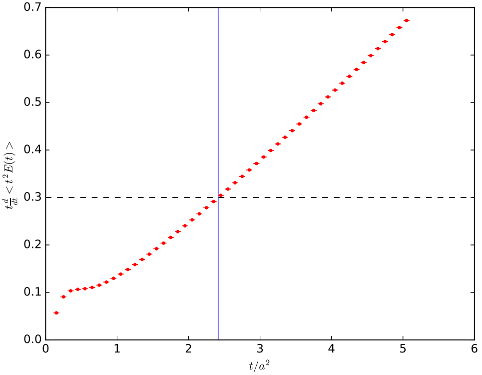


Scale setting uncertainty appears in several ways:

- Definition of the physical point
- Translation of the result in MeV.
- ...

Introduction

Very promising for high precision scale setting: w_0.



- Apply Wilson flow to "smooth out" the gauge fields and bring them closer to the classical solution. [1]
- Monitor the action density.
- Define w_0 to be the value of $\sqrt{t}$ where $t \frac{\mathrm{d}}{\mathrm{d}t} \langle t^2 E(t) \rangle = 0.3$. [2]
- Closely related to t_0. [1]

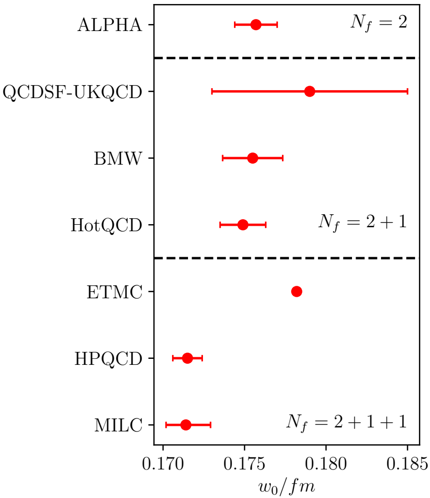
[1] M. Lüscher, "Properties and uses of the Wilson flow in lattice QCD," JHEP **1008** (2010) 071 Erratum: [JHEP **1403** (2014) 092] [arXiv:1006.4518 [hep-lat]].
[2] S. Borsanyi *et al.*, "High-precision scale setting in lattice QCD," JHEP **1209** (2012) 010 [arXiv:1203.4469 [hep-lat]].

Introduction

w_0 can not be determined expertimentally. $\Rightarrow$ It has to be determined once on the lattice.

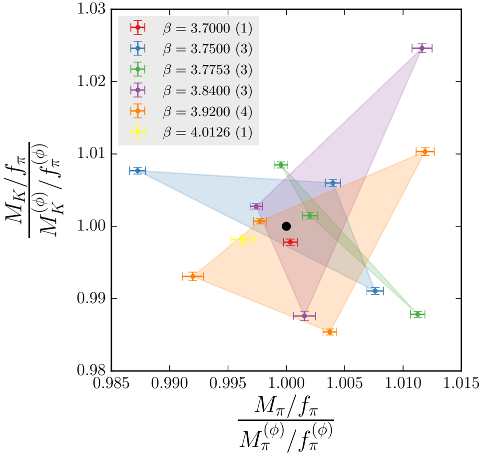
In this talk I will present an ongoing effort to determine w_0 with high precision in a blind analysis.

[ALPHA] M. Bruno *et al.* [ALPHA Collaboration], PoS LATTICE **2013** (2014) 321 [arXiv:1311.5585 [hep-lat]].
[QCDSF-UKQCD] R. Horsley *et al.* [QCDSF-UKQCD Collaboration], PoS LATTICE **2013** (2014) 249 [arXiv:1311.5010 [hep-lat]].
[BMW] S. Borsanyi *et al.*, "High-precision scale setting in lattice QCD," JHEP **1209** (2012) 010 [arXiv:1203.4469 [hep-lat]].
[HotQCD] A. Bazavov *et al.* [HotQCD Collaboration], Phys. Rev. D **90** (2014) 094503 [arXiv:1407.6387 [hep-lat]].
[ETMC] A. Deuzeman and U. Wenger, PoS LATTICE **2012** (2012) 162.
[HPQCD] R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, Phys. Rev. D **88** (2013) 074504 [arXiv:1303.1670 [hep-lat]].
[MILC] A. Bazavov *et al.* [MILC Collaboration], Phys. Rev. D **93** (2016) no.9, 094510 [arXiv:1503.02769 [hep-lat]].



(Plot based on [MILC])

Lattice ensembles



The analysis is based on 18 ensembles at 6 different gauge couplings.

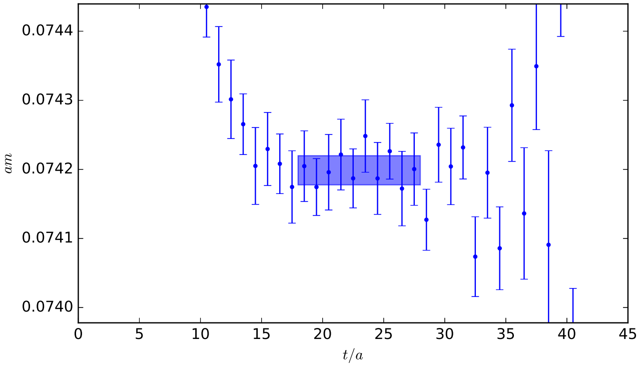
The ensembles are generated with a $N_f = 2+1+1$ staggered fermion action on stout-smeared gauge configurations. The gauge action is a tree-level Symmanzik improved action.

The ensembles are tuned to bracket the physical point (determined via f_π).

Masses and decay constants

Masses of pions and kaons are extracted by fitting appropriate correlators by staggered standard anstzes:

$$C_\pi(t) = A\cosh(-M_a(t - {}^{N_t}/_2))$$
$$C_K(t) = A\cosh(-M_a(t - {}^{N_t}/_2)) + B(-1)^t\cosh(-M_b(t - {}^{N_t}/_2))$$



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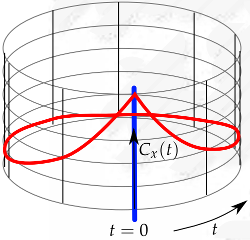
pion decay constant is defined via

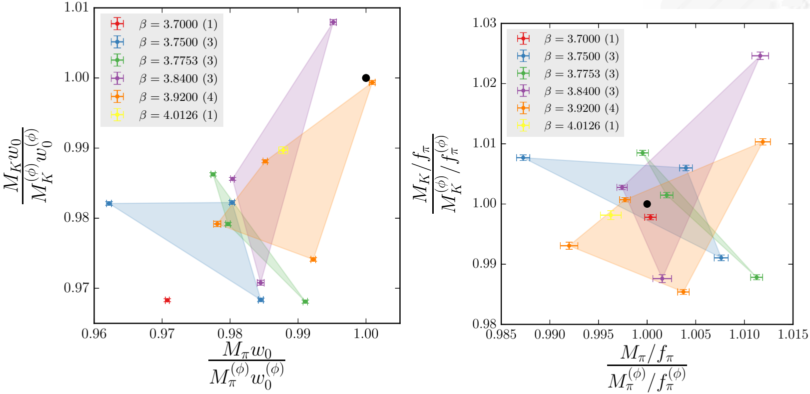
$$(af_\pi)^2 = \frac{A}{2M_a^3}\frac{1-\exp\left(-N_t M_a\right)}{\exp\left(-\frac{N_t}{2}M_a\right)}$$

The $1-\exp\left(-N_t m\right)$ accounts for contribution wrapping multiple times around the lattice.



Analysis strategy

Two different strategies have been followed:



f_π based analysis

Define the four quantities

$$x_1 = M_\pi / f_\pi - x_1^{\text{target}} \,, \quad x_2 = M_K / f_\pi - x_2^{\text{target}} \,, \quad x_3 = a^2 f_\pi^2 \,, \quad x_4 = f_\pi w_0$$

where x_i^{target} is the respective quantity at the physical point.

Fit the data with the following ansatz:

$$x_4 = c_{00} f_\pi^{(\Phi)} + c_{10} x_1 + c_{20} x_2 + c_{30} x_3 + c_{12} x_1 x_2 + c_{23} x_2 x_3 + c_{13} x_1 x_3 + c_{11} x_1^2 + c_{22} x_2^2 + c_{33} x_3^2$$

Always enable the c_{00} (w_0) and the c_{30} (lattice spacing) term. Switch other terms on and off to estimate systematic error.

w_0 based analysis

Define the four quantities

$$x_1 = M_\pi w_0 - x_1^{\text{target}} \,, \quad x_2 = M_K w_0 - x_2^{\text{target}} \,, \quad x_3 = \frac{a^2}{w_0^2} \,, \quad x_4 = f_\pi w_0$$

where x_i^{target} is the respective quantity at the physical point.

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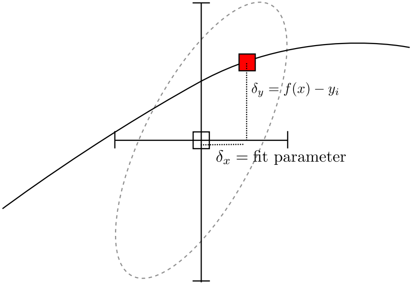
Physical value of w_0 is not known prior to the analysis $\Rightarrow$ Replace it self-consistently with the value from the fit:

$$x_1 = M_\pi w_0 - M_\pi^{(\Phi)} c_{00} \,, \quad x_2 = M_K w_0 - M_K^{(\Phi)} c_{00} \,, \quad x_3 = \frac{a^2}{w_0^2} \,, \quad x_4 = f_\pi w_0$$

Correlated fit



For each ensemble there are several quantities with a statistical error.
All of these quantities are correlated.
Consider 2d case:

Calculate χ^2 via

$$\vec{\delta} = \begin{pmatrix} f(x_i) - y_i \\ \delta_{x,i} \end{pmatrix} \quad \chi^2 = \sum_i \vec{\delta}^T C^{-1} \vec{\delta}$$

where C is the covariance matrix.

Generalizes to the case of several channels.

Estimation of errors

Statistical errors are estimated by a bootstrap procedure.

Systematic Errors are estimated by the Histogram method [1,2]: Several correct analysis are performed and a histogram is constructed. The spread of the histogram is an estimate for the systematic error.

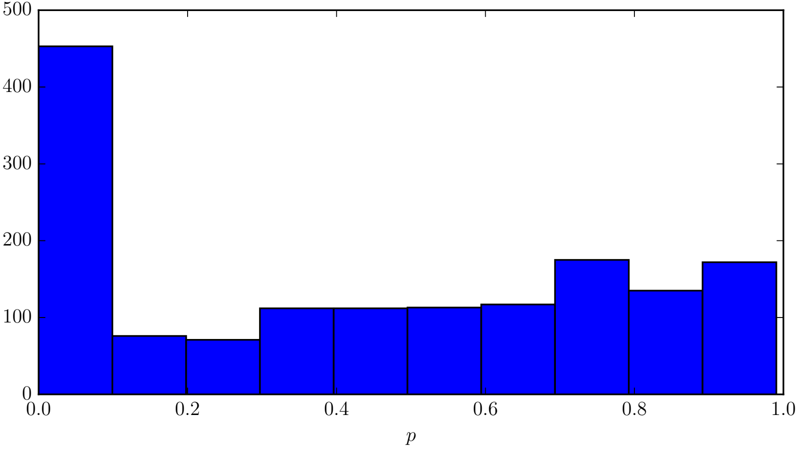
We have altogether 1536 analyses:

- 2^8 for the variation of the fit parameters
- 2 for the two different approaches
- 3 for the removing non, the coarsest and the two coarsest lattice spacings.

[1] S. Durr *et al.*, "Ab-Initio Determination of Light Hadron Masses," Science **322** (2008) 1224 [arXiv:0906.3599 [hep-lat]].
[2] S. Borsanyi *et al.*, "Ab initio calculation of the neutron-proton mass difference," Science **347** (2015) 1452 [arXiv:1406.4088 [hep-lat]].

Fit quality

Check: Are fit qualities ok?



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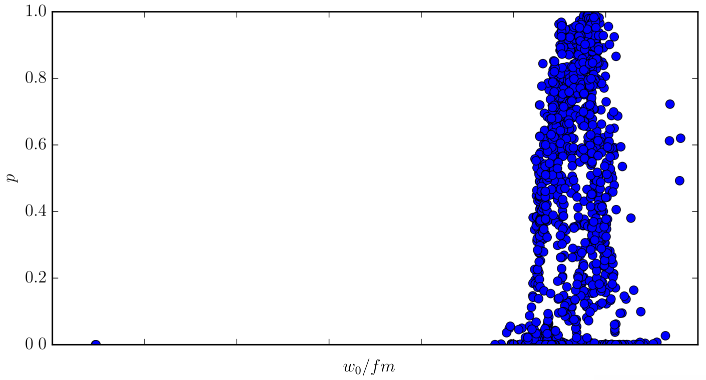
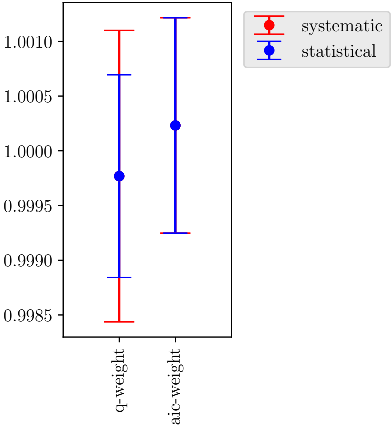


Except one outlier with very poor fit quality a sensible distribution.

Results



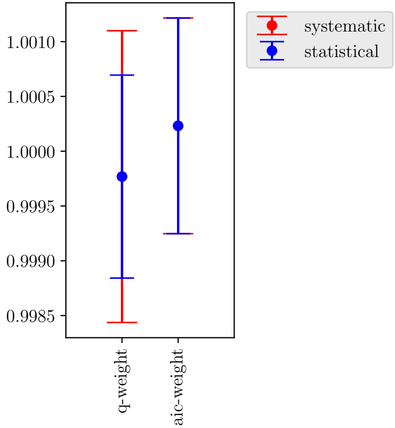
preliminary

Histograms where weighted by either the fit quality or by the aic weight.

Errors due to the finite lattice extent have not yet been fully included.

Final values can not be shown to allow for an independent crosscheck.

Results



preliminary

Experimental value from PDG for f_π is: 130.41(0.20) MeV. This has a 0.15% uncertainty. $\Rightarrow$ Determination of w_0 based on f_π can not be more accurate.

Plan is to replace f_π by the Ω-mass.

Conclusion

I have presented a ongoing analysis aiming to determine w_0 with high precision.

The available data seems to allow for high precision.

Finite lattice extend effects are not fully included.

f_π should be replaced by the Ω-mass due to experimental uncertainty $\Rightarrow$ work in progress.

Thank you for your attention!