Energy-momentum tensor correlation function in Nf=2+1 full QCD at finite temperature

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for
WHOT QCD collaboration

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Our motivation

Energy momentum tensor ⟺ Poincare symmetry

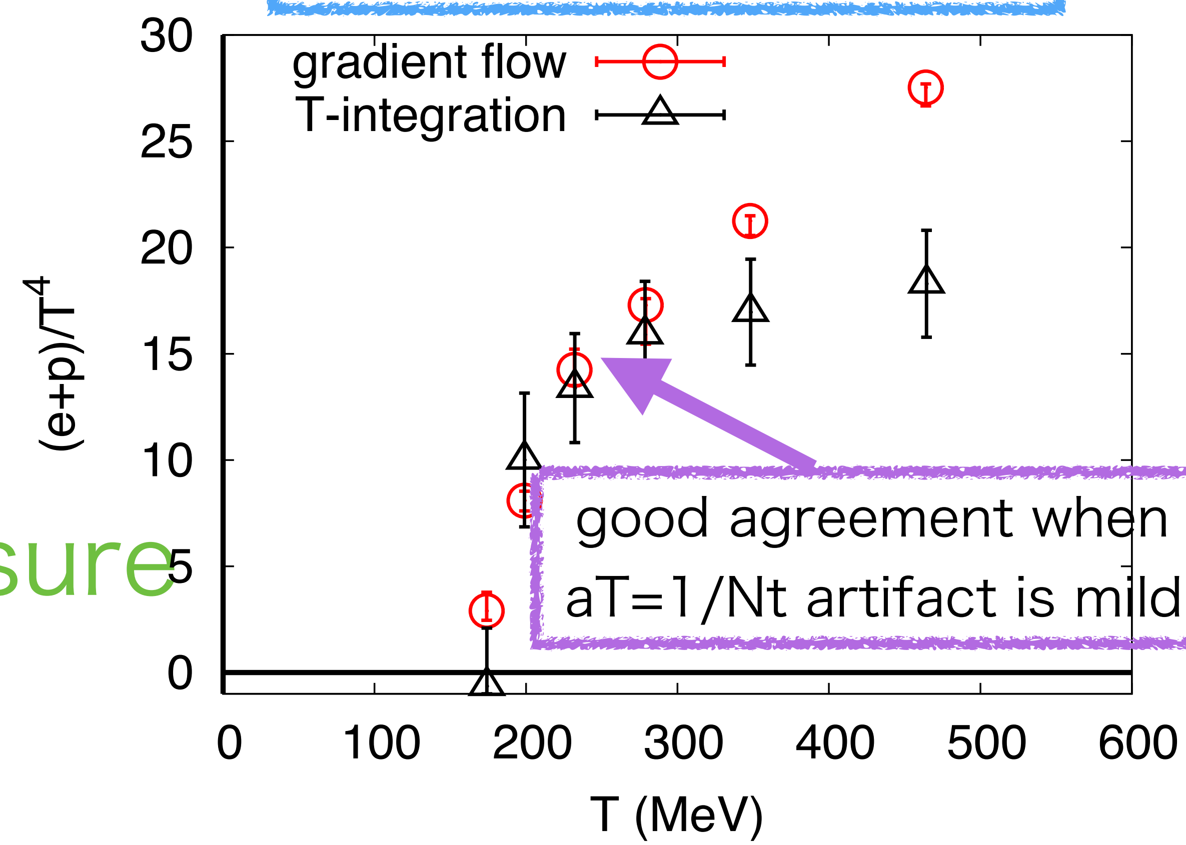
$T_{\mu\nu}$

energy momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress pressure

entropy density



- If we have $T_{\mu\nu}$

➡ direct measurement of thermodynamic quantity

Our topics of Lattice 2016

Successful for 1 point function

Our motivation

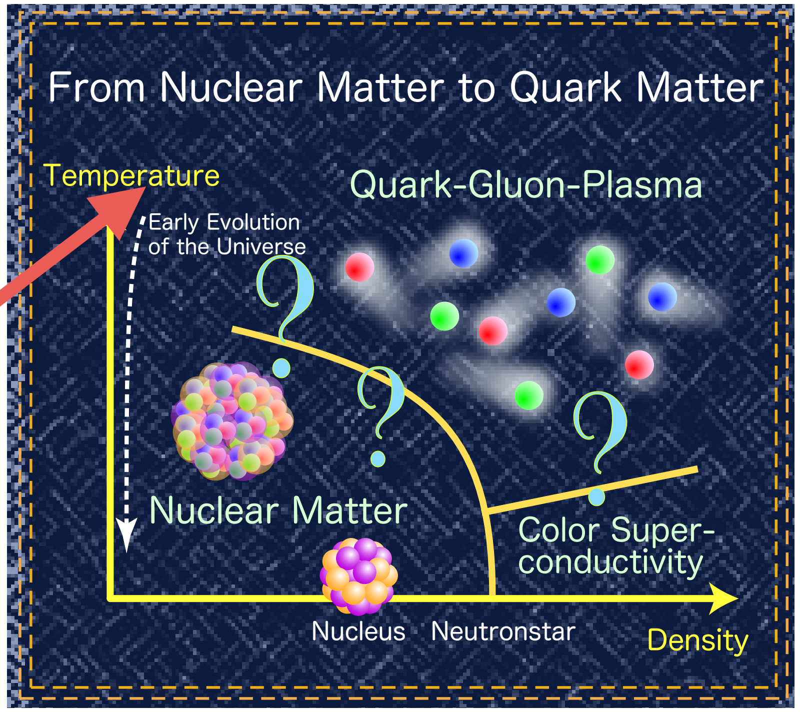
Energy momentum tensor ⟺ Poincare symmetry

$T_{\mu\nu}$

energy momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

stress pressure



hot topics in QGP

- If we have $T_{\mu\nu}$
  - Fluctuations and correlations of $T_{\mu\nu}$

➡ specific heat, viscosity, $\cdots$

Our target of this summer!

How to calculate $T_{\mu\nu}$ on lattice?

- Measure operators on lattice

terms in QCD Lagrangian

$\delta_{\mu\nu} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x)$ $\delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x)$ $\delta_{\mu\nu} \bar{\psi}(x) \psi(x)$

terms in QCD Lagrangian when trace is taken

$F^a_{\mu\rho}(x) F^a_{\nu\rho}(x)$ $\bar{\psi}(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi(x)$

- Renormalization

Well established for E and P

Karsch coefficients

problems

- non universal (No Poincare symmetry)
  - depends on: lattice action, operator
- additive correction for $\delta_{\mu\nu} \bar{\psi}(x) \psi(x)$

How to calculate T$_{\mu\nu}$ on lattice?

Easier method for renormalization?

Gradient Flow as a renormalization scheme

Narayanan-Neuberger(2006), Lüscher(2010), Lüscher-Weisz(2011)

Gauge operators with flowed field $A_\mu(t,x)$

- does not have UV divergence
- operators are renormalized

scale: $\sqrt{8t}$

visit t<<1/Λ_{QCD}^2 region non-perturbatively

easier than SSF

NP renormalized operator

gradient flow

lattice operator

universal

finite ren.

$F^a_{\mu\nu}F^a_{\mu\nu}(x,t)$

Tr<1− □>

take a→0 limit safely

$\overline{\text{MS}}$ scheme

Matching coefficients are calculable perturbatively

How to calculate $T_{\mu\nu}$ on lattice?

Five steps to calculate $T_{\mu\nu}$

1. Flow the gauge link and quark fields

2. Calculate expectation value of flowed operators

3. Multiply the matching coefficients

4. Take a→0 limit

5. Take t→0 limit

H.Suzuki, PTEP 2013, 083B03 (2013)

Makino-Suzuki, PTEP 2014, 063B02 (2014)

two reasons

Solve an operator mixing

$$\{T_{\mu\nu}\}(x,t,a) = \{T_{\mu\nu}\}_{\rm WT}(x)$$ ~~+t(dim6 operator)~~

Use of perturbative matching coefficients

What to calculate this summer?

2 point correlation function of fluctuation

$$C_{\mu\nu;\rho\sigma}(t;x_0) = \frac{1}{T^5}\int_{V_3} d^3x \left(\langle \delta T_{\mu\nu}(t;x_0,\vec{x})\delta T_{\rho\sigma}(t;0)\rangle\right)$$

fluctuation: $\delta T_{\mu\nu}(t;x) = T_{\mu\nu}(t;x) - \langle T_{\mu\nu}(t;x)\rangle$

Highlights?

- Conservation law $\frac{d}{dx_0}C_{0\nu;\rho\sigma}(x_0) = 0$
- Restoration of spatial rotation symmetry

$$C_{ij;kl}(x_0) = c_1(x_0)\delta_{ij}\delta_{kl} + c_2(x_0)\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$$

- Linear response relations

$$C_{0i;0i} = C_{00;ii} = -\frac{\epsilon + p}{T^4} \qquad C_{00;00} = \frac{c_V}{T^3}$$

Technical details of correlation function

- Calculate correlations among five operators

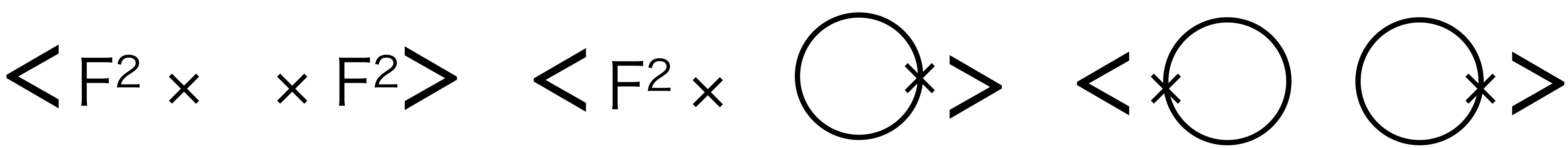
Operators in QCD Lagrangian

$$\delta_{\mu\nu} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x) \qquad \delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x) \qquad \delta_{\mu\nu} \bar{\psi}(x) \psi(x)$$

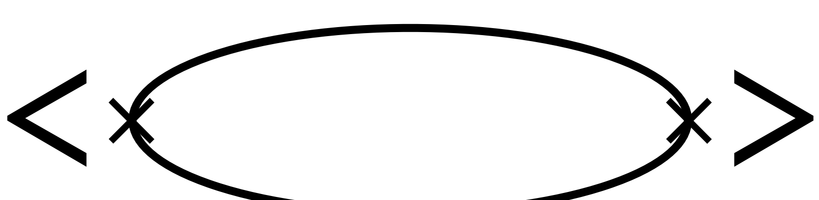
Operators in QCD Lagrangian when trace is taken

$$F^a_{\mu\rho}(x) F^a_{\nu\rho}(x) \qquad \bar{\psi}(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi(x)$$

☆Disconnected contributions



☆Connected contributions



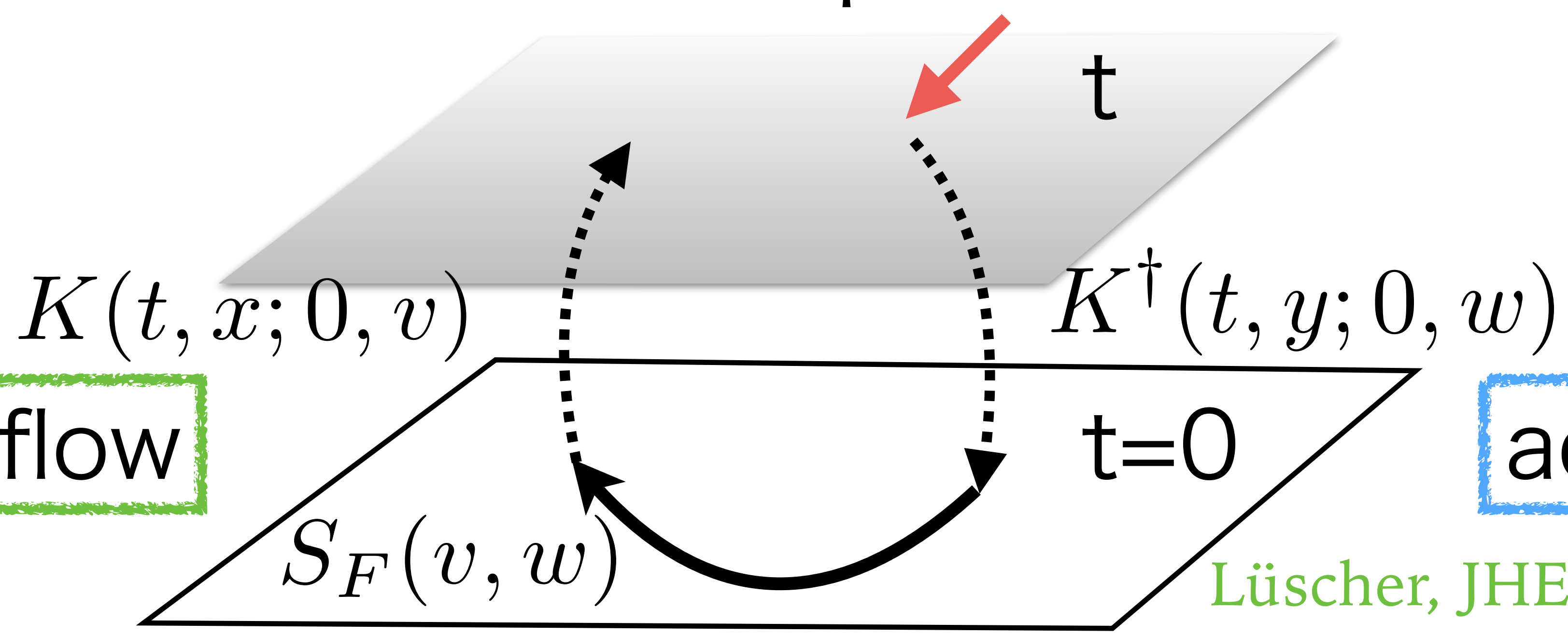
Technical details of correlation function

- Quark connected diagram

Quark propagator at flow time t $\langle \chi(t,x)\bar{\chi}(t,y)\rangle_{\rm Wick}$

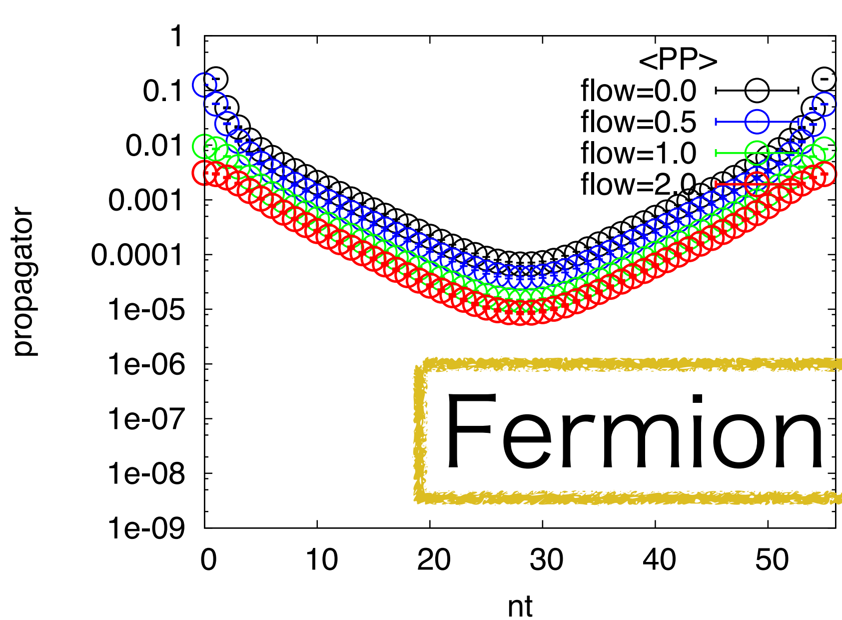
set point source at flow time t

standard flow



adjoin flow

Lüscher, JHEP 1304, 123 (2013)

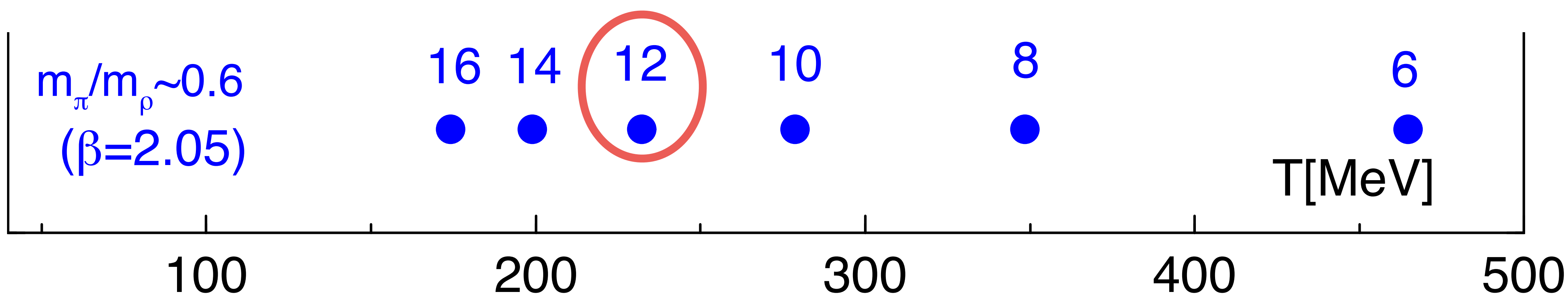


Pion propagator with flowed quark fields

Fermion flow does not change meson mass!

Numerical setups

- Iwasaki gauge action
  - β=2.05 : a∼0.07 [fm]
- Fixed scale method
  - T=1/(aNt)=232 MeV, Nt=12 only in this talk



32^3xNt for T≠0

28^3x56 for T=0

- Nf=2+1
- NP improved Wilson fermion
- On an equal quark mass line

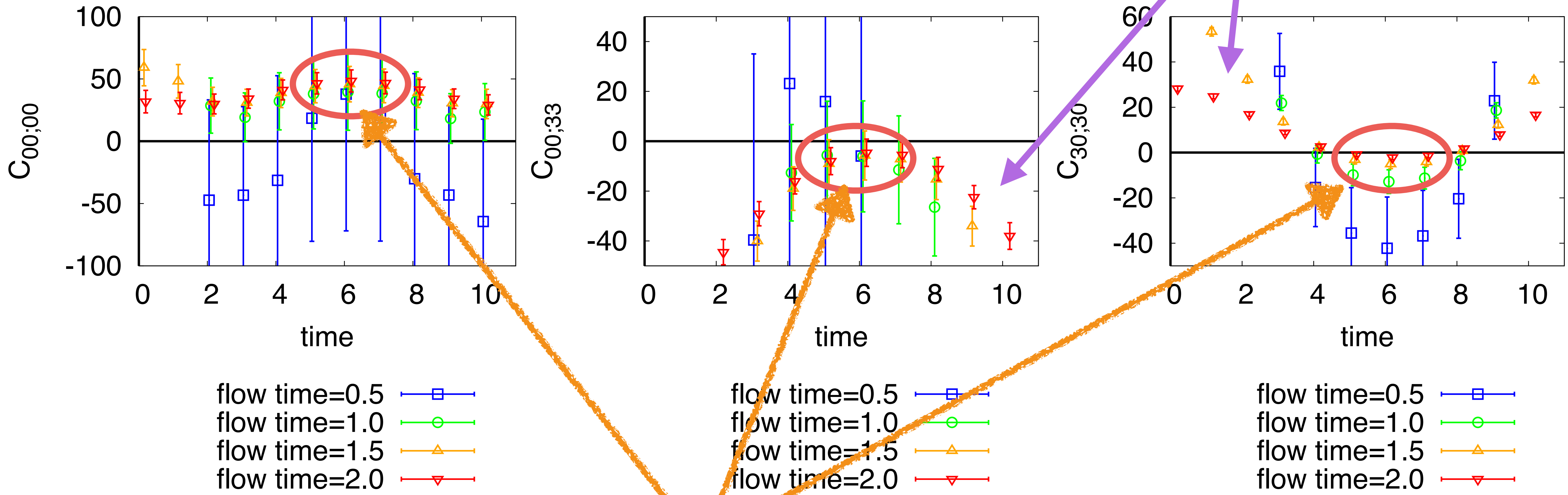
$$\frac{m_\pi}{m_\rho} \sim 0.6 \quad \frac{m_{\eta_{ss}}}{m_\phi} \sim 0.74$$

Conservation law

$$\frac{d}{dx_0}\underbrace{\int_{V_3} d^3x\,(\langle\delta T_{0\nu}(t;x_0,\vec{x})}_{P_\mu}\delta T_{\rho\sigma}(t;0)\rangle\rangle) = 0$$

lattice artifact is severe

$\langle\delta T_{00}\delta T_{00}\rangle$ $\langle\delta T_{00}\delta T_{33}\rangle$ $\langle\delta T_{03}\delta T_{03}\rangle$

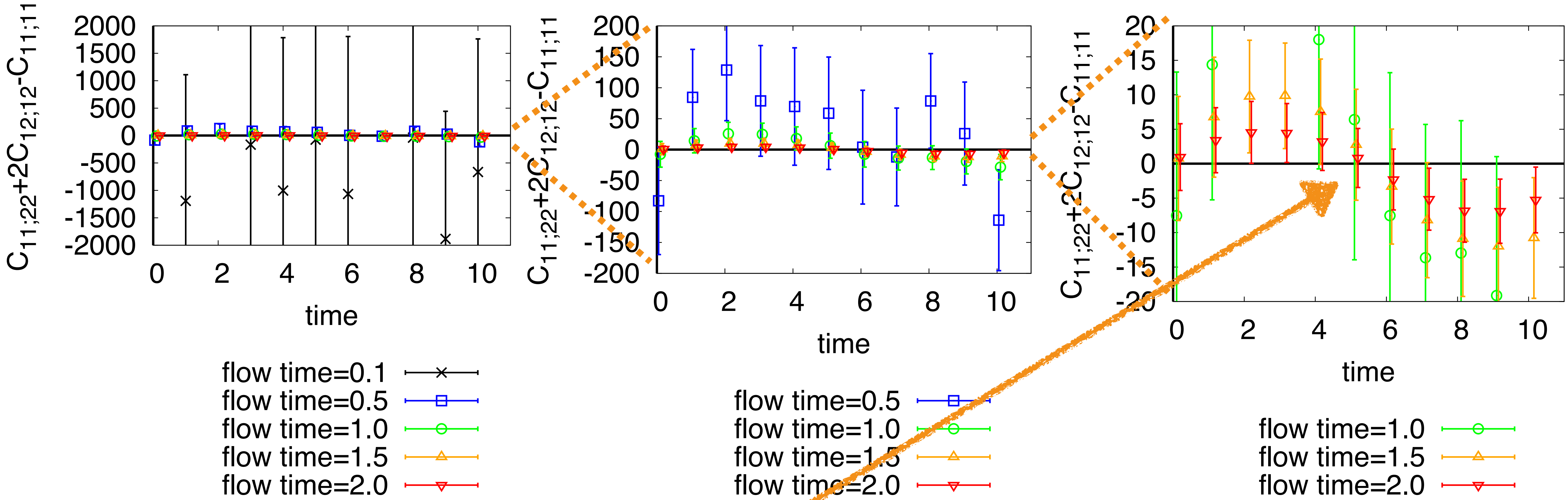


correlation function is very flat in the middle

Spatial rotational symmetry

$$\langle \delta T_{11} \delta T_{22} \rangle + 2 \langle \delta T_{12} \delta T_{12} \rangle - \langle \delta T_{11} \delta T_{11} \rangle = 0$$

T=232 MeV (Nt=12)



spatial rotational symmetry restores
but more statistics needed?

Entropy density

Thermodynamical relation

compare

Maxwell's relation

$$s = \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\epsilon + p}{T} = \frac{\langle T_{00} + T_{ii}\rangle}{T}$$

integrable condition of entropy

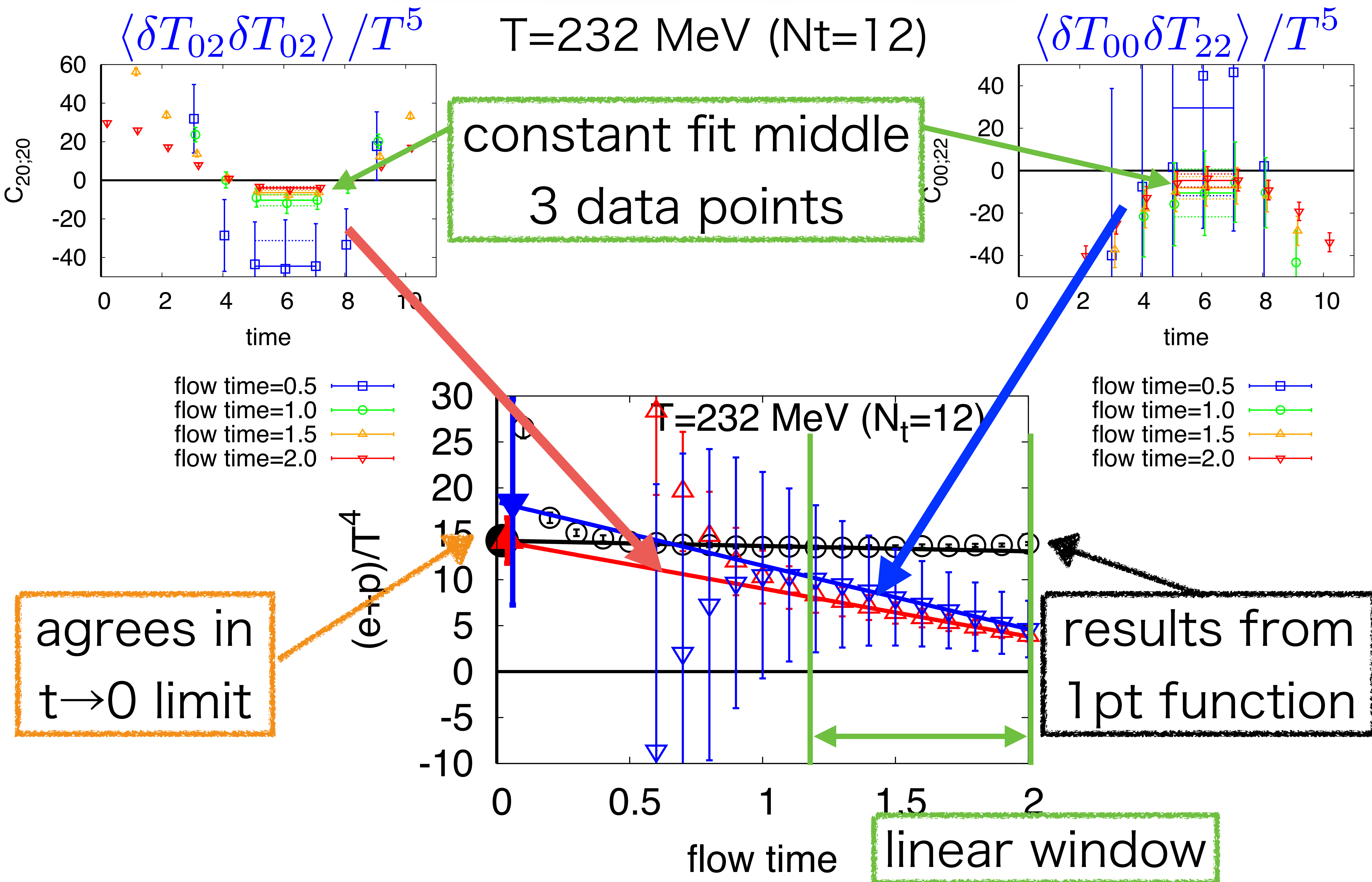
Linear response relation

$$s = \left(\frac{\partial p}{\partial T}\right)_V = \frac{\partial \langle T_{ii}\rangle}{\partial T} \qquad \langle T_{ii}\rangle = \frac{1}{Z}\mathrm{Tr}\left(T_{ii}e^{-H/T}\right)$$

$$s = \frac{1}{T^2}\langle \delta H \delta T_{ii}\rangle = \frac{1}{T^2}\int_{V_3} d^3x \left(\langle \delta T_{00}(t; x_0, \vec{x})\delta T_{ii}(t; 0)\rangle\right)$$

$$\epsilon + p = \left.\frac{\partial \langle T_{01}\rangle}{\partial v_1}\right|_{\vec{v}=0} = \frac{1}{T}\int_{V_3} d^3x \left(\langle \delta T_{0i}(t; x_0, \vec{x})\delta T_{0i}(t; 0)\rangle\right)$$

Entropy density



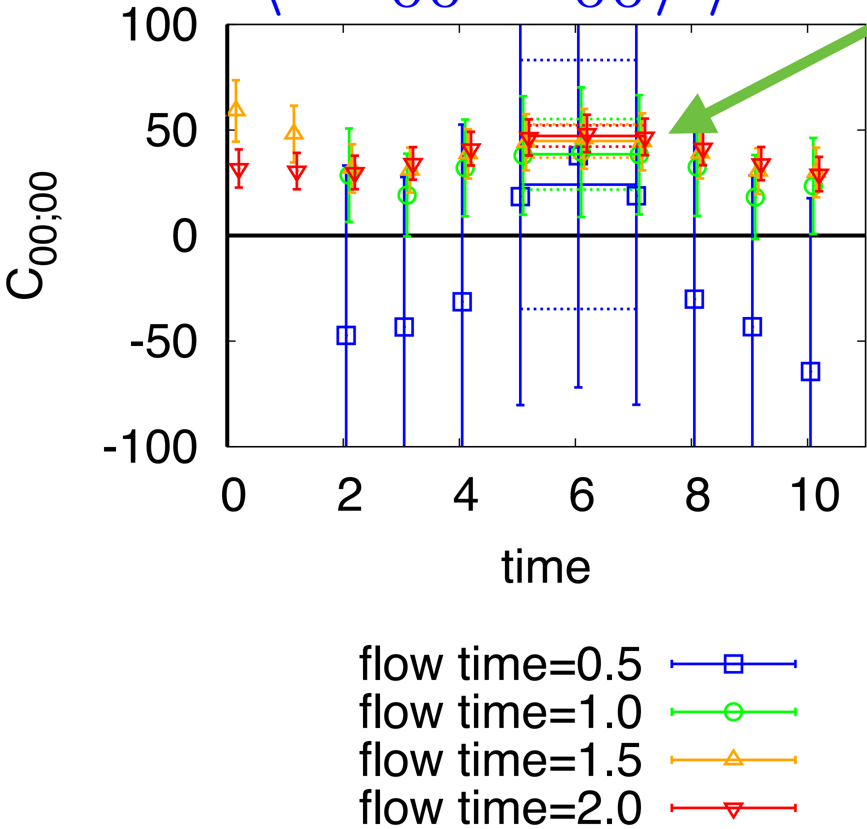
Specific heat

Linear response relation

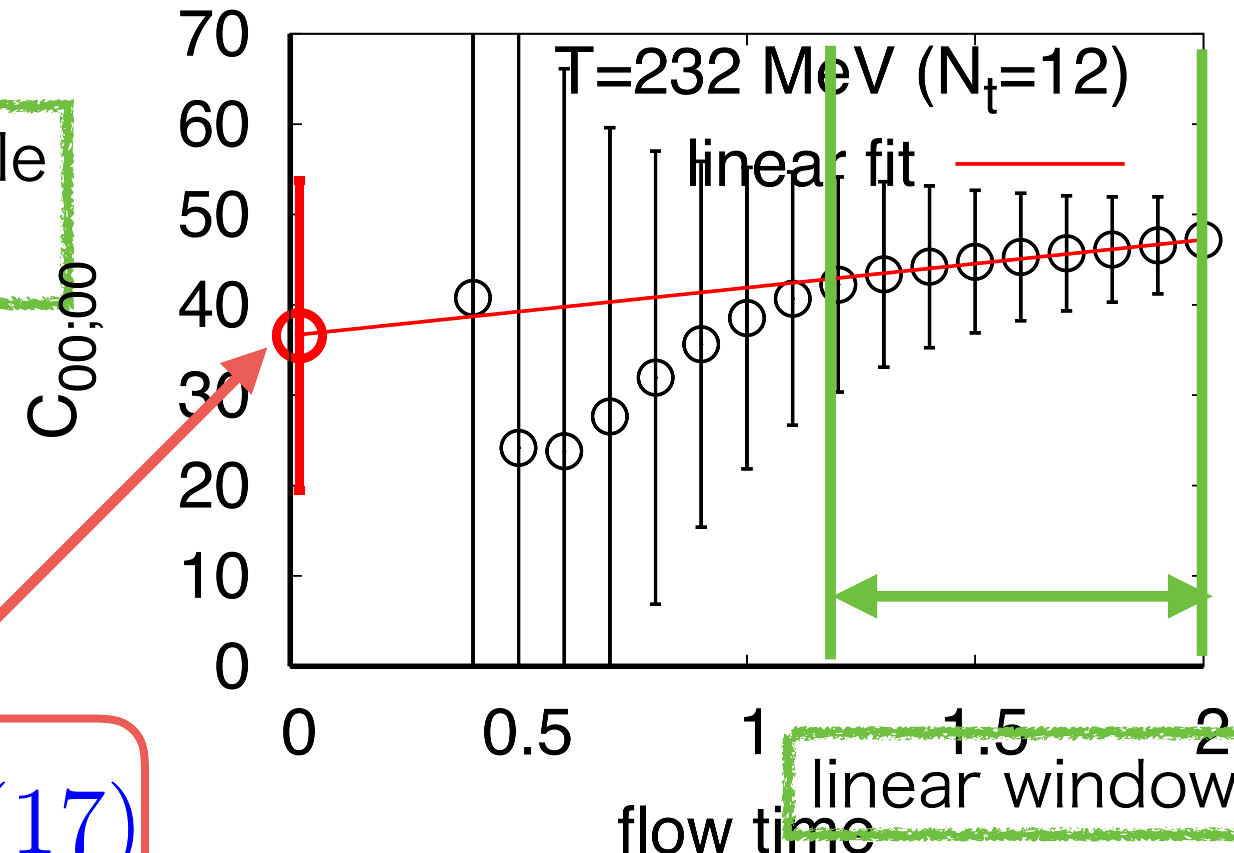
$$c_V = \frac{1}{V}\frac{d\langle H\rangle}{dT} \qquad \frac{1}{V}\langle H\rangle = \frac{1}{Z}\mathrm{Tr}\left(T_{00}e^{-H/T}\right)$$

$$c_V = \frac{1}{T^2}\int_{V_3} d^3x\,\left(\langle \delta T_{00}(t;x_0,\vec{x})\delta T_{00}(t;0)\rangle\right)$$

$\langle \delta T_{00}\delta T_{00}\rangle / T^5$



constant fit middle 3 data points



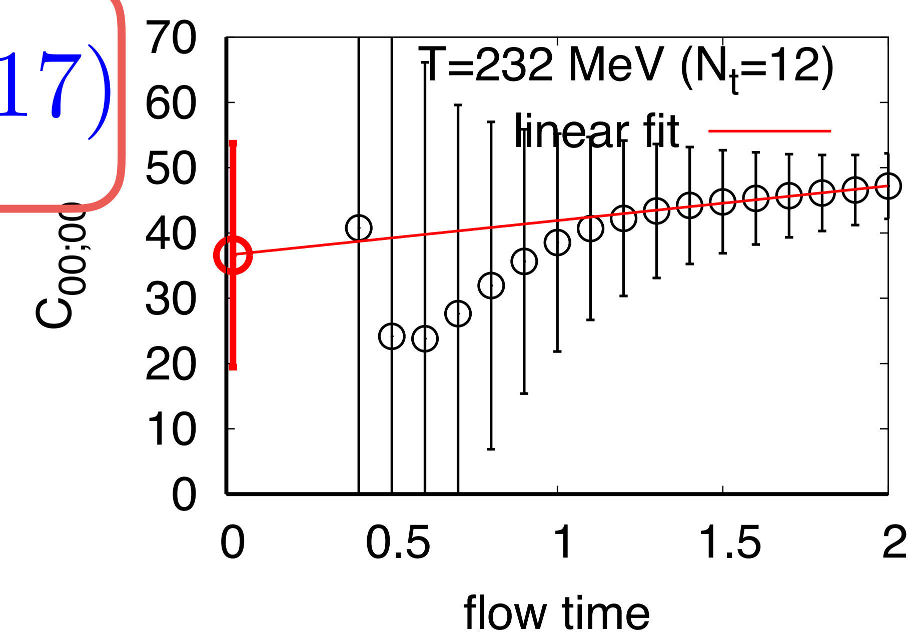
$$\frac{c_V}{T^3} = 37(17)$$

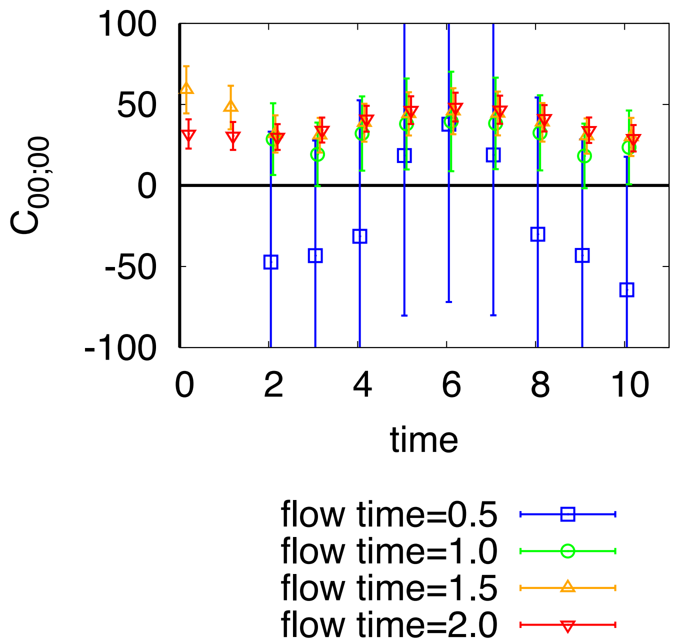
T=232 MeV (Nt=12)

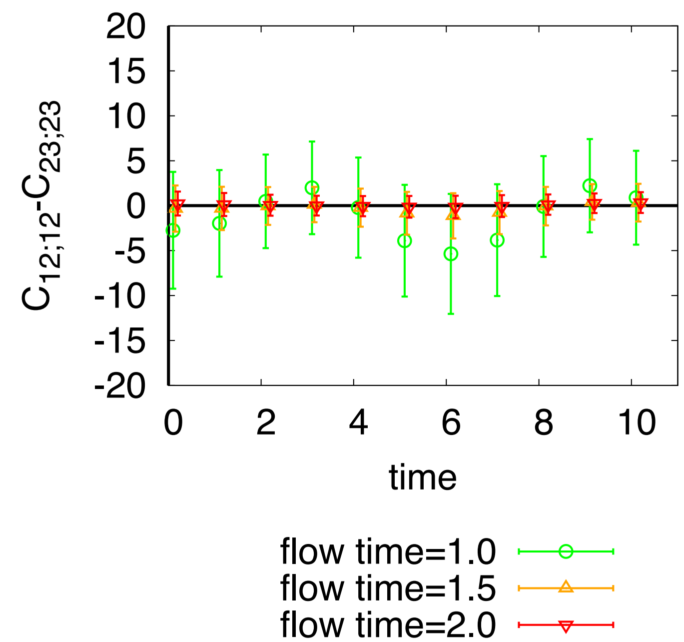
Summary

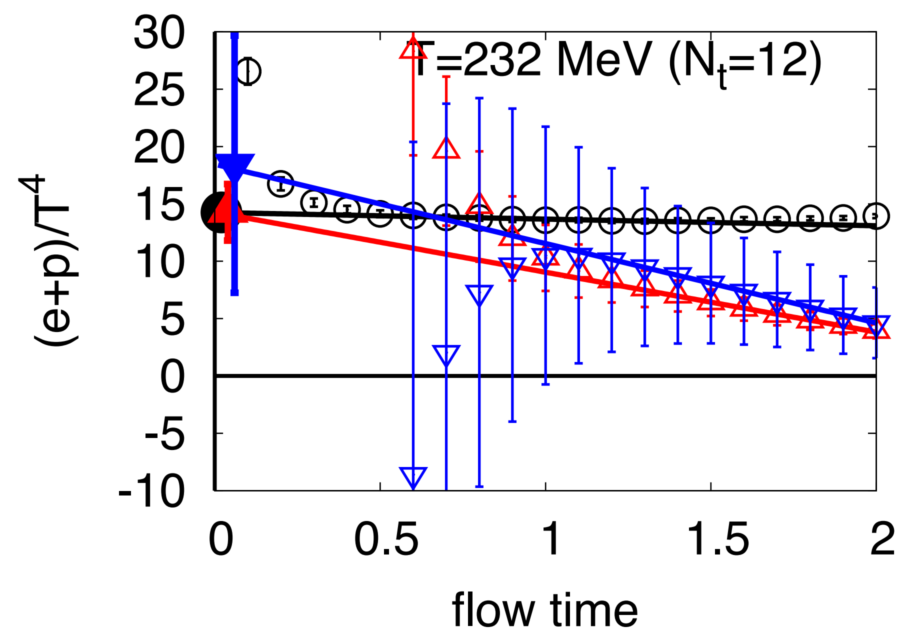
- Gradient flow works well for EMT correlation function
- We have good results:
  - Conservation law
  - Spatial rotational symmetry
  - Linear response relation

$$\frac{c_V}{T^3} = 37(17)$$









Future plan

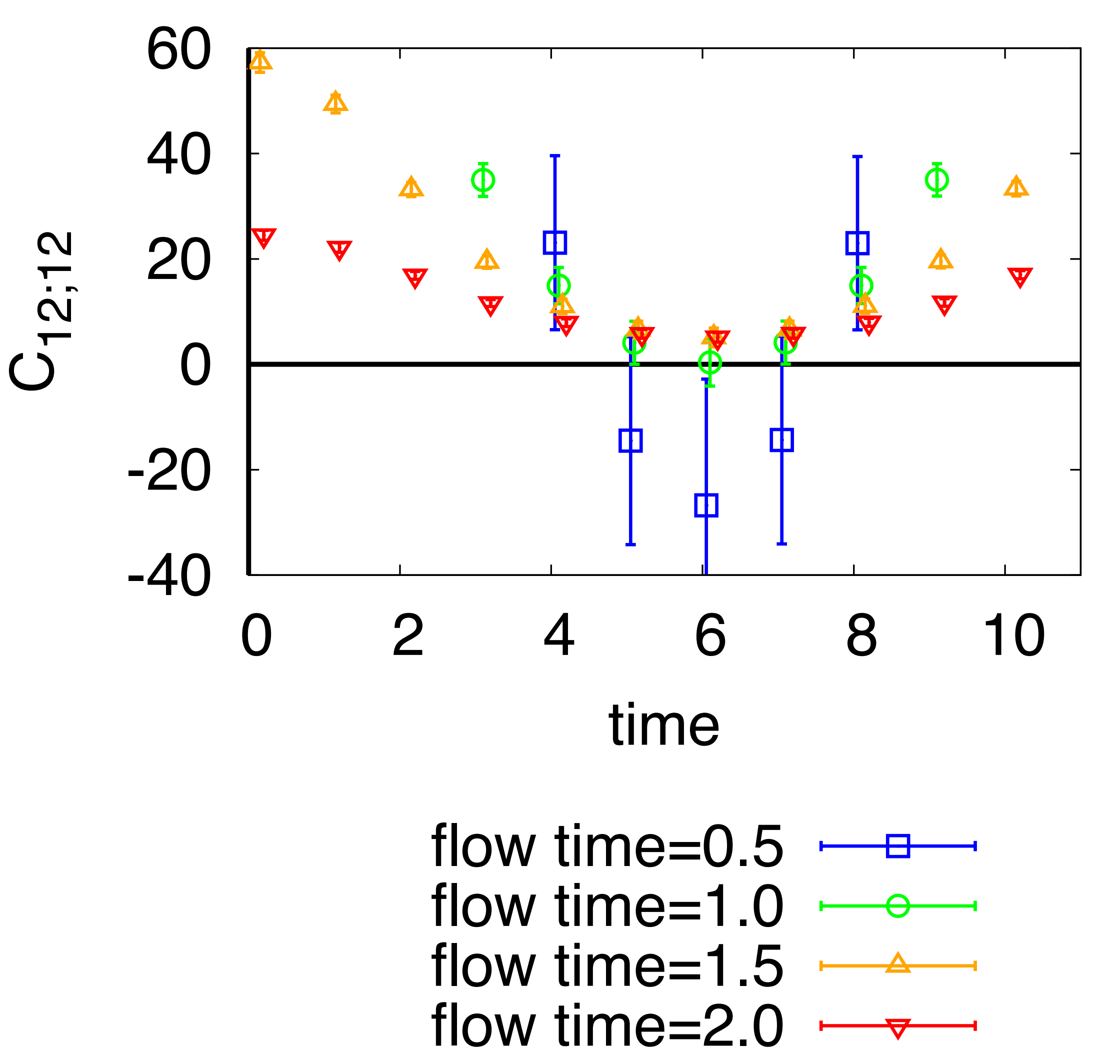
- We want to work for viscosity in future.

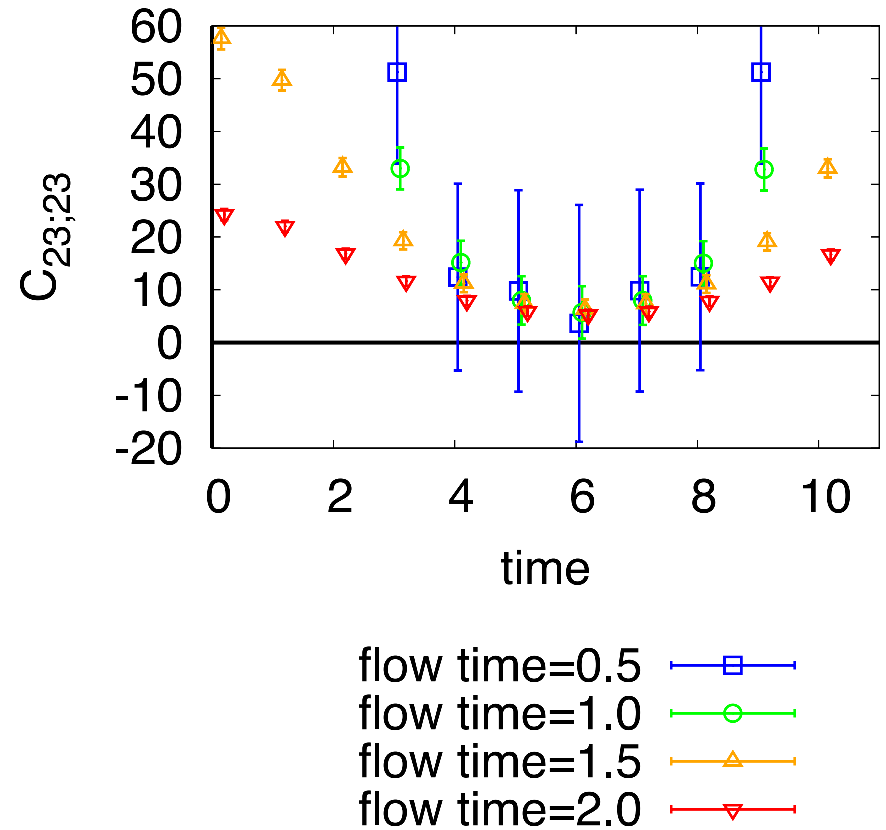
Shear viscosity

$$\eta = \lim_{\omega \to 0} \frac{\rho_{\rm shear}(\omega)}{\omega}$$

$$\int_0^\infty d\omega K(x_0,\omega)\rho_{\rm shear}(\omega) = C_{12;12}(x_0)$$

$$K(x_0,\omega) = \frac{\cosh\left(x_0 - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$





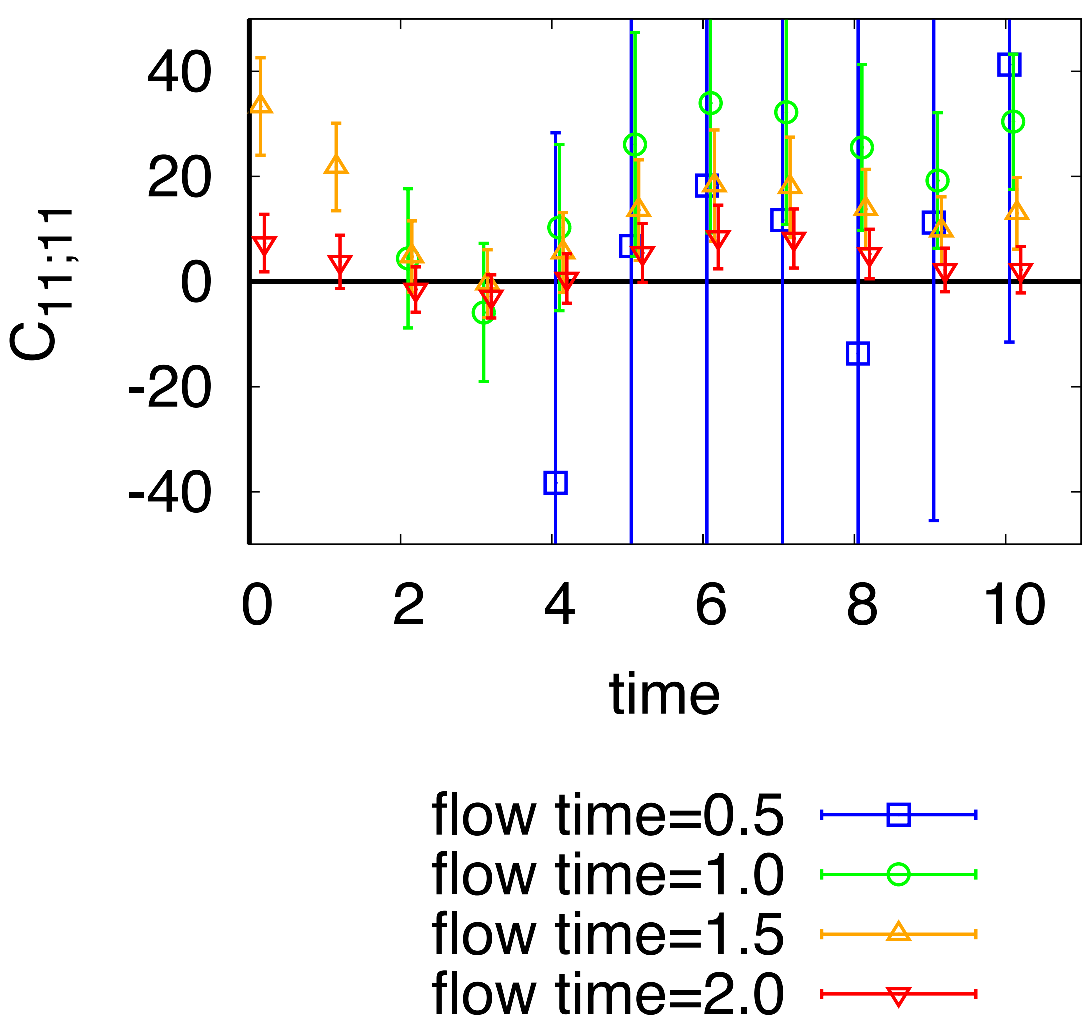
Future plan

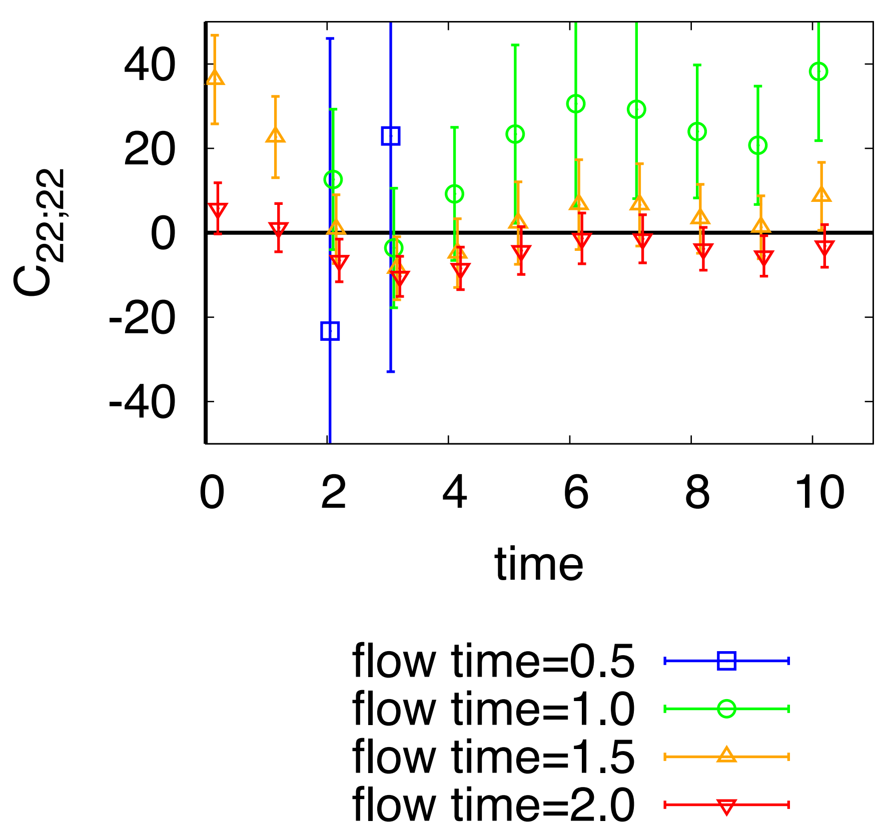
- We want to work for viscosity in future.

Bulk viscosity

$$\zeta = \lim_{\omega \to 0} \frac{\rho_{\mathrm{bulk}}(\omega)}{\omega} \qquad \int_0^\infty d\omega K(x_0,\omega)\rho_{\mathrm{bulk}}(\omega) = \sum_{i=1}^{3} C_{ii;ii}(x_0)$$

$$K(x_0,\omega) = \frac{\cosh\left(x_0 - \frac{1}{2T}\right)\omega}{\sinh\frac{\omega}{2T}}$$



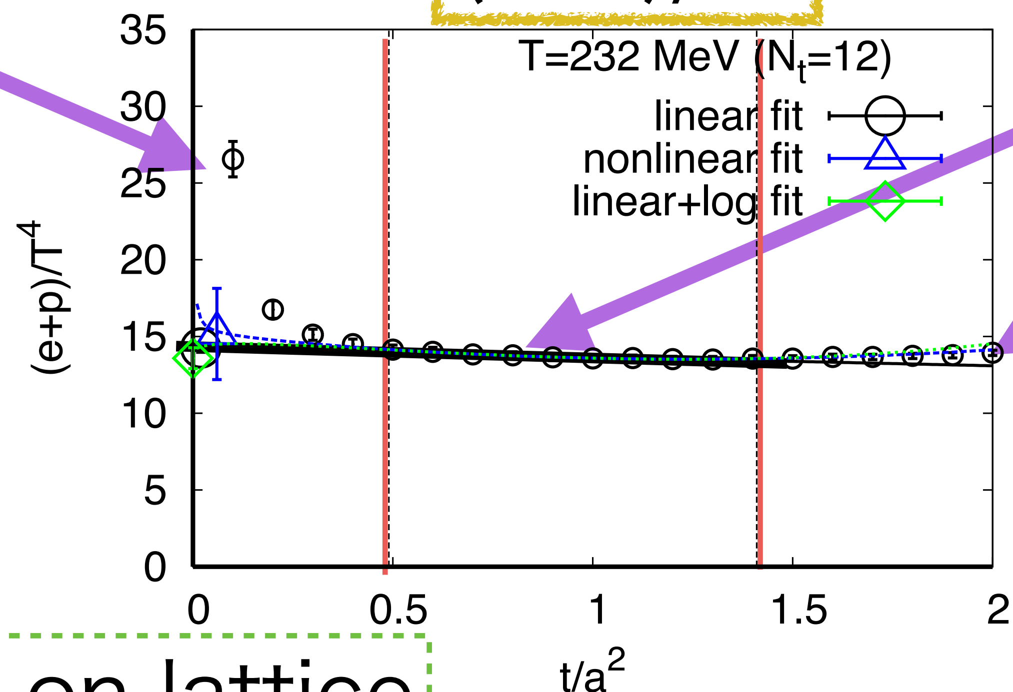


Can we extract physics before a→0 ?

Previous works and lessons

(E+P)/T^4

$O\left(\frac{a^2}{t}\right)$



$O(t)$

$O(t^2)$

WHOT QCD Collaboration (PRD 2017)

flowed operator on lattice

$\{T_{\mu\nu}\}(x,t,a) = \{T_{\mu\nu}\}_{\rm WT}(x)$ +t(dim6 operator)

the window region

$+\frac{a^2}{t}$ (dim4 operator) tamed at large t

+1/log^2(t)(dim4) +t^2(dim8 operator) tamed at small t

$+O(a^2T^2, a^2m^2, a^2\Lambda^2_{\rm QCD})$ → need to take a→0 limit

Technical details of correlation function

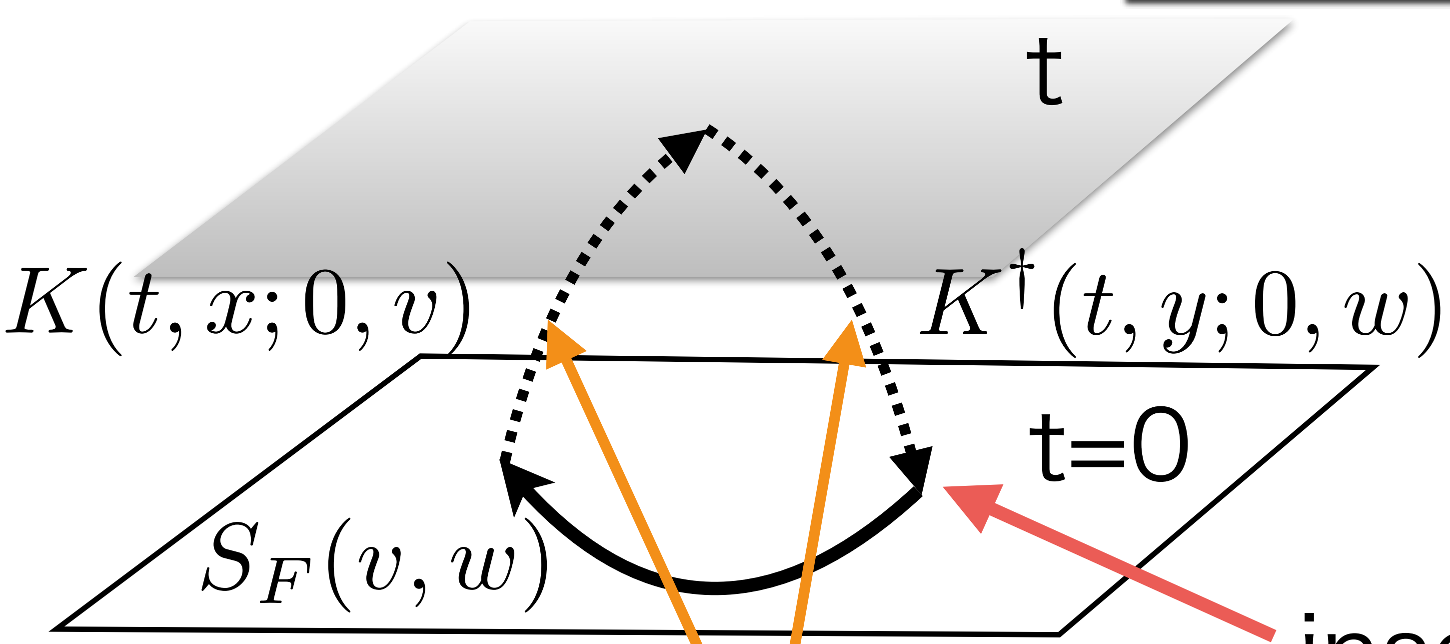
☆Quark 1 point function for disconnected contributions

Wick contraction is a complication

$$\langle \chi(t,x)\bar{\chi}(t,y)\rangle_{\mathrm{Wick}} \neq \left(D(A_\mu(t,x))+m\right)^{-1}$$

meson mass changes as we flow Dirac operator (Ukita'16)

This is not an effective Dirac operator at t



noise method for $\sum_x \langle \bar{\chi}(t,x)\chi(t,x)\rangle$

insert noise vector here $\eta(w)\eta^\dagger(w)$

we know the Dirac operator!

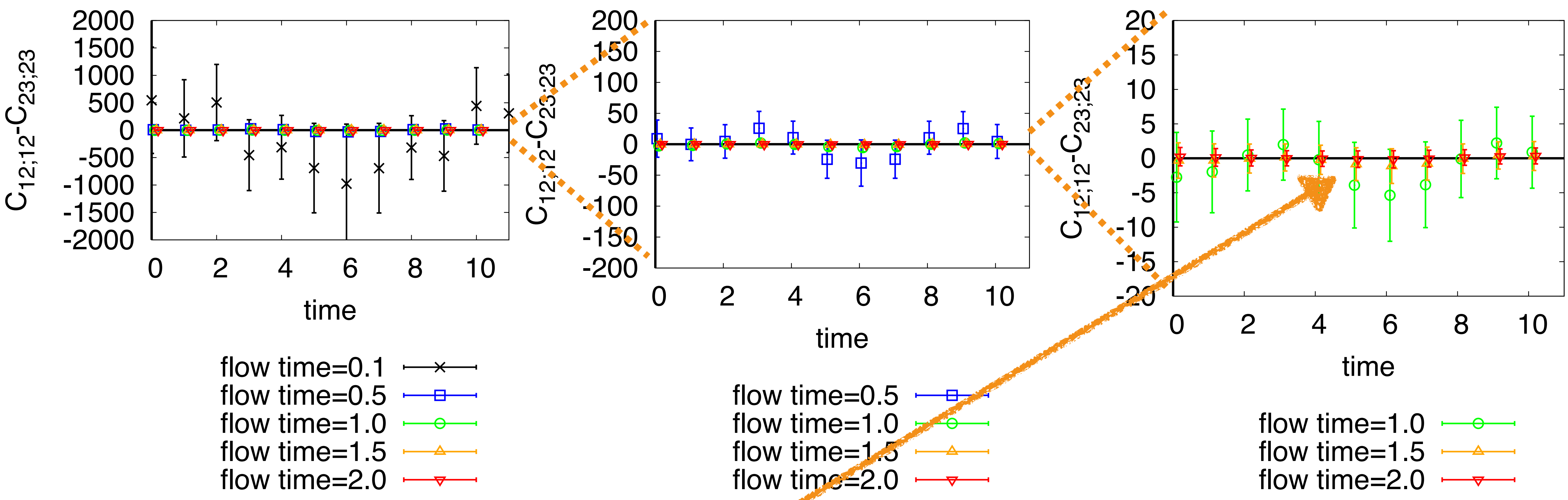
standard fermion flow

Lüscher, JHEP 1304, 123 (2013)

Spatial rotational symmetry

$$\langle \delta T_{12} \delta T_{12} \rangle - \langle \delta T_{32} \delta T_{32} \rangle = 0$$

T=232 MeV (Nt=12)



spatial rotational symmetry restores!