Heavy-light tetraquarks from Lattice QCD

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OUTLINE

- Introduction
- Tetraquark operators
- Review of status
- Lattice Details
- Results
- Conclusion

Exotic Spectroscopy

Standard paradigm of Hadron spetroscopy: Quark model,

$$\text{Strong attractive color forces in } \mathbf{1}_c \longrightarrow \underbrace{(\bar{q}q)}_{\text{Mesons}} \qquad \underbrace{(qqq)}_{\text{Baryons}}$$

Discovery of copious exotics, $X(3872), X(3915), Z_c^0(3900)$ motivates a new mechanism.

EXOTIC SPECTROSCOPY

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DIQUARK CORRELATIONS

"Next most attractive channel"[a] with $(qq) \to (\bar{\mathbf{3}}_c, 0^+, \bar{\mathbf{3}}_f)$.

[a] *Jaffe R.L, Exotica,* ***Phys.Rept.*** ***409 (2005)***

Phenomenological evidence seen in Baryon spectrum, weak non-leptonic decays and in ratio of nucleon PDF's.

DIQUARK CLASSIFICATION

Diquarks are classified in SU(3) as:

$$\mathbb{Q}(\mathbf{3} \otimes \mathbf{3}) \longrightarrow \underbrace{\bar{\mathbf{3}}}_{\text{anti-symmetric}} \oplus \underbrace{\mathbf{6}}_{\text{symmetric}}$$

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One gluon exchange dictates $\bar{\mathbf{3}}_c$ is attractive color configuration. From Fermi statistics,

$$\underbrace{\mathbb{Q}(\bar{\mathbf{3}}_c, 0^+, \bar{\mathbf{3}}_f)}_{\substack{\text{Scalar} \\ \text{Good diquark}}} \qquad \underbrace{\mathbb{Q}(\bar{\mathbf{3}}_c, 1^+, \bar{\mathbf{6}}_f)}_{\substack{\text{Vector} \\ \text{Bad diquark}}}$$

Since diquarks are colored,

$$\mathbb{Q} \to \bar{q} \quad \bar{\mathbb{Q}} \to q \qquad \bar{q}q \longrightarrow \underbrace{\bar{\mathbb{Q}}\mathbb{Q}}_{\substack{\text{Construct} \\ \text{exotic states}}}$$

Tetraquarks from diquarks

Diquark framework employed in the context heavy tetraquarks.

- Effective Hamiltonian
  - Effective quark model spin-spin interaction of $\bar{\mathbb{Q}}\mathbb{Q}$ used to compute exotic spectrum.[1]
  - States studied $X(3872), X(3940), D_s(2317)$.
- QCD Sum rules for exotics
  - Compute $\langle 0|\mathcal{O}_{\bar{\mathbb{Q}}\mathbb{Q}}\mathcal{O}^{\dagger}_{\bar{\mathbb{Q}}\mathbb{Q}}|0\rangle$ by applying sum rules.[2]
  - States studied X(5568) as open flavor $su\bar{b}\bar{d}$
- Lattice Calculations.
  - Computation of potentials for $ud\bar{b}\bar{b}$.
  - Computing energy levels for $ud\bar{b}\bar{b}$.
  - This work.

[1] Maiani et-al, *Phys.Rev. D71 (2005) 014028*

[2] Chen et-al, *Phys.Rev.Lett. 117 (2016) 022002*

BOTTOM TETRAQUARKS ON LATTICE

- Computation of static $\bar{b}\bar{b}$ potential in presence of ud [3] with fit ansatz,

$$V(r) = -\frac{\alpha}{r} e^{-\left(\frac{r}{d}\right)^2}$$

α, d obtained from fitting to lattice data.
Lattice action :

  - Twisted mass fermions $m_\pi \sim 380 - 650$ MeV.
  - Bound state for $(I = 0, J = 0)$ at 90^{+43}_{-30} MeV.

[3] Bicudo et al, *PhysRev.D.93.034501*

[4] Bicudo et-al, *arXiv:1704.02383*

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  Lattice action :
  - Twisted mass fermions $m_\pi \sim 380 - 650$ MeV.
  - Bound state for $(I = 0, J = 0)$ at 90^{+43}_{-30} MeV.
- Emergent wave Method,[4]
  - Solve Schrödinger equation for above potential and determine δ_l
  - Prediction for $(I = 1, J = 1)$ for m = 10576(4)MeV $\Gamma = 114^{+90}_{-113}$ MeV.

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[3] Bicudo et al, *PhysRev.D.93.034501*

[4] Bicudo et-al, *arXiv:1704.02383*

BOTTOM TETRAQUARKS ON LATTICE

Following Jaffe's suggestion, using $(\bar{\mathbf{3}}_c, S, \bar{\mathbf{3}}_f)$ diquarks as,

$$\mathbb{Q}_{ia} = \epsilon_{ijk}\epsilon_{abc}\, q_{jb}\, (C\Gamma)\, q_{kc}$$

$$\underbrace{(\bar{Q}\bar{Q}) \to (\mathbf{3}_c, 1)}_{\text{Heavy sector}} \qquad \underbrace{(qq') \to (\bar{\mathbf{3}}_c, 0, \bar{\mathbf{3}}_f)}_{\text{Light sector}}$$

[5] Francis et-al, *Phys.Rev.Lett. 118 (2017) 142001*

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Construct operator for $(I = 0, J = 1)$[5]:

$$D(x) = u^a_\alpha(x)\,(C\gamma_5)_{\alpha\beta}\, q^b_\beta(x)\, \bar{b}^a_\kappa(x)(C\gamma_i)_{\kappa\rho}\, \bar{b}^b_\rho(x)$$

$$M(x) = B^+(x)B^{0*}(x) - B^0(x)B^{+*}(x)$$

[5] Francis et-al, *Phys.Rev.Lett. 118 (2017) 142001*

Bottom Tetraquarks on Lattice

Compute correlator matrix from the operators as :

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(t)|0\rangle \quad i,j \subset \mathrm{D,M}$$

and solve the generalized eigenvalue problem (GEVP),

$$C_{ij}(t+\Delta t)v_j(t) = \lambda(t)C_{ij}(t)v_j(t)$$

and obtain ground state effective masses.

Binding energy results

$$(\Delta E)_{ud\bar{b}\bar{b}} = 189 \text{ MeV}^a \qquad (\Delta E)_{us\bar{b}\bar{b}} = 90 \text{ MeV}^a$$

[a] Francis et-al, *Phys.Rev.Lett. 118 (2017) 142001*

Gauge configurations

$2+1+1$ HISQ configurations by MILC

- Three sets Ensembles, $\underline{48^3 \times 144}$, $32^3 \times 96$ and $24^3 \times 64$.
- $a = \underline{0.05}, 0.08, 0.12$ fm, $m_\pi = 305 \sim 319$ MeV, $m_\pi L = 4.5$
- Gauge fields employ HYP smearing.
- Employ wall sources on Coulomb gauge fixed configurations.

Overlap fermions in the valence sector

- Automatic $\mathcal{O}(a)$ improvement
- Use of Multimass algorithm allows a range of masses to be computed.

LATTICE SET-UP

STRANGE QUARKS

- Strange quark mass tuned by pseudoscalar $\bar{s}s = 685$ MeV.
- Precise lattice spacing determination from $M_\Omega(sss) = 1675$ GeV.

CHARM QUARKS

- Employ Relativistic Fermilab formulation.
- Charm quark mass tuned with kinetic mass of

$$\frac{1}{4}(M_{\eta_c} + 3m_{J/\psi})$$

Lattice Set-up

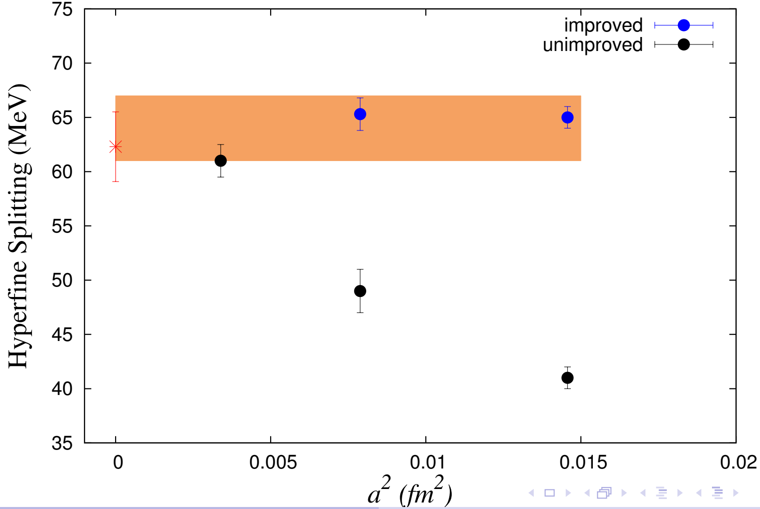
Bottom quarks

- Employ NRQCD formulation $H = H_0 + \Delta H$,

$$\begin{aligned}\Delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^3}(\nabla \cdot \tilde{E} - \tilde{E} \cdot \nabla) \\ &\quad - c_3 \frac{1}{8(m_b)^2} \sigma \cdot (\nabla \times \tilde{E} - \tilde{E} \times \nabla) - c_4 \frac{1}{2am_b} \sigma \cdot \tilde{B} \\ &\quad + c_5 \frac{(\Delta^{(4)})}{24am_b} - c_6 \frac{(\Delta^{(2)})^2}{16(am_b)^2}\end{aligned}$$

- For coarser lattices, use improved coefficients $c_1..c_6$ computed by HPQCD. Finer lattice, use tree level.
- Bottom quark mass tuned by setting spin averaged $1S$ bottomonium to experiment.

HYPERFINE SPLITTING



TETRAQUARKS IN THIS WORK

In this work we study tetraquarks in Bottom and Charm sectors.

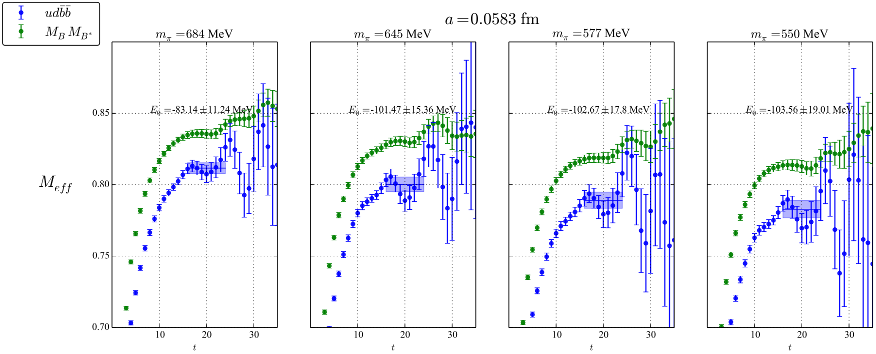
(I = 0, J = 1) TETRAQUARKS

$$48^3 \times 144 \quad m_\pi = 550 \sim 680 \text{ MeV}, \quad a = 0.0583 \text{ fm},$$

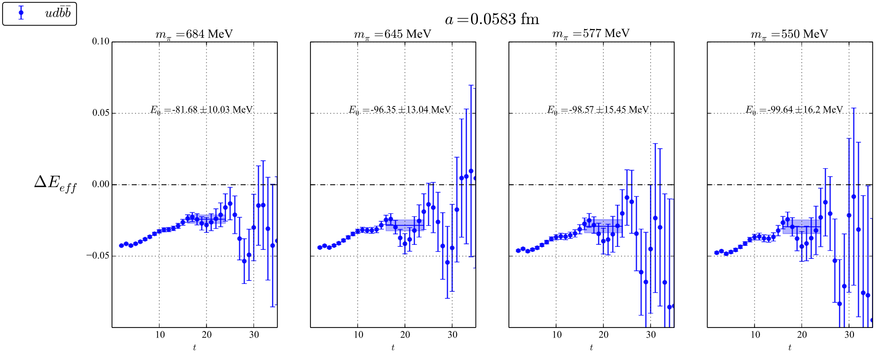
$$ud\bar{b}\bar{b} \qquad ud\bar{c}\bar{c}$$

$$BB^* \qquad DD^*$$

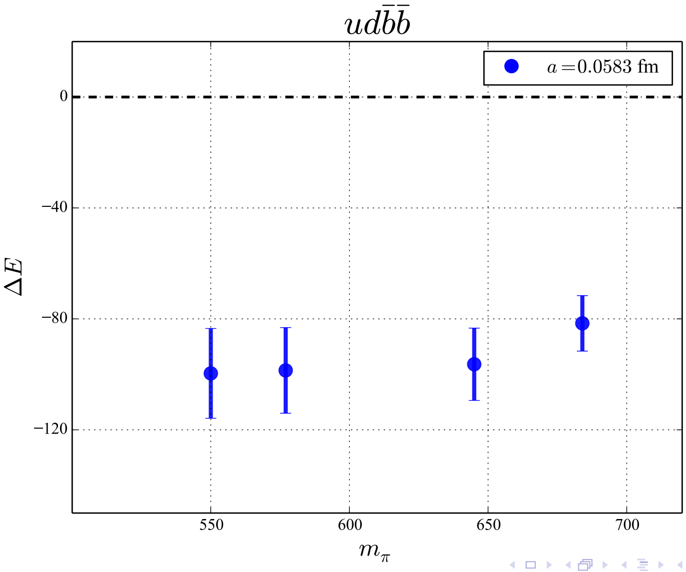
PRELIMINARY RESULTS ON $ud\bar{b}\bar{b}$



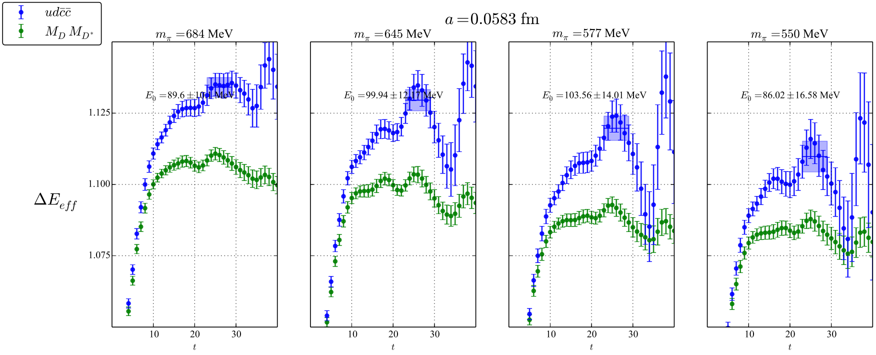
Preliminary results on $ud\bar{b}\bar{b}$



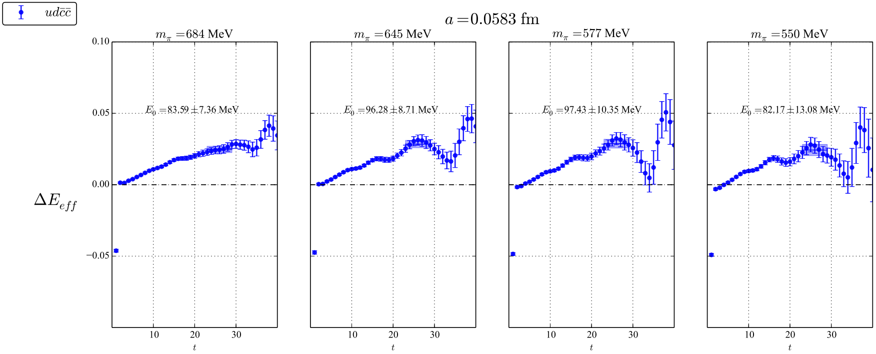
Summary on $ud\bar{b}\bar{b}$



PRELIMINARY RESULTS ON $ud\bar{c}\bar{c}$



PRELIMINARY RESULTS ON $ud\bar{c}\bar{c}$



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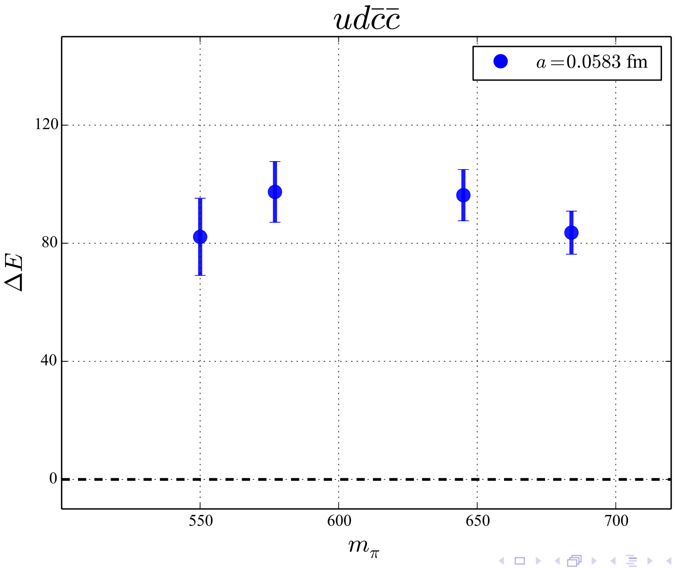

Conclusions and Outlook

- In the bottom sector, clear evidence is seen for bound state of $ud\bar{b}\bar{b}$ type tetraquark
- In the charm sector, a scattering state is found for $ud\bar{c}\bar{c}$ type tetraquark.
- Need more statistics and lower pion masses to explore pion mass trends in both sectors.
- Possibly explore other flavour structures.

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