Isospin splittings of pseudoscalar mesons from lattice QCD and quenched QED

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Outline

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Motivation

- Lattice computations nowadays reach below 1 % precision
- $m_u \neq m_d$ and QED effects become relevant
- Parameters can be calibrated using experimental data
  - meson masses $M_{\pi^+}, \quad M_{K^+}, \quad \ldots$
  - mass splittings $\Delta M_\pi^2 = M_{\pi^+}^2 - M_{\pi^0}^2$
    $\Delta M_K^2 = M_{K^0}^2 - M_{K^+}^2$
    $\Delta M_D^2 = M_{D^+}^2 - M_{D^0}^2$
- Quenched QED
  $\longrightarrow$ existing $N_f = 2+1(+1)$ configurations can be reused

Outline

QCD+QED

- Noncompact photon action

$$S_\gamma[A] = \frac{1}{4} \sum_{\mu,\nu,x} (\partial_\mu A_{x,\nu} - \partial_\nu A_{x,\mu})^2$$

- Gauge fixing: Coulomb gauge $\underline{\partial}^\dagger \cdot \underline{\mathbf{A}}_x = 0$
- Zero mode subtraction: QED_L [Hayakawa & Uno, 2008]

$$\sum_{\underline{\mathbf{x}}} A_{\underline{\mathbf{x}},t,\mu} = 0 \qquad \text{for all } t \text{ and } \mu$$

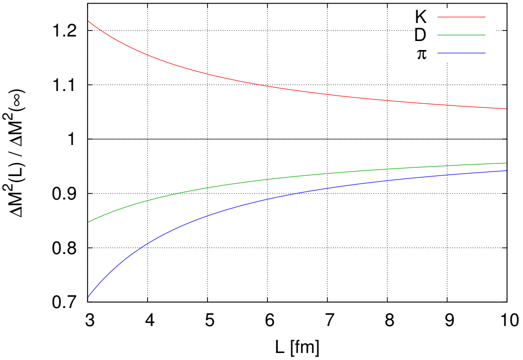
- Coupling to quarks

$$U_{x,\mu} \longrightarrow U_{x,\mu} \cdot e^{i\, e\, q\, A_{x,\mu}}$$

FV correction in QED$_L$

$$m(T,L) \sim m \cdot \left(1 - \frac{q^2\,\alpha\,\kappa}{2\,m\,L}\left[1 + \frac{2}{m\,L}\right] + \mathcal{O}\left(\frac{\alpha}{L^3}\right)\right)$$

with $\kappa \approx 2.837297$ [Borsanyi *et.al.*, 2015]



Quenched QED

- QCD + QED

$$\langle \mathcal{O} \rangle_{U,A} = \frac{1}{Z} \int \mathrm{d}U \int \mathrm{d}A \ \mathcal{O}(U,A) \ e^{-S_{\mathrm{g}}[U]} \, e^{-S_\gamma[A]} \ \det M(U,A)$$

- Quenched QED

$$\langle \mathcal{O} \rangle_{\substack{U,\text{dynamical} \\ A,\text{quenched}}} = \frac{1}{Z} \int \mathrm{d}U \underbrace{\int \mathrm{d}A \ \mathcal{O}(U,A) \ e^{-S_\gamma[A]}}_{\langle \mathcal{O}(U,A) \rangle_{A,\text{q.}}} e^{-S_{\mathrm{g}}[U]} \ \det M(U)$$

$$= \Big\langle \langle \mathcal{O}(U,A) \rangle_{A,\text{q.}} \Big\rangle_U$$

- In practice
  - Generate $SU(3)$ configurations
  - For each $SU(3)$ configuration, generate $U(1)$ configurations with S_γ
  - Measure $\mathcal{O}(U,A)$

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Expansion of correlators

$$m_u = m_l - \frac{\delta m}{2} \qquad m_d = m_l + \frac{\delta m}{2}$$

- 1^{st} order in δm and e^2

$$\langle C(\delta m, e)\rangle \approx \Big\langle C_0(U) \Big\rangle_U + \delta m \cdot \Big\langle C_m(U) \Big\rangle_U + \frac{e^2}{2} \cdot \Big\langle \langle C_2(U,A)\rangle_{A,\text{q.}} \Big\rangle_U$$

where

$$C_0(U) = C(\delta m, e, U, A)\Big|_{\substack{\delta m=0\\ e=0}}$$

$$C_m(U) = \frac{\mathrm{d}}{\mathrm{d}(\delta m)} C(\delta m, e, U, A)\Big|_{\substack{\delta m=0\\ e=0}}$$

$$C_2(U,A) = \frac{\mathrm{d}^2}{\mathrm{d}e^2} C(\delta m, e, U, A)\Big|_{\substack{\delta m=0\\ e=0}}$$

- Advantage: δm and e contributions can be separated

Computation in practice

- Approximate derivatives

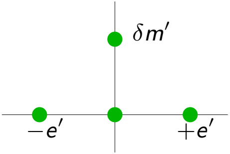
$$C_m(U) \approx \frac{C(\delta m',0) - C(0,0)}{\delta m'}$$

$$C_2(U,A) \approx \frac{C(0,e') + C(0,-e') - 2\,C(0,0)}{e'^2}$$

$$\langle C(\delta m, e)\rangle \approx \Big\langle C(0,0) \Big\rangle_U + \frac{\delta m}{\delta m'} \cdot \Big\langle C(\delta m',0) - C(0,0) \Big\rangle_U + \\ + \frac{e^2}{2\,e'^2} \cdot \Big\langle \big\langle C(0,e') + C(0,-e') \big\rangle_{A,\text{q.}} - 2\,C(0,0) \Big\rangle_U$$

- Correlators to be computed

$C(0,0)$
$C(\delta m',0)$
$C(0,e')$
$C(0,-e')$



- Advantage: $\mathcal{O}(e)$ noise cancellation

Computation using random vectors

Pseudoscalar meson correlator built from flavors f and f'

$$C_{f,f'}(t,\bar{t}) = \sum_{\underline{\mathbf{x}},\underline{\bar{\mathbf{x}}}} \mathrm{Tr}_c \left[\left(M^{(f)\dagger} \right)^{-1}_{\underline{\mathbf{x}},t\,;\,\underline{\bar{\mathbf{x}}},\bar{t}} \left(M^{(f')} \right)^{-1}_{\underline{\bar{\mathbf{x}}},\bar{t}\,;\,\underline{\mathbf{x}},t} \right]$$

ξ_t: random wall source on timeslice t

$$\left\langle (\xi_t)^*_x \, (\xi_{t'})_{x'} \right\rangle = \delta_{t,x_4} \, \delta_{t',x'_4} \, \delta_{x,x'}$$

$$C_{f,f'}(t,\bar{t}) = \left\langle \left(M^{(f)} \right)^{-1} \xi_t \,\middle|\, \left(M^{(f')} \right)^{-1} \xi_t \right\rangle_{\bar{t}}$$

13 inversions on each ξ_t

	$m_l - \frac{\delta m}{2}$	$m_l + \frac{\delta m}{2}$	m_l	m_s	m_c
$q=0$	2	2	1	1,2	1,2
$q=\frac{1}{3}$			4	4	
$q=-\frac{1}{3}$			3	3	
$q=\frac{2}{3}$			3		3
$q=-\frac{2}{3}$			4		4

1: $C(0,0)$
2: $C(\delta m',0)$
3: $C(0,e')$
4: $C(0,-e')$

Outline

Ensembles

- Tree-level Symanzyk gauge action
- $N_f = 2+1+1$ staggered fermions
  stout smearing 4 steps, $\varrho = 0.125$
- m_l and m_s fixed via
  $\overline{M}_\pi = 134.8(3)$ MeV
  $\overline{M}_K = 494.2(3)$ MeV
  [FLAG, 2017]
- m_c is fixed via $\frac{m_c}{m_s} = 11.85$
- $e = \sqrt{4\pi\alpha}$ is set to Thomson limit
- δm is fixed to give

$$\frac{m_u}{m_d} = 0.485(11)(8)(14)$$

[Fodor *et.al.*, 2016]

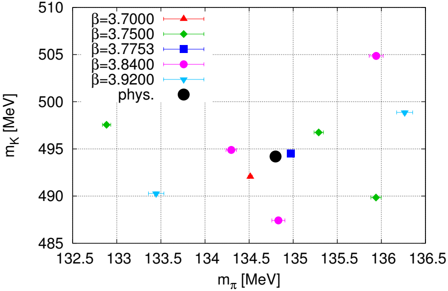
- other possible choice e.g.

$$\frac{m_u}{m_d} = 0.4582(38)_{\mathrm{stat}}\left({}^{+12}_{-82}\right)_{a^2}(1)_{\mathrm{FV_{QCD}}}(110)_{\mathrm{EM}}$$

[Gottlieb *et.al.*, 2016]

- Subset of [arXiv:1612.02364]

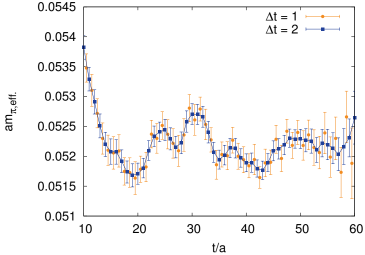
β	a [fm]	size	L [fm]	T [fm]	#conf
3.7000	0.134	$48^3 \times 64$	6.4	8.6	160
3.7500	0.118	$56^3 \times 96$	6.6	11.3	204
3.7753	0.111	$56^3 \times 84$	6.2	9.3	128
3.8400	0.095	$64^3 \times 96$	6.1	9.1	198
3.9200	0.078	$80^3 \times 128$	6.2	10.0	184

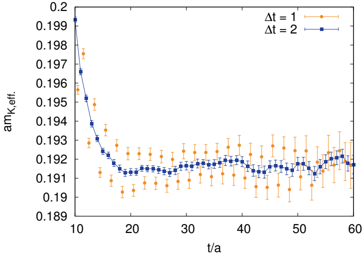


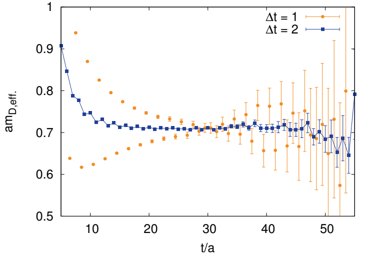
Effective masses, $\beta = 3.9200$

$$\frac{C(t+\Delta t)}{C(t)} = \frac{\cosh\left(m \cdot \left(t + \Delta t - \frac{T}{2}\right)\right)}{\cosh\left(m \cdot \left(t - \frac{T}{2}\right)\right)}$$

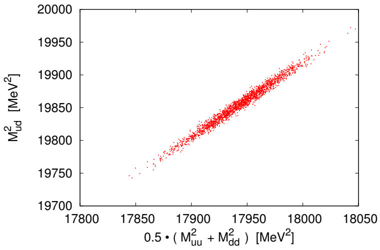
solve for $m \quad \longrightarrow \quad m_{\text{eff.}}\left(t + \frac{\Delta t}{2}\right)$

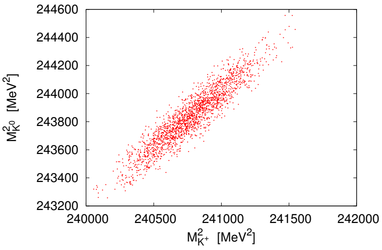


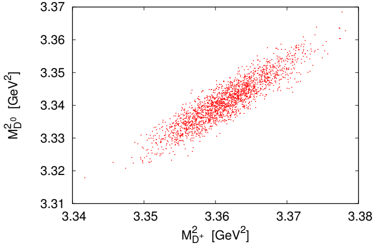




Mass correlation, $\beta = 3.9200$

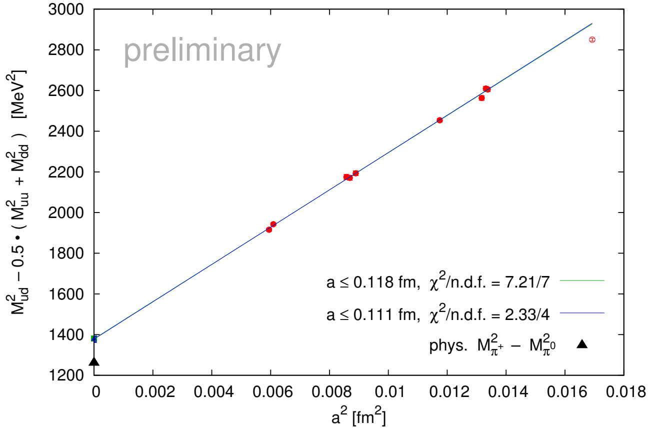






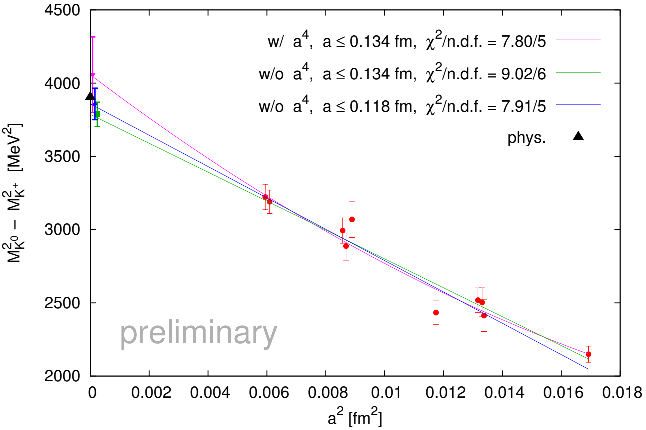
- Masses are highly correlated
- Error of ΔM^2 is smaller than that of M^2

Continuum limit



$$\Delta M_\pi^2(\text{lat.}) = \Delta M_\pi^2 + A_2 \cdot a^2$$

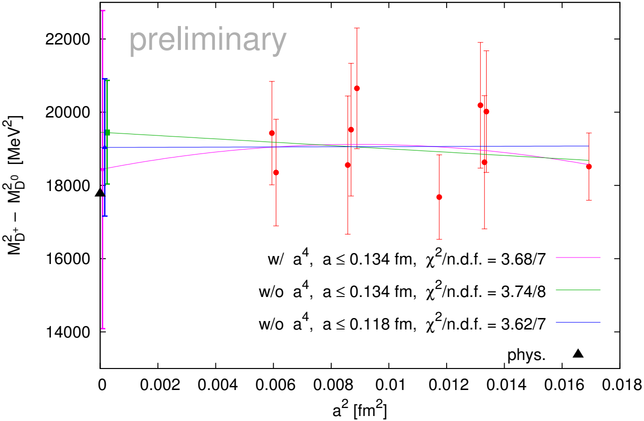
Continuum limit



$$\Delta M_K^2(\text{lat.}) = \Delta M_K^2 + A_2 \cdot a^2 + B \cdot (M_\pi^2 - M_{\pi,\text{phys.}}^2) + C \cdot (M_K^2 - M_{K,\text{phys.}}^2)$$

$$\Delta M_K^2(\text{lat.}) = \Delta M_K^2 + A_2 \cdot a^2 + A_4 \cdot a^4 + B \cdot (M_\pi^2 - M_{\pi,\text{phys.}}^2) + C \cdot (M_K^2 - M_{K,\text{phys.}}^2)$$

Continuum limit



$$\Delta M_D^2(\text{lat.}) = \Delta M_D^2 + A_2 \cdot a^2$$
$$\Delta M_D^2(\text{lat.}) = \Delta M_D^2 + A_2 \cdot a^2 + A_4 \cdot a^4$$

Outline

Conclusions & Outlook

- Conclusions
  - Computed ΔM_π^2, ΔM_K^2 and ΔM_D^2 using quenched QED
  - ΔM_K^2 and ΔM_D^2 are compatible with experimental value
  - ΔM_π^2 disagrees with experimental value
    $\longleftarrow$ lack of disconnected contribution / quenching effects
  - Large lattice artefacts
- Outlook
  - Include disconnected contributions to π^0
  - Use gained experience to compute other observables