

# Higgs compositeness in $Sp(2N)$ gauge theories - Part II : The pure gauge model

Davide Vadicchino<sup>1</sup>

(with E. Bennett<sup>1</sup> D.-K. Hong<sup>2</sup> J.-W. Lee<sup>2</sup> D. Lin<sup>3</sup> B. Lucini<sup>1</sup>  
M. Piai<sup>1</sup> )

<sup>1</sup>Department of Physics - Swansea University - UK

<sup>2</sup>Department of Physics - Pusan National University - South Korea

<sup>3</sup>Department of Physics - National Chiao-Tung University - Taiwan

35th International Symposium on Lattice field theory - Granada,  
18 June 2017

# Outline

## General methods for $Sp(2N)$

- HB algorithm for  $Sp(2N)$ .
- Resymplectization.
- Variational calculus and smoothing routines.

## $Sp(4)$

- String tension.
- Glueball Spectrum.

# The $Sp(2N)$ LGT

## Definition

On a  $\mathbb{Z}^4$  lattice with lattice spacing  $a$

$$S_g[U] = \beta \sum_{x, \mu < \nu} \left( 1 - \frac{1}{N} \text{Re Tr } \mathcal{P}_{\mu\nu}(x) \right),$$

with

$$\mathcal{P}_{\mu\nu}(x) = U_\mu(x) U_\nu(x + \hat{\mu}) U_\mu^\dagger(x + \hat{\nu}) U_\nu^\dagger(x) \in Sp(2N)$$

and  $\beta = 2N/g^2$ .

## Algorithms

- Heathbath (this talk)
- Hybrid MonteCarlo (Talk by E. Bennett)

## Algorithms

HB for  $Sp(2N)$ The  $Sp(2N)$  group

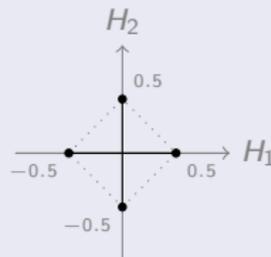
$$U \in Sp(2N) \iff \left\{ U \in SU(2N) / \Omega^\dagger U \Omega = U^* \right\}$$

$$U = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix}, \text{ with } \begin{cases} A^\dagger A + B^\dagger B = \mathbb{I}, \\ A^T B = B^T A, \end{cases} \quad A, B \in \mathbb{C}^{N \times N}$$

The HB - Update on  $SU(2)$  subgroups (Cabibbo-Marinari)

To choose the  $SU(2)$  subgroups:

- $Sp(2N) \subset SU(2N)$
- Block structure of  $Sp(2N)$  matrices.
- Weight diagram of the fundamental representation ( $Sp(4)$  in figure).



# Resymplecticization

Modified Gram-Schmidt algorithm

## GramSchmidt for $SU(N)$

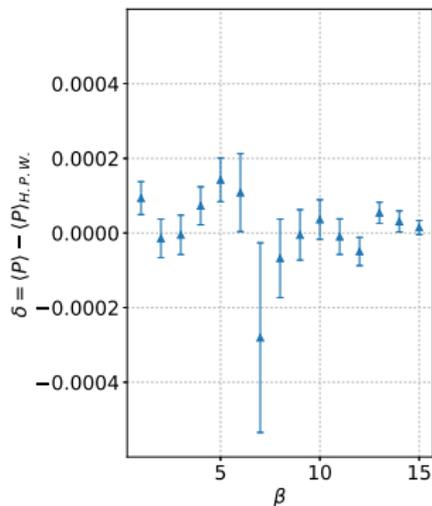
normalize col(1).  
    ortogonalize col(2) w.r.t col(1).  
normalize col(2).  
    ortogonalize col(3) w.r.t col(1).  
    ortogonalize col(3) w.r.t col(2).  
normalize col(3).  
...

## GramSchmidt for $Sp(2N)$

normalize col(1).  
    column  $N$  is set to  $\Omega \times \text{col}(1)^*$   
    ortogonalize col(2) w.r.t col(1).  
    ortogonalize col(2) w.r.t col( $N$ ).  
normalize col(2)  
...

- Fully generalizable to any  $N$ .
- Could reformulate as row-wise.
- Same numerical stability as Gram-Schmidt.
- Other possible approach in E. Bennett's Talk.

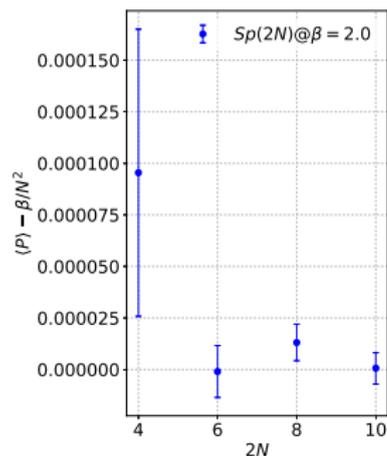
## Simulation Algorithm

A comparison with existing results for  $Sp(4)$ . $Sp(4)$ 

[K. Holland, M. Pepe, U.-J. Wiese('03)]

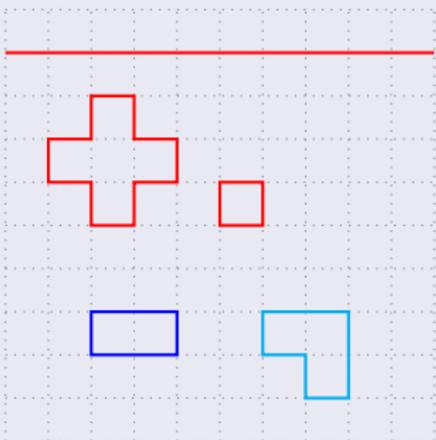
Str. Coupling for  $Sp(2N)$ 

$$\langle P \rangle \sim \frac{\beta}{N^2} + \dots$$



## Gauge invariant operators

On a timeslice...



Zero momentum symmetrized operators

$$\phi_C(t) = \sum_{\vec{x}} \phi_C(x, t) = \sum_{\vec{x}} \text{Tr} \left( \prod_C U_l \right)$$

- $C$  closed loop: contractible (glueballs) or non-contractible (fluxtubes).
- Symmetry of  $|i\rangle$  dictated by  $C$ :  
subduced reps of the rotation group:

$$R^P = A1^\pm, A2^\pm, T1^\pm, T2^\pm, E^\pm$$

- Sum over  $\vec{x}$  for zero momentum operators,

To be measured...

$$\Gamma_C(t) = \frac{\langle \phi_C^\dagger(0) \phi_C(t) \rangle}{\langle \phi_C^\dagger(0) \phi_C(0) \rangle} = \frac{\sum_i |\langle i | \phi_C(0) | 0 \rangle|^2 e^{-m_i t}}{\sum_i |\langle i | \phi_C | 0 \rangle|^2} \sim A_C^{(0)} e^{-m t}$$

## Exponential decay of signal to noise ratio

- Problem 1: How to Maximize  $A_C^{(0)}$ ?
- Problem 2:  $A_C^{(0)} \rightarrow 0$  when  $a \rightarrow 0$ .

### Solution to 1 - Variational calculus

$$C_{ij}(t) = \frac{\langle 0 | \phi_i^\dagger(0) \phi_j(t) | 0 \rangle}{\langle 0 | \phi_i^\dagger(0) \phi_j(0) | 0 \rangle}$$

Diagonalize  $C$  with

$$\Phi(t) = \sum_{i \in R^P} \alpha_i \phi_i(t),$$

obtain  $\tilde{C}$ ,

$$m_{\text{eff}}(t) = -\log \frac{\tilde{C}_{ii}(t)}{\tilde{C}_{ii}(t-1)}.$$

[K. Ishikawa, M. Teper, G. Schierholz ('82)]

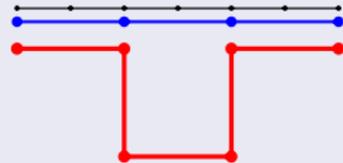
### Solution to 2 - Improved Smearing

- Blocking:  $U_l \rightarrow U_l^b, (N_b, \delta_b)$ .
- Smearing:  $U_l \rightarrow \tilde{U}_l, (N_s, \delta_s)$ .

Combine the two:

$$U_l \rightarrow U_l^b \rightarrow \tilde{U}_l^b$$

and use Cooling to reproject on  $Sp(2N)$ .



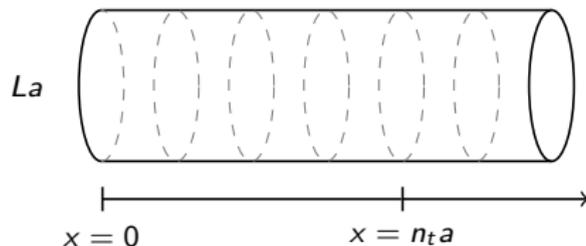
[M. Teper, B. Lucini, U. Wenger('04)]

# General setting

## Lattice parameters

- We implemented the tools to simulate  $Sp(2N)$  into the *HiRep* code.
- Variational analysis fully automatized. [B. Lucini, A. Rago, E. Rinaldi ('10)]
- $L = 10 - 32$
- $\beta = 7.7 - 8.3$
- Update pattern: 1/4 sweeps of *HB/OR*,
- Thermalization and decorrelation: 1000 – 5000 initial sweeps, 20 decorrelation sweeps.
- 10000 configurations at each  $L$  and each  $\beta$ .
- Improved smearing: 200 operators,  $\delta_b = 0.4$ ,  $\delta_s = 0.16$ .
- Cooling: 15 steps.

## Closed fluxtubes



## Closed Fluxtubes

- Non-contractible loops create space-like and time-like propagating fluxtubes.
- With the algorithm above we can measure their masses  $m_s(L)$ ,  $m_t(L)$ .

## Effective String Theory

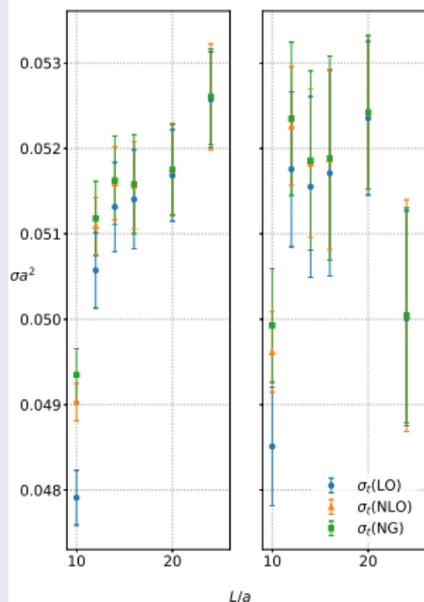
For a closed fluxtube of length  $L$ ,

$$m(L) = \sigma_{LO} L - \frac{\pi}{3L} = \sigma_{NLO} L - \frac{\pi}{3L} - \left(\frac{\pi}{3}\right)^3 \frac{1}{2\sigma_{NLO} L^3} = \sigma_{NG} L \sqrt{1 - \frac{2\pi}{3\sigma_{NG} L}}$$

[M. Lüscher, P. Weisz, O. Aharony, M. Field...]

## The string tension

## Numerical results



## Observations

- At this level of precision, we cannot detect the difference between NLO and NG.
- At  $L \geq 14a$  we are in the infinite volume regime.
- $\sigma_s$  and  $\sigma_t$  are compatible.
- Systematical errors at  $L = 24a$ .

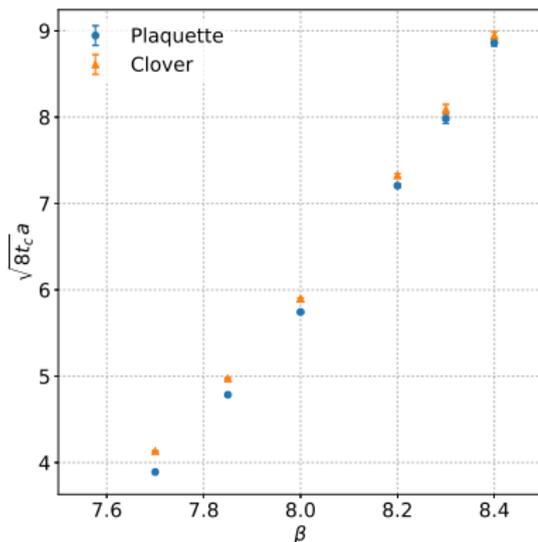
Estimate  $\sqrt{\sigma}$  with

$$\sqrt{\sigma} = \frac{\frac{\sqrt{\sigma_s}}{(\Delta\sqrt{\sigma_s})^2}}{\frac{1}{(\Delta\sqrt{\sigma_s})^2} + \frac{1}{(\Delta\sqrt{\sigma_s})^2}}$$

$$\textcircled{\beta = 7.7, \sigma a^2 = 0.0517(30)}$$

## Scale setting

## Using the Wilson flow. . .



## Settings &amp; Observations

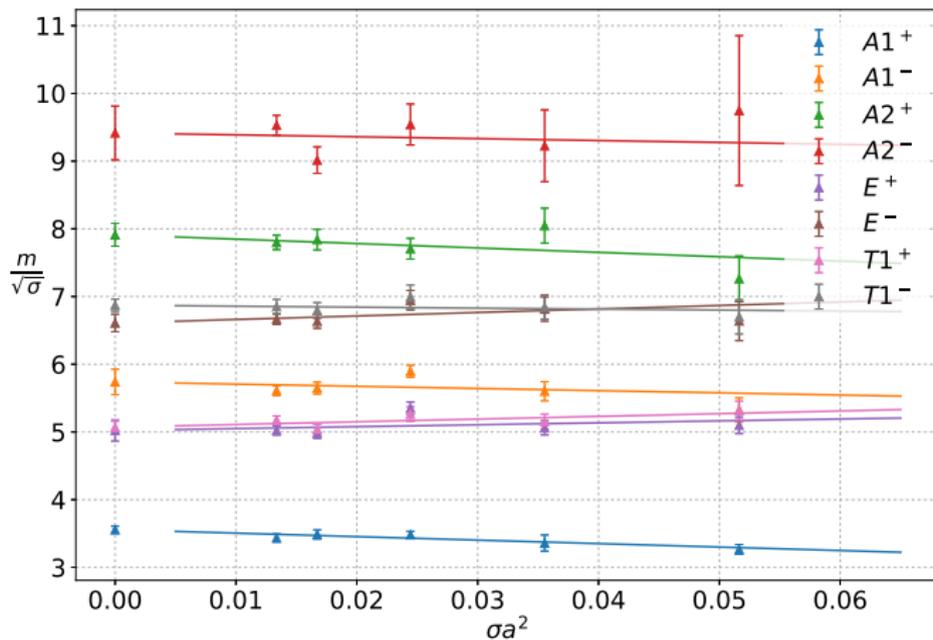
- 200 pure gauge configurations for each point
- $L/a$  ranging from 16 to 32.
- $c_\tau = \frac{\sqrt{8t_c}}{L} \leq 4$  to avoid finite size effects.
- See E.J. Bennet's talk for more detailed account.

## Settings & Observations

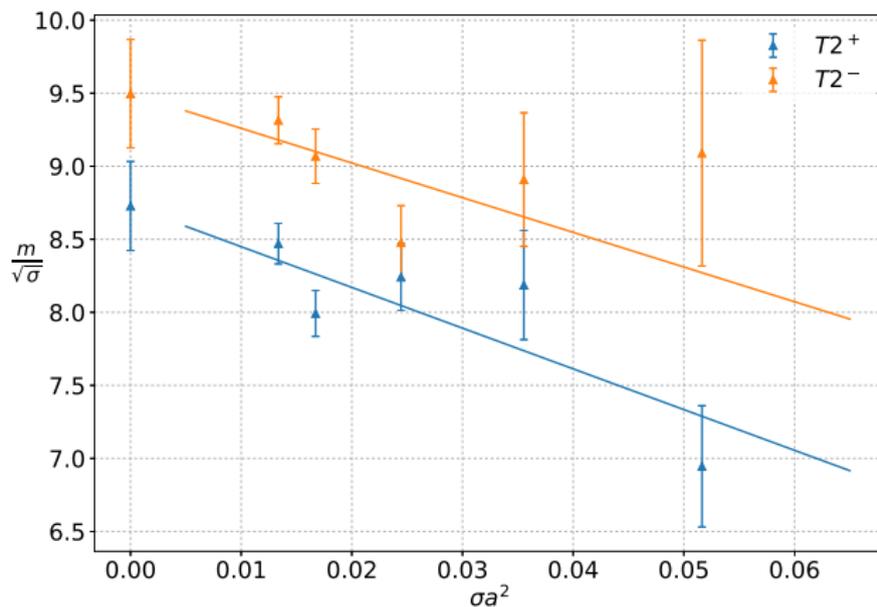
- We compute  $m(R^P)/\sqrt{\sigma}$  in all the symmetry channels.
- Since  $SP(2N)$  has only pseudoreal representations, no charge conjugation.
- $\sim 200$  operators in every symmetry channel were used.
- Physical volume  $\sqrt{\sigma}L$  was kept approximately constant while  $a \rightarrow 0$ .
- Glueball masses have  $\sim \sigma$  discretization effects.
- We expect that, in the continuum limit:

$$m(E^\pm) = m(T1^\pm)$$

## Continuum extrapolations - 1



## Continuum extrapolations - 2



## Numerical Results

## The glueball spectrum in the continuum limit

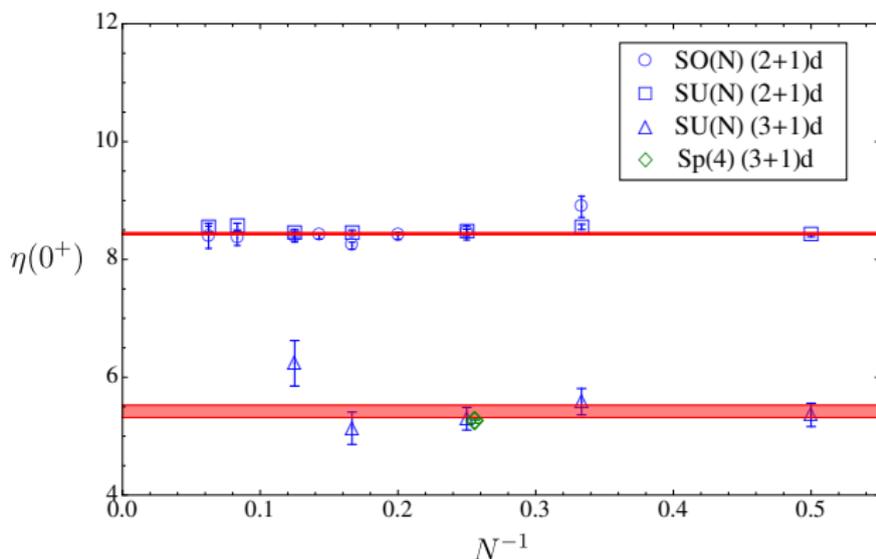
$R^P$	$m(R^P)/\sqrt{\sigma}$
$A1^+$	3.557(52)
$A1^-$	5.74(19)
$A2^+$	7.91(17)
$A2^-$	9.42(40)
$E^+$	5.02(16)
$E^-$	6.61(13)
$T1^+$	5.070(91)
$T1^-$	6.872(89)
$T2^+$	8.73(30)
$T2^-$	9.50(37)

Agreement with  $\eta$  conjecture

We conjecture that

$$\eta = \frac{m_{0^{++}}}{\sqrt{\sigma}} \frac{C_2(F)}{C_2(A)}$$

is a **universal** quantity that only depends on the dimension of spacetime and on the group:  $SU(N)$ ,  $SO(N)$ ,  $Sp(2N)$ . [B. Lucini, M. Piai, D.-K. Hong, J.-W. Lee, D.V.('17)]



## Conclusions

- The tools to simulate pure gauge  $Sp(2N)$  were developed: HeathBath, Resymplecticization, ...
- The glueball spectrum for  $Sp(4)$  was obtained by including these into the *HiRep* package for BSM on the lattice.
- The value of  $\eta$  for  $Sp(4)$  is in agreement with its conjectured universality.
- Future directions: use the developed tools to study  $Sp(2N)$  at large  $N$ .

Thank you for your attention!

(To be continued...)