The pion mass dependence of the nucleon

Lattice 2017: Granada
35th International Symposium on Lattice Field Theory
18-24 June 2017

André Walker-Loud

BERKELEY LAB

Team

UNC	Casey Berger
Jülich	Evan Berkowitz
LBL/UCB	David Brantley, Chia Cheng (Jason) Chang, Thorsten Kurth, Henry Monge Camacho, Amy Nicholson
Glasgow	Chris Bouchard
NVIDIA	Kate Clark
Liverpool/Plymouth	Nicolas Garron
JLab	Balint Jóo
Rutgers	Chris Monahan
William and Mary	Kostas Orginos
RIKEN/BNL	Enrico Rinaldi
LLNL	Pavlos Vranas

red = postdoc
blue = grad student

Outline

- Review Chiral Perturbation Theory
- M_n - M_p
- M_N

Motivation

- Several motivations to understand the pion mass dependence of the nucleon
  - perform extrapolation of nucleon quantities computed with lattice QCD (LQCD) from the input quark masses used to those of nature (still important for nucleons)
  - pion mass dependence of 2+ nucleon systems will also depend upon nucleon
  - determine range of convergence for baryon chiral perturbation theory (HBχPT), and determine low-energy constants (LECs)
  - The LECs are universal, so connected to other quantities of interest: notably, the pion-nucleon sigma term which is important for understanding constraints on dark matter interactions with nuclei

$$\sigma_{\pi N} = \sum_{q=u,d} \langle N|\bar{q} m_q q|N\rangle = \sum_{q=u,d} m_q \frac{\partial}{\partial m_q} m_N(m_q)$$

Chiral Perturbation Theory Overview

- Chiral Perturbation Theory (χPT) is the low-energy Effective Field Theory (EFT) of QCD
- QCD has an approximate chiral symmetry, broken explicitly by the light quark masses

$$\mathcal{L}_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L$$

$$\mathcal{P}_L = \frac{1-\gamma_5}{2} \qquad \mathcal{P}_R = \frac{1+\gamma_5}{2}$$

- In the limit $m_q=0$, QCD has an exact chiral symmetry

$$U(1)_{L+R} \times SU(N_f)_L \times SU(N_f)_R$$

- But this is not readily evident in the spectrum. If the chiral symmetry were respected - the negative parity nucleon would be degenerate with the nucleon

$$M_N \sim 940 \text{ MeV} \qquad M_{N^*} \sim 1535 \text{ MeV}$$

Chiral Perturbation Theory Overview

$$\mathcal{L}_{QCD} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L$$

- We also observe 3 light degrees of freedom, pseudo-scalar pions

$$m_\pi \ll m_N \qquad m_\pi \sim 135 \text{ MeV}$$

- We postulate the vacuum of QCD spontaneously breaks the full chiral symmetry down to the vector subgroup, giving rise to three (pseudo) Nambu-Goldstone bosons associated with the broken Axial generators (here, I am focussed on 2 light flavors, u & d)

$$U(1)_{L+R} \times SU(2)_L \times SU(2)_R \longrightarrow U(1)_V \times SU(2)_V$$

- The up and down quark masses are small, but not zero, so the pions are not massless, and so they are *pseudo* Nambu-Goldstone bosons

Chiral Perturbation Theory Overview

Langacker & Pagels PRD 8 (1973)
Gasser & Leutwyler Annals Phys 158 (1984)

- χPT is then constructed by writing down all operators consistent with the approximate symmetry of QCD, in terms of the observed degrees of freedom - the pions
- The pions are derivatively coupled to themselves and other hadrons due to their Nambu-Goldstone nature.
- This allows for an organization of the operators into different orders in a perturbative expansion in small momentum

$$\mathcal{L}_{\chi\mathrm{PT}}^{\mathrm{LO}} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\Sigma\chi^\dagger + \chi\Sigma^\dagger\right)$$

$$\Sigma = e^{i\sqrt{2}\phi/F} \quad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}$$

- Analogous to SSB of complex ϕ^4-theory, the pions parameterize rotations about the minimum of the vacuum

Chiral Perturbation Theory Overview

Langacker & Pagels PRD 8 (1973)
Gasser & Leutwyler Annals Phys 158 (1984)

- χPT is then constructed by writing down all operators consistent with the approximate symmetry of QCD, in terms of the observed degrees of freedom - the pions
- The pions are derivatively coupled to themselves and other hadrons due to their Nambu-Goldstone nature.
- This allows for an organization of the operators into different orders in a perturbative expansion in small momentum

$$\mathcal{L}_{\chi\mathrm{PT}}^{\mathrm{LO}} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right) + \frac{F^2}{4}\mathrm{Tr}\left(\Sigma\chi^\dagger + \chi\Sigma^\dagger\right)$$

F = LO pion decay constant $\qquad \chi = 2Bm_q$

$$m_\pi^2[\mathrm{LO}] = B(m_u + m_d)$$

$$B = -\frac{\langle\Omega|\bar{q}q|\Omega\rangle}{F^2}$$

$$\epsilon_\pi^2 = \frac{m_\pi^2}{(4\pi F_\pi)^2}$$ perturbative expansion parameter

Chiral Perturbation Theory Overview

Langacker & Pagels PRD 8 (1973)
Gasser & Leutwyler Annals Phys 158 (1984)

- The pion mass and decay constant are then determined through NLO

$$\hat{m} = \frac{1}{2}(m_u + m_d)$$

$$m_\pi^2 = 2B\hat{m}\left\{1 + \frac{2B\hat{m}}{(4\pi F)^2}\left[\frac{1}{2}\ln\left(\frac{2B\hat{m}}{\mu^2}\right) + 2(4\pi)^2 l_3^r(\mu)\right]\right\}$$

LO NLO

$$F_\pi = F_0\left\{1 - \frac{2B\hat{m}}{(4\pi F_0)^2}\left[\ln\left(\frac{2B\hat{m}}{\mu^2}\right) + (4\pi)^2 l_4^r(\mu)\right]\right\}$$

LO NLO

NLO = 1 loop from LO Lagrangian
+ tree level from NLO Lagrangian

- Radiative pion loop corrections generate non-analytic dependence upon the quark masses, in this case, logarithmic
- These are the “predictions” from χPT, the analytic terms are the same as from a Taylor expansion about $\epsilon_\pi = 0$

Chiral Perturbation Theory Overview

Jenkins & Manohar PhysLettB 255 (1991)

- Including matter fields (nucleons) poses additional issues to overcome

$$\mathcal{L}_N = \bar{\psi}\left[i\partial\!\!\!/ - M_0\right]\psi$$

- In loop corrections, the large nucleon mass term, M_0, appears
- $M_0/(4\pi F)\simeq 1$ and so the loop power-counting breaks down
- Introduce phase redefined fields

$$\psi = e^{iv\cdot x M_0}(N_v + H_v) \qquad \frac{1+v\!\!\!/}{2}N_v = N_v \qquad \frac{1-v\!\!\!/}{2}H_v = H_v$$

v_μ = four velocity of nucleon

- In nucleon rest frame, $v_\mu = (1,\mathbf{0})$ and these are particle anti-particle projectors

$$\mathcal{L}_N = \bar{N}_v iv\cdot\partial N_v + \bar{H}_v\left[iv\cdot\partial - 2M_0\right]H_v$$

Chiral Perturbation Theory Overview

Jenkins & Manohar PhysLettB 255 (1991)

- Integrate out the heavy $2M_0$ field, introducing power series in $1/M_0$ operators, restoring power counting

$$\mathcal{L}_N = \bar{N}_v iv \cdot \partial N_v + \mathrm{O}(1/M_0)$$

- Nearly on-shell nucleon has momentum $p_\mu = M_0 v_\mu + k_\mu$
- derivatives acting on N_v field bring down powers of this soft, residual momentum

$$\partial_\mu N_v = -ik_\mu N_v$$

- This is known as Heavy Baryon χPT (HBχPT)

Chiral Perturbation Theory Overview

Jenkins & Manohar PhysLettB 255 (1991)

- HBχPT for M_N up to NNLO

$$\mathcal{L}_{N\pi} = \bar{N} i v \cdot D N - \frac{\alpha_N}{4\pi F} \bar{N} \mathcal{M}_+ N - \frac{\sigma_N}{4\pi F} \bar{N} N \,\mathrm{tr}(\mathcal{M}_+) + 2 g_A^{\mathrm{LO}} \bar{N} S \cdot \mathcal{A} N$$

$$- \bar{N} \frac{D_\perp^2}{2M_0} N + g_A^{\mathrm{LO}} \left(\bar{N} \frac{i\overleftarrow{D} \cdot S}{M_0} v \cdot \mathcal{A} N + \bar{N} v \cdot \mathcal{A} \frac{S \cdot i\overrightarrow{D}}{M_0} N \right) + \frac{b_A}{4\pi F} \bar{N} N \,\mathrm{tr}(\mathcal{A} \cdot \mathcal{A}) + \frac{b_{vA}}{4\pi F} \bar{N} N \,\mathrm{tr}(v \cdot \mathcal{A} v \cdot \mathcal{A})$$

$$+ \frac{b_1}{(4\pi F)^3} \bar{N} \mathcal{M}_+^2 N + \frac{b_2}{(4\pi F)^3} \bar{N} N \,\mathrm{tr}(\mathcal{M}_+^2) + \frac{b_3}{(4\pi F)^3} \bar{N} \mathcal{M}_+ N \,\mathrm{tr}(\mathcal{M}_+) + \frac{b_4}{(4\pi F)^3} \bar{N} N [\mathrm{tr}(\mathcal{M}_+)]^2$$

$$\mathcal{M}_+ = \frac{1}{4} \left(\xi 2 B m_q \xi + \xi^\dagger 2 B m_q \xi^\dagger \right) \qquad \xi^2 = \Sigma = e^{i\sqrt{2}\phi/F}$$

$$\mathcal{A}_\mu = \frac{i}{2} \left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) \qquad S_\mu = \text{spin vector}$$

- This Lagrangian is modified from the literature such that all LECs are dimensionless (extra powers of 4πF are used to normalize the operators)
- One can also add the nearby delta-resonances as explicit degrees of freedom (DOF) - large N_c puts this on sound theoretical ground
- Still open how necessary they are in general as explicit DOF

Chiral Perturbation Theory Overview

Jenkins & Manohar PhysLettB 255 (1991)

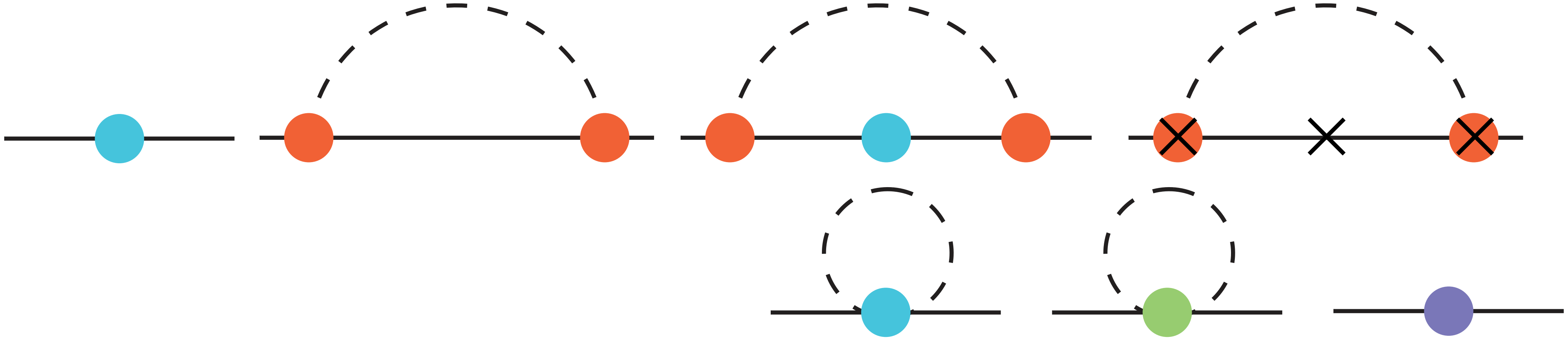
- HBχPT for M_N up to NNLO

$$\mathcal{L}_{N\pi} = \bar{N} i v\cdot D N - \frac{\alpha_N}{4\pi F}\bar{N}\mathcal{M}_+ N - \frac{\sigma_N}{4\pi F}\bar{N}N\,\mathrm{tr}(\mathcal{M}_+) + 2g_A^{\mathrm{LO}}\bar{N}S\cdot\mathcal{A}N$$

$$- \bar{N}\frac{D_\perp^2}{2M_0}N + g_A^{\mathrm{LO}}\left(\bar{N}\frac{i\overleftarrow{D}\cdot S}{M_0}v\cdot\mathcal{A}N + \bar{N}v\cdot\mathcal{A}\frac{S\cdot i\overrightarrow{D}}{M_0}N\right) + \frac{b_A}{4\pi F}\bar{N}N\,\mathrm{tr}(\mathcal{A}\cdot\mathcal{A}) + \frac{b_{vA}}{4\pi F}\bar{N}N\,\mathrm{tr}(v\cdot\mathcal{A}v\cdot\mathcal{A})$$

$$+ \frac{b_1}{(4\pi F)^3}\bar{N}\mathcal{M}_+^2 N + \frac{b_2}{(4\pi F)^3}\bar{N}N\,\mathrm{tr}(\mathcal{M}_+^2) + \frac{b_3}{(4\pi F)^3}\bar{N}\mathcal{M}_+N\,\mathrm{tr}(\mathcal{M}_+) + \frac{b_4}{(4\pi F)^3}\bar{N}N[\mathrm{tr}(\mathcal{M}_+)]^2$$

$$\Delta M_N =$$



LO	NLO	NNLO	
$\frac{m_\pi^2}{4\pi F}$	$-\frac{3\pi g_A^2 m_\pi^3}{2(4\pi F)^2}$	$\frac{m_\pi^4}{(4\pi F)^3}\ln\left(\frac{m_\pi^2}{\mu^2}\right)$	$\frac{m_\pi^4}{(4\pi F)^3}$

Chiral Perturbation Theory Overview

Jenkins & Manohar PhysLettB 255 (1991)

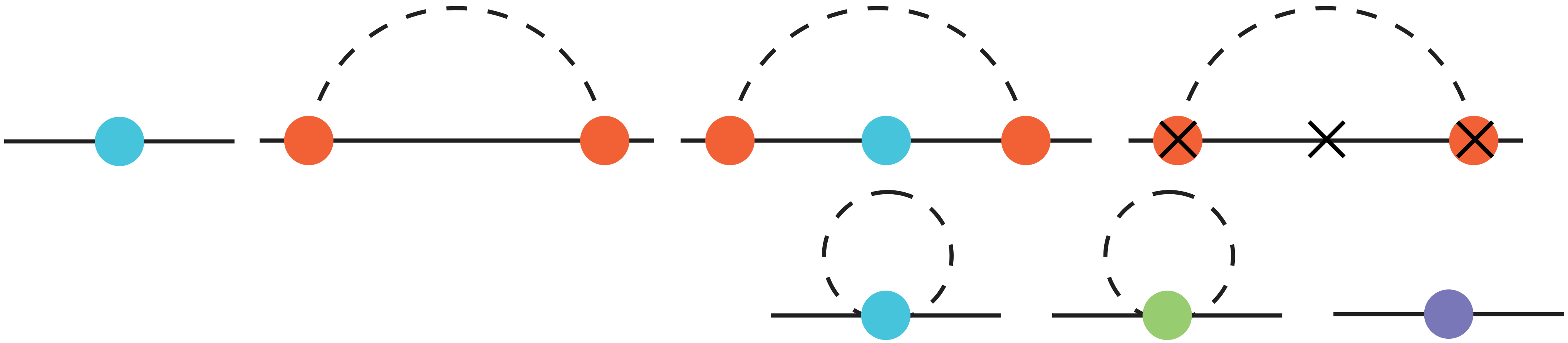
- HBχPT for M_N up to NNLO

$$\mathcal{L}_{N\pi} = \bar{N} i v\cdot D N - \frac{\alpha_N}{4\pi F}\bar{N}\mathcal{M}_+ N - \frac{\sigma_N}{4\pi F}\bar{N}N\,\mathrm{tr}(\mathcal{M}_+) + 2g_A^{\mathrm{LO}}\bar{N}S\cdot\mathcal{A}N$$

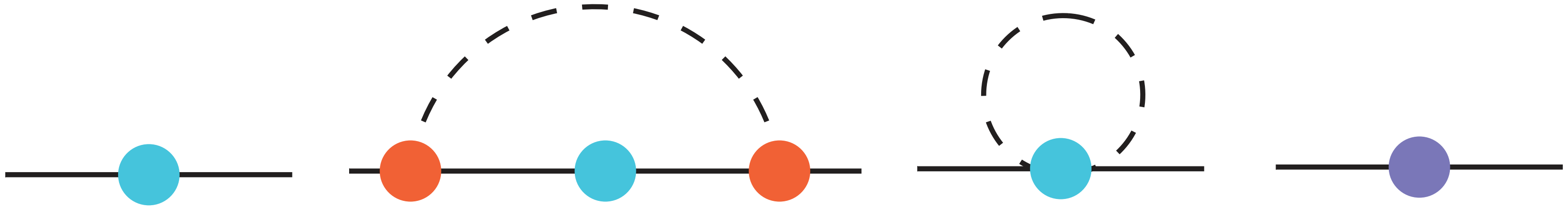
$$- \bar{N}\frac{D_\perp^2}{2M_0}N + g_A^{\mathrm{LO}}\left(\bar{N}\frac{i\overleftarrow{D}\cdot S}{M_0}v\cdot\mathcal{A}N + \bar{N}v\cdot\mathcal{A}\frac{S\cdot i\overrightarrow{D}}{M_0}N\right) + \frac{b_A}{4\pi F}\bar{N}N\,\mathrm{tr}(\mathcal{A}\cdot\mathcal{A}) + \frac{b_{vA}}{4\pi F}\bar{N}N\,\mathrm{tr}(v\cdot\mathcal{A}v\cdot\mathcal{A})$$

$$+ \frac{b_1}{(4\pi F)^3}\bar{N}\mathcal{M}_+^2 N + \frac{b_2}{(4\pi F)^3}\bar{N}N\,\mathrm{tr}(\mathcal{M}_+^2) + \frac{b_3}{(4\pi F)^3}\bar{N}\mathcal{M}_+N\,\mathrm{tr}(\mathcal{M}_+) + \frac{b_4}{(4\pi F)^3}\bar{N}N[\mathrm{tr}(\mathcal{M}_+)]^2$$

$$\Delta M_N =$$



$$M_n - M_p =$$



$$= \frac{2B\delta}{4\pi F}\left\{\alpha_N\left[1 - \frac{m_\pi^2}{2(4\pi F)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta(\mu)\frac{m_\pi^2}{(4\pi F)^2}\right\}$$

$$\delta = \frac{1}{2}(m_d - m_u)$$

Walker-Loud arXiv:0904.2404

arXiv.org > hep-lat > arXiv:1612.07733

High Energy Physics - Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

(Submitted on 22 Dec 2016)

☐ Add isospin breaking in valence sector through *Symmetric Isospin Breaking*

Walker-Loud arXiv:0904.2404

$$m_{u,d}^{sea} = m_l$$

$$m_u^{val} = m_l - \delta \,, \quad m_d^{val} = m_l + \delta$$

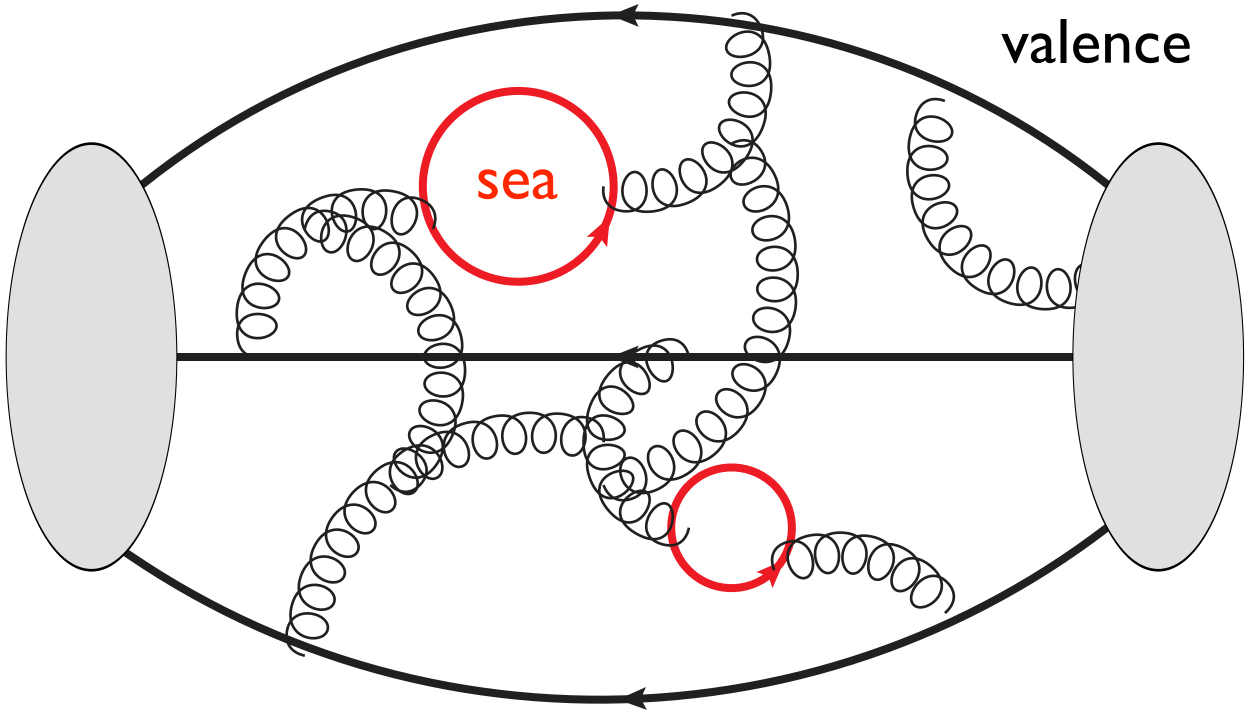
partially quenched lattice QCD

$$m_q^{val} \neq m_q^{sea}$$

☐ With this symmetric splitting, the error in such a calculation scales as

$\mathcal{O}(\delta^2)$ iso-scalar quantities

$\mathcal{O}(\delta^3)$ iso-vector quantities



arXiv.org > hep-lat > arXiv:1612.07733

High Energy Physics – Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

(Submitted on 22 Dec 2016)

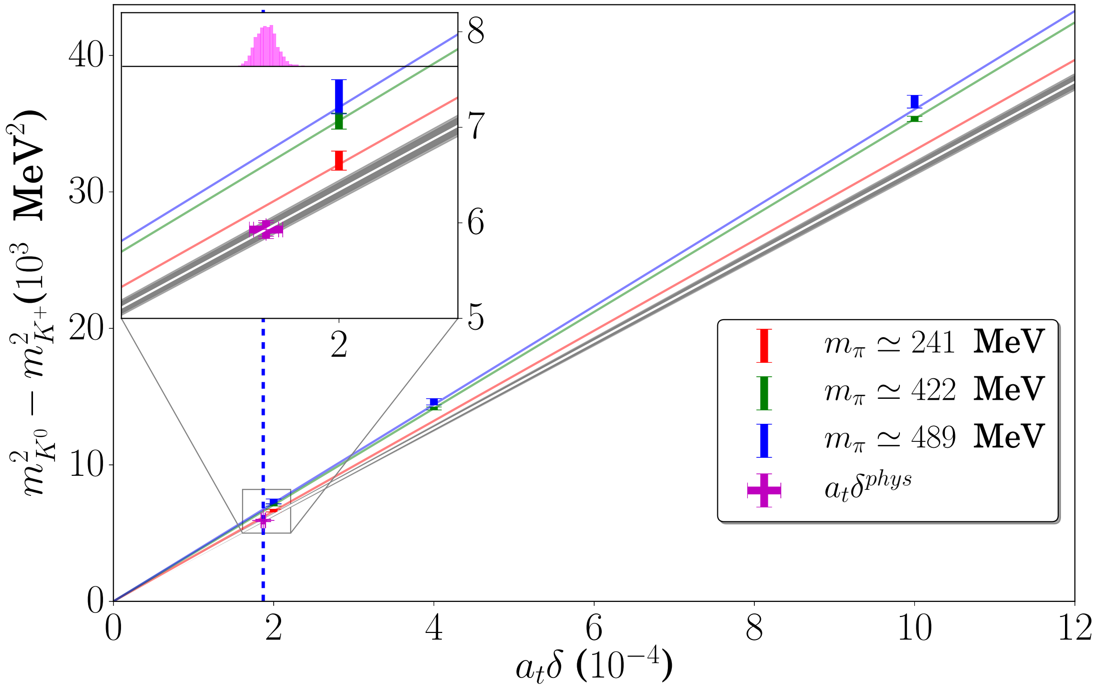
☐ Our calculation was performed with 3 values of the pion mass and we chose 3 values of the isospin breaking parameter $2\delta = m_d - m_u$

m_π [MeV]	m_K [MeV]	$N_{cfg} \times N_{src}$		
		$\mathring{\delta} = 1.22$ MeV	$\mathring{\delta} = 2.44$ MeV	$\mathring{\delta} = 6.11$ MeV
490	629	207×16	207×16	207×16
421	588	291×10	291×10	291×10
241	506	802×10.5	–	–

☐ The physical value of δ was determined from the kaon spectrum (after subtracting the electromagnetic corrections)

$$m_{K^0}^2 - m_{K^\pm}^2 = 2B\delta \left\{ 1 + \frac{m_\pi^2}{(4\pi f_\pi)^2} \left[\alpha(\mu) - \ln\left(\frac{m_\pi^2}{\mu^2}\right) \right] \right\}$$

$$\mathring{\delta}^{phys} = 1.14(1)(1)\ \text{MeV}$$



arXiv.org > hep-lat > arXiv:1612.07733

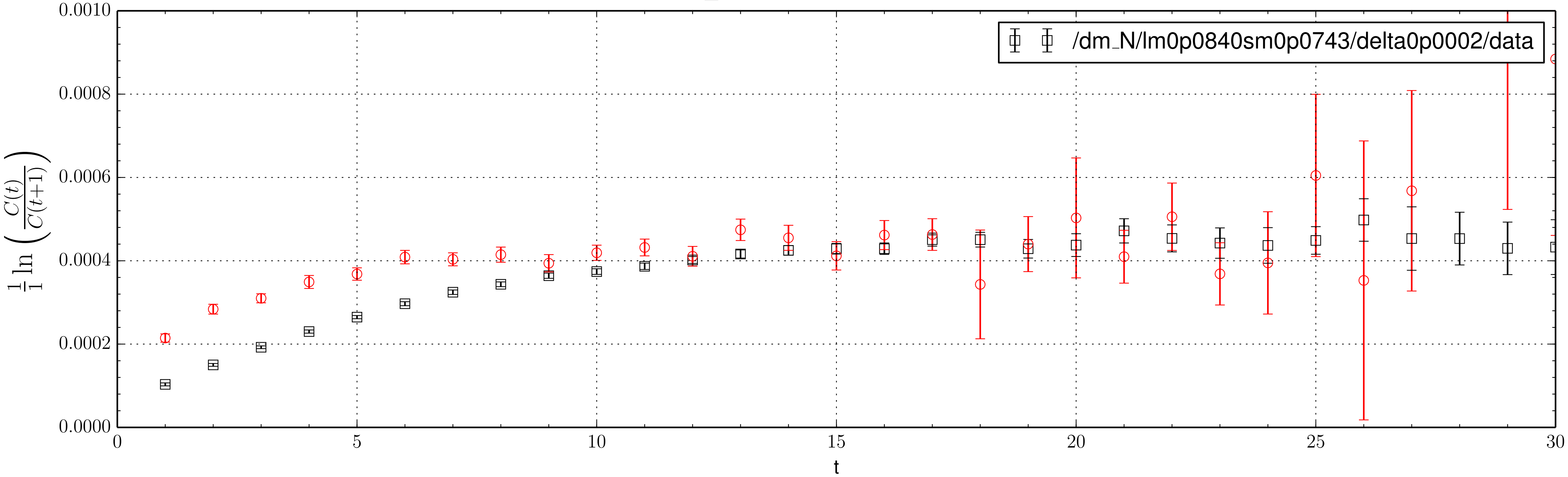
High Energy Physics - Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

(Submitted on 22 Dec 2016)

- ☐ Here is the ratio of the neutron / proton correlation function



$$\frac{C_n(t)}{C_p(t)} = e^{-\delta M_N t}\,\frac{A_0+\delta_0^n+(A_1+\delta_1^n)e^{-(\Delta+\delta\Delta^n)t}+\cdots}{A_0+\delta_0^p+(A_1+\delta_1^p)e^{-(\Delta+\delta\Delta^p)t}}$$

$$= e^{-\delta M_N t}\left\{1+(\delta_0^n-\delta_0^p)+\left[\delta_1^n-\delta_1^p-A_1(\delta\Delta^n-\delta\Delta^p)t\right]e^{-\Delta t}\right\}$$

In ratio, excited state mass gap is the nucleon excited state, $\Delta >> M_n - M_p$

arXiv.org > hep-lat > arXiv:1612.07733

Search

(Help | Adv

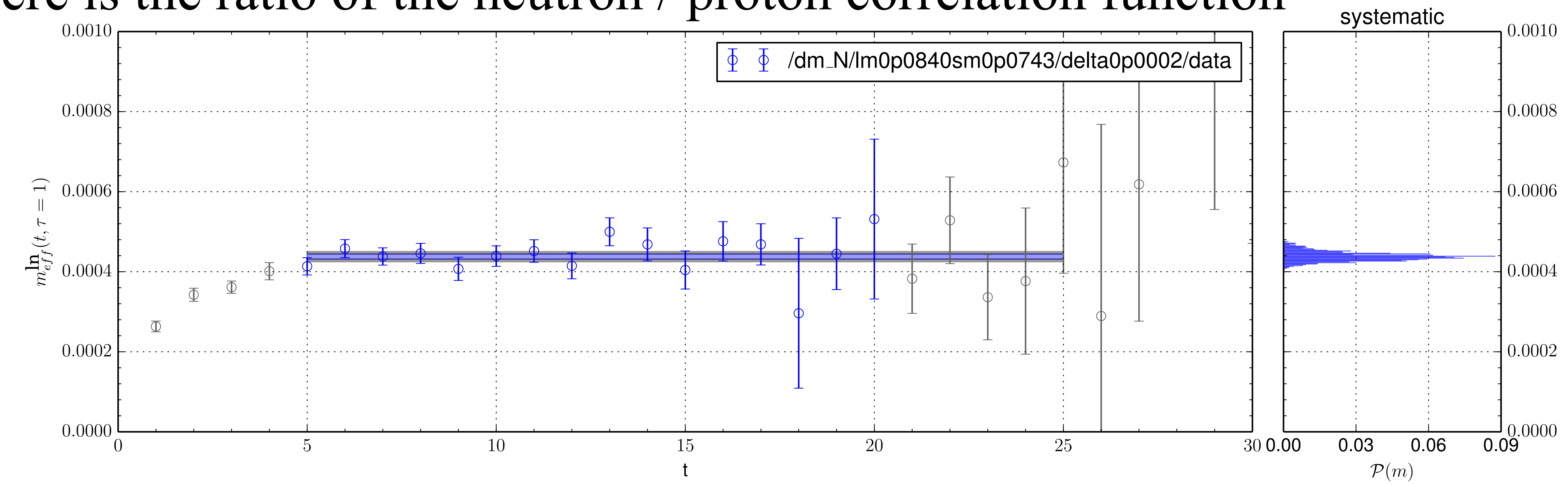
High Energy Physics - Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

(Submitted on 22 Dec 2016)

☐ Here is the ratio of the neutron / proton correlation function



$$\frac{C_n(t)}{C_p(t)} = e^{-\delta M_N t}\frac{A_0+\delta_0^n+(A_1+\delta_1^n)e^{-(\Delta+\delta\Delta^n)t}+\cdots}{A_0+\delta_0^p+(A_1+\delta_1^p)e^{-(\Delta+\delta\Delta^p)t}}$$

$$= e^{-\delta M_N t}\left\{1+(\delta_0^n-\delta_0^p)+\left[\delta_1^n-\delta_1^p-A_1(\delta\Delta^n-\delta\Delta^p)t\right]e^{-\Delta t}\right\}$$

In ratio, excited state mass gap is the nucleon excited state, $\Delta >> M_n - M_p$

arXiv.org > hep-lat > arXiv:1612.07733

Search

(Help | Adv

High Energy Physics - Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

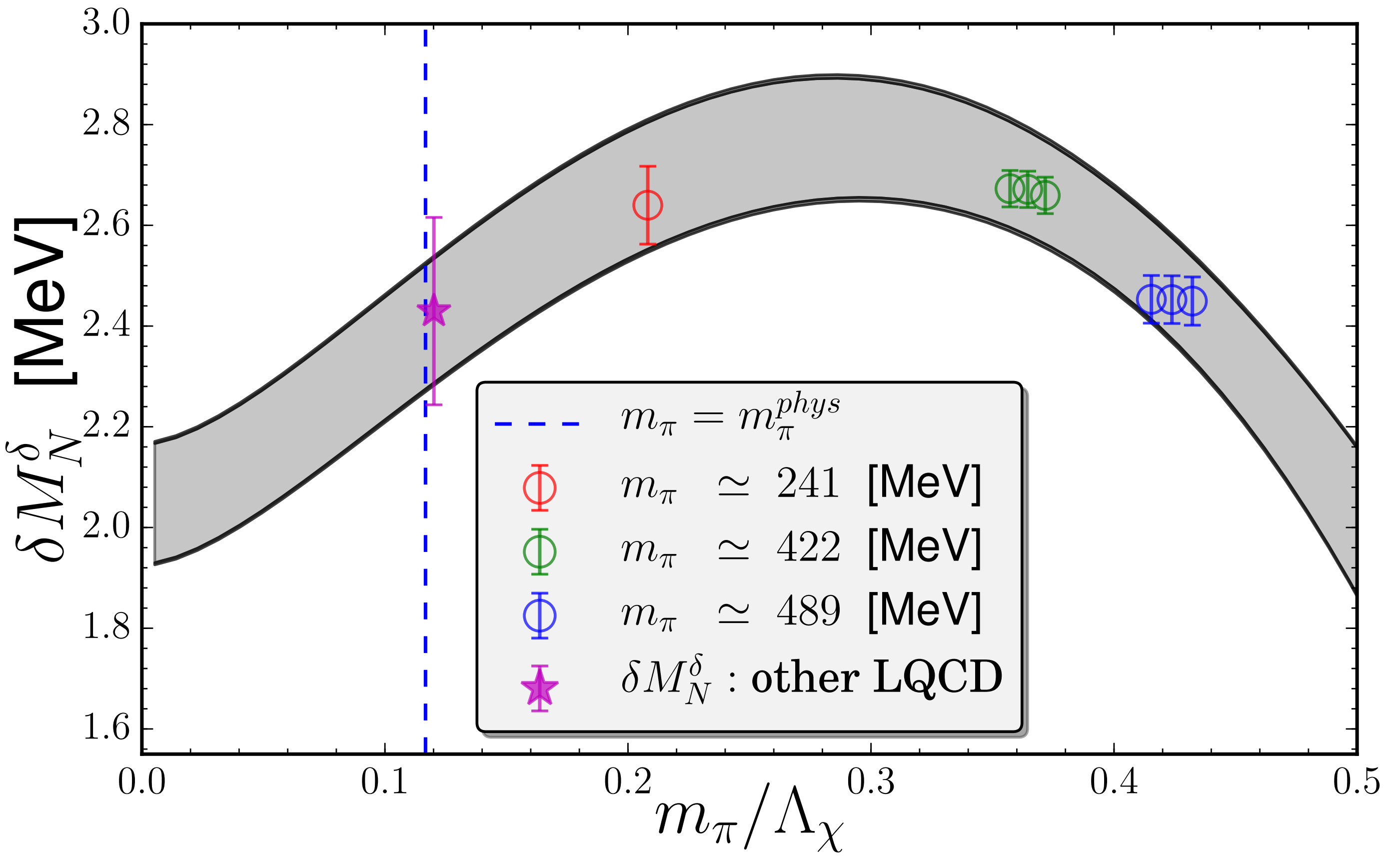
(Submitted on 22 Dec 2016)

☐ Here is the resulting extrapolation

☐ Fit-function is prediction from chiral-perturbation theory

$$\delta M_N^\delta = \delta\left\{\alpha_N\left[1-\frac{m_\pi^2}{(4\pi f_\pi)^2}(6g_A^2+1)\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + 2\beta(\mu)\frac{m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

$g_A =$ nucleon axial coupling



☐ curvature driven by competition between log and local counter-term $\beta(\mu)$

☐ This was first conclusive evidence for the presence of such logarithmic behavior in the baryon spectrum from lattice QCD (with one assumption)


arXiv.org > hep-lat > arXiv:1612.07733

High Energy Physics - Lattice

Strong isospin violation and chiral logarithms in the baryon spectrum

David A. Brantley, Balint Joo, Ekaterina V. Mastropas, Emanuele Mereghetti, Henry Monge-Camacho, Brian C. Tiburzi, Andre Walker-Loud

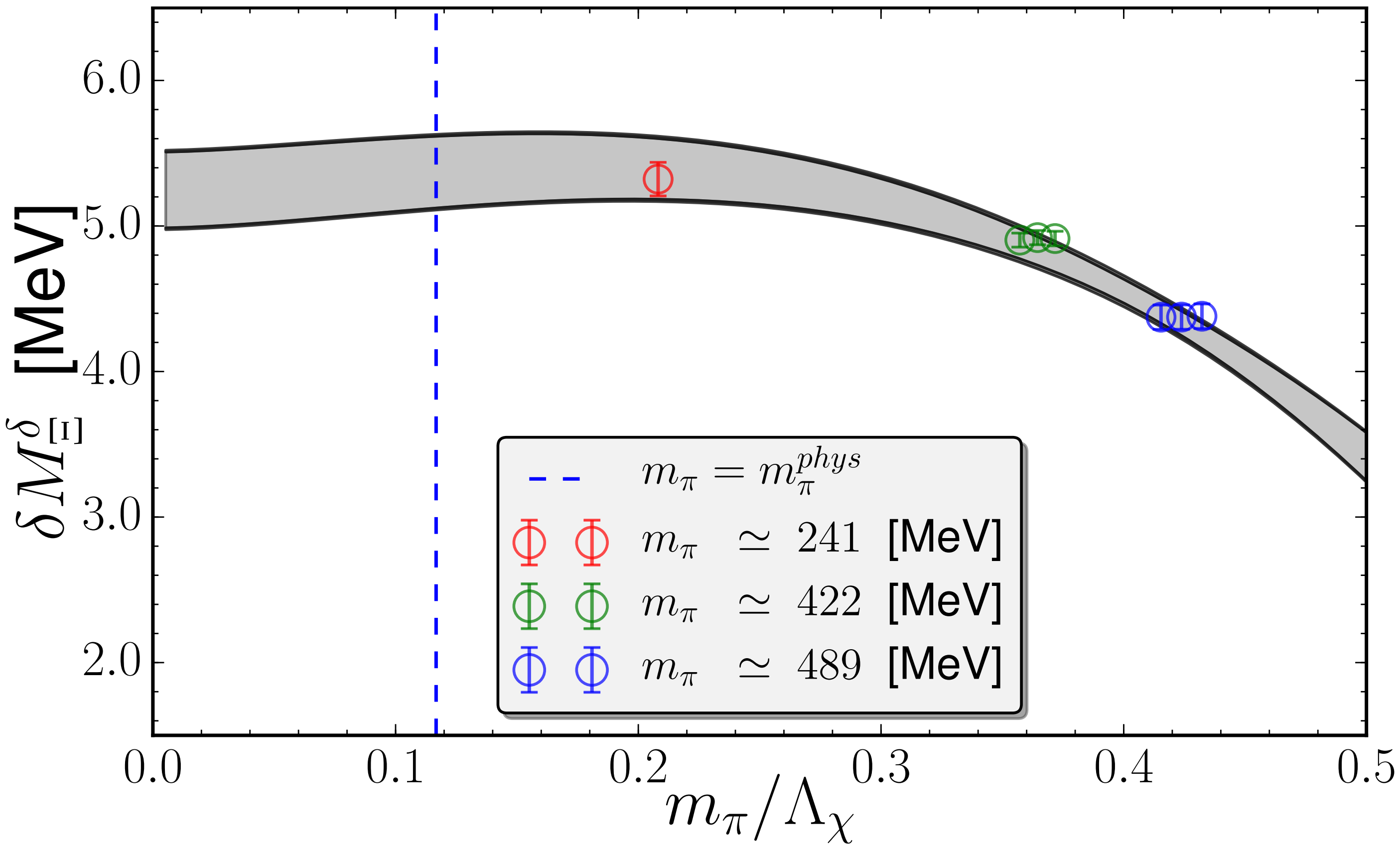
(Submitted on 22 Dec 2016)

- ☐ Same calculation, but for the Cascade Baryon mass splitting
- ☐ Form of Lagrangian identical - only difference is value of LECs

$$\delta M_{\Xi}^{\delta} = \delta\left\{\alpha_{\Xi}\left[1-(6g_{\pi\Xi\Xi}^2+1)\frac{m_\pi^2}{(4\pi f_\pi)^2}\ln\left(\frac{m_\pi^2}{\mu^2}\right)\right] + \beta_{\Xi}(\mu)\frac{2m_\pi^2}{(4\pi f_\pi)^2}\right\}$$

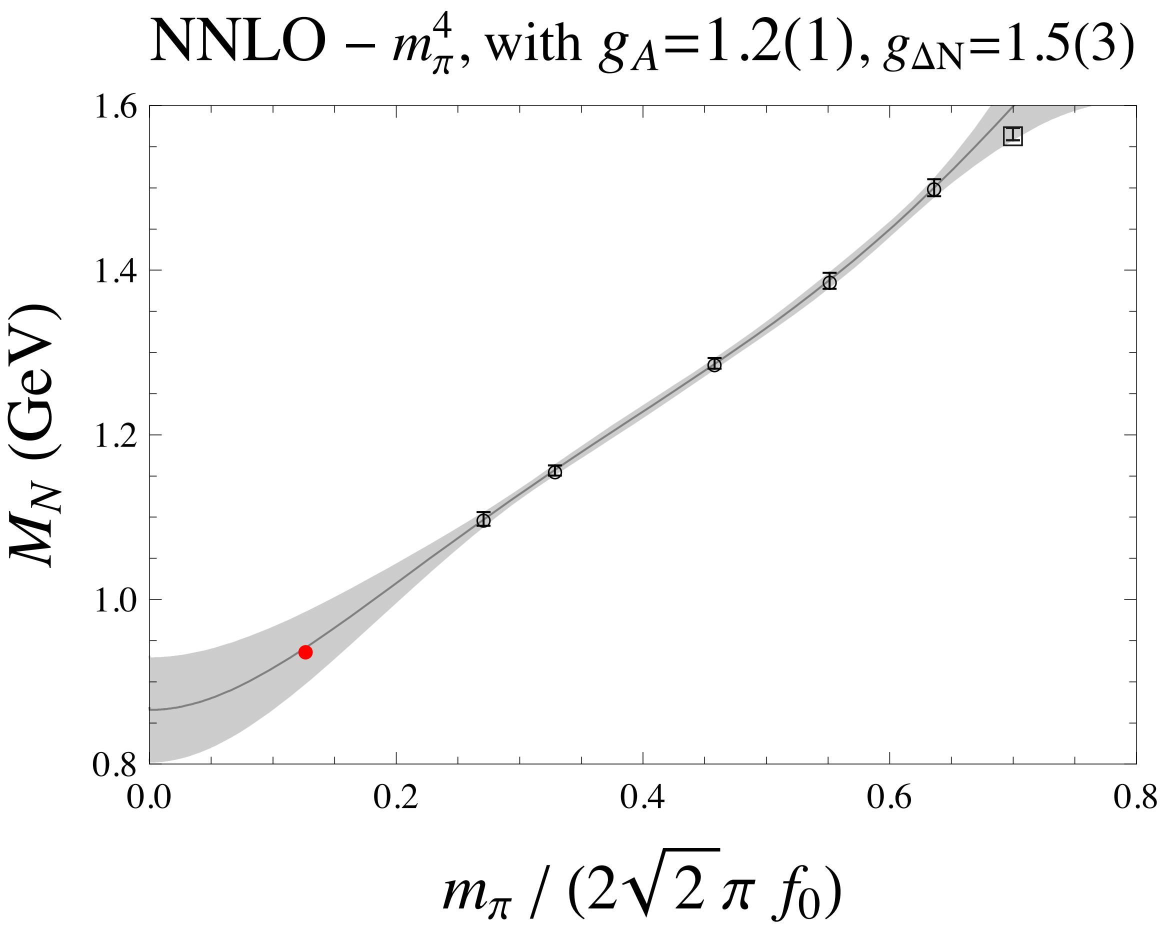
$g_{\pi\Xi\Xi}$ = cascade axial coupling

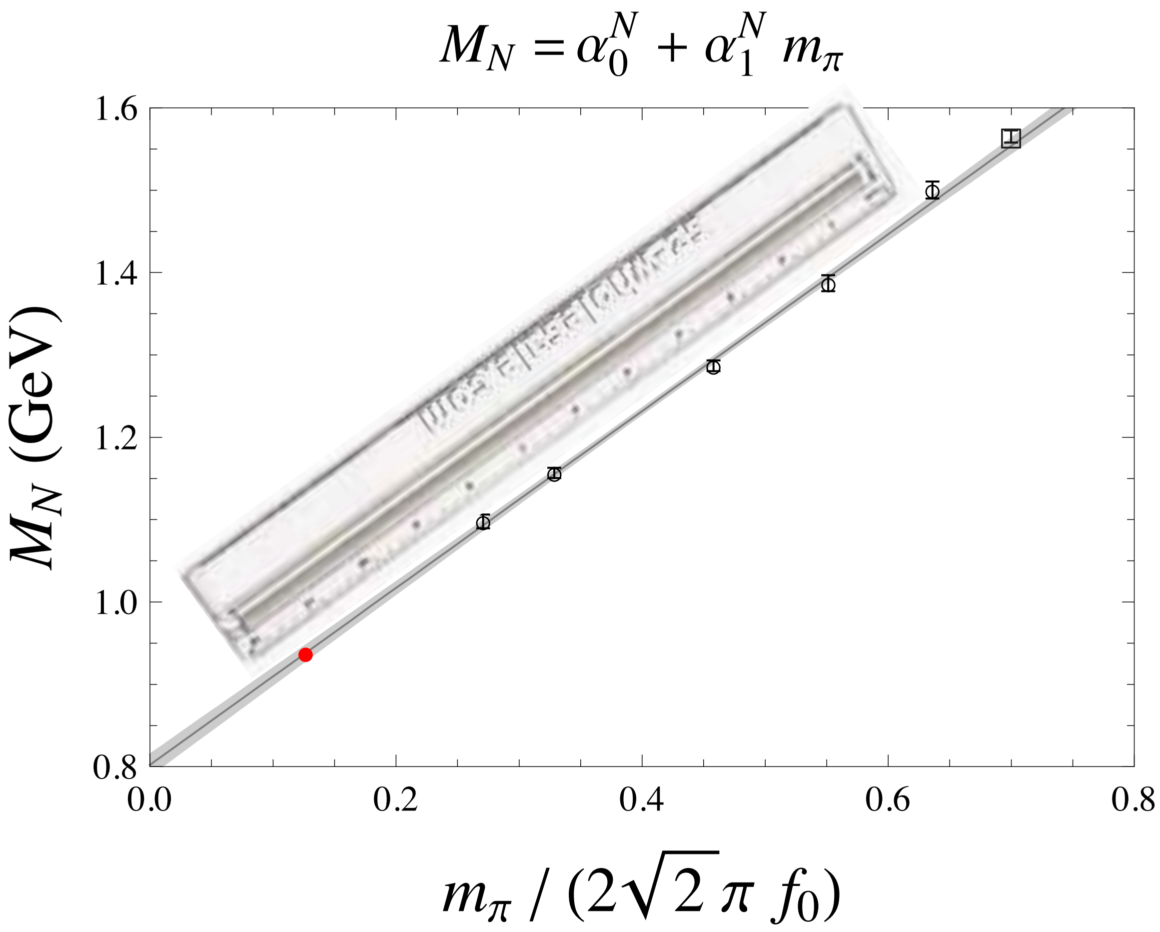
$g_{\pi\Xi\Xi} \simeq g_A/5$



- ☐ We observe the curvature is now "washed out" because the coefficient of the chiral logarithm is much smaller for the Ξ

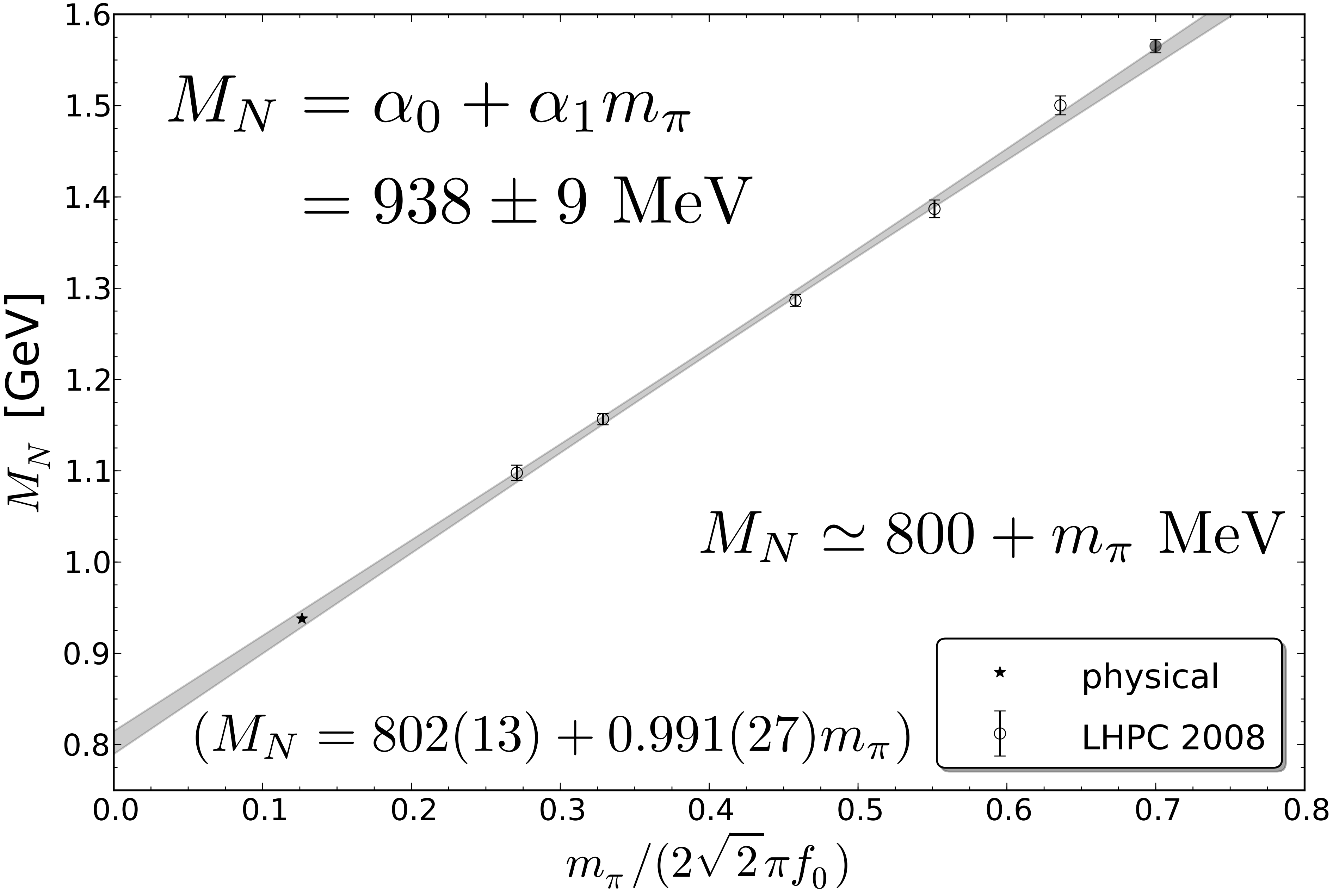
M_N OLD 2008 work: LHPC 0806.4549


NNLO – m_π^4, with g_A=1.2(1), $g_{\Delta N}$=1.5(3)


$M_N = \alpha_0^N + \alpha_1^N m_\pi$

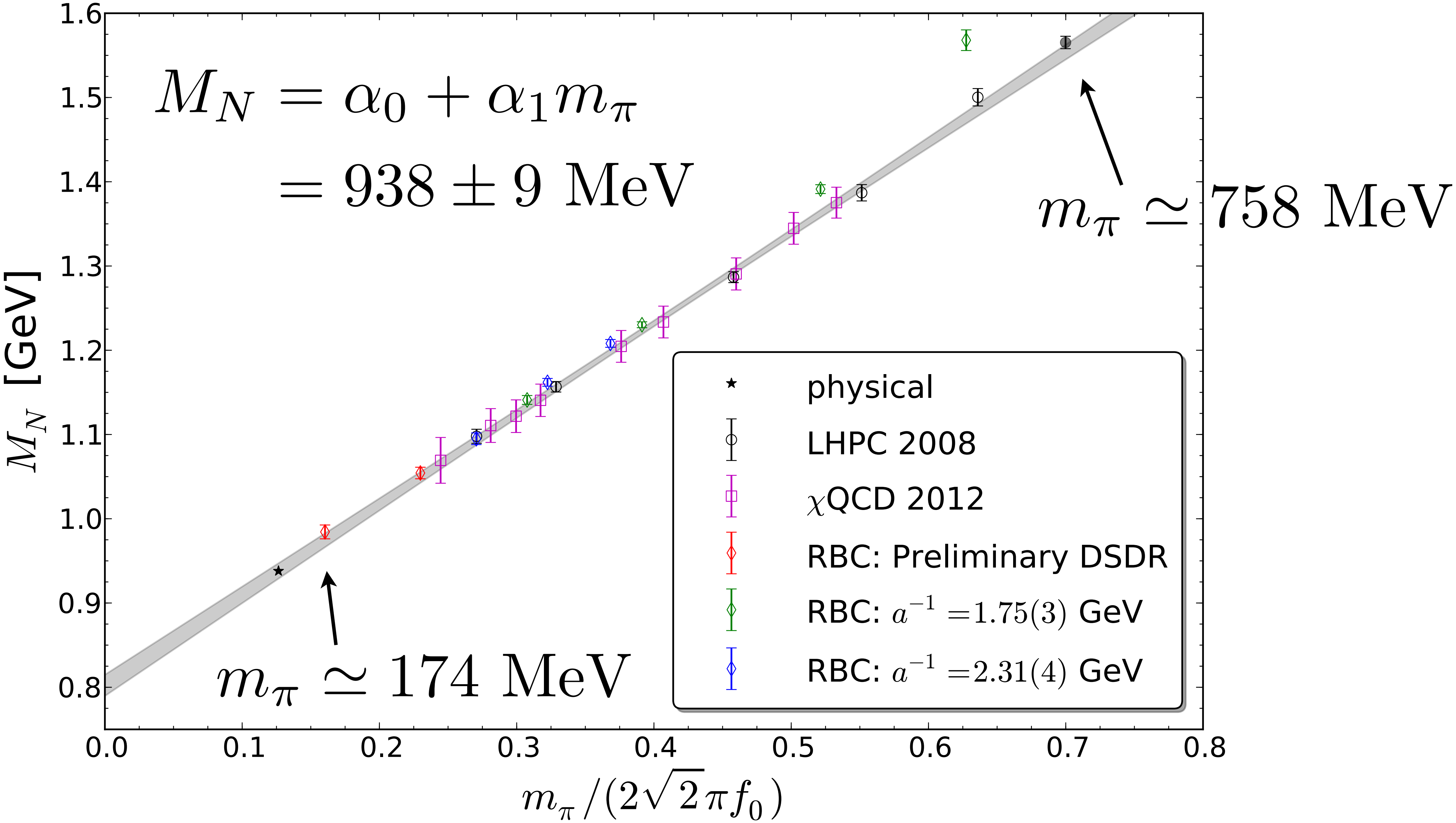
- Is this a lattice artifact? Or a conspiracy of QCD?
- All lattice calculations in 2008 with 2+1 dynamical fermion showed this linear pion mass dependence

What is the status now (2012)?



Physical point NOT included in fit

What is the status now (2012)?



Taking this seriously yields

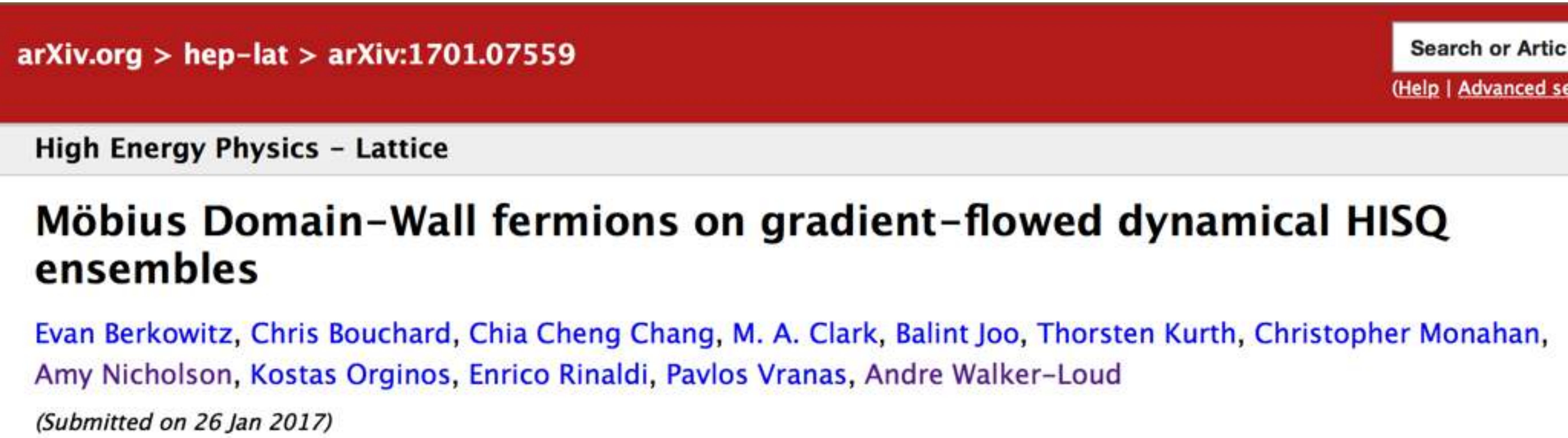
$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

$$M_N \simeq 800 + m_\pi \text{ MeV}$$

$$(M_N = 802(13) + 0.991(27) m_\pi)$$

M_N

work in progress



arXiv.org > hep-lat > arXiv:1701.07559

Search or Artic

(Help | Advanced se

High Energy Physics – Lattice

Möbius Domain-Wall fermions on gradient-flowed dynamical HISQ ensembles

Evan Berkowitz, Chris Bouchard, Chia Cheng Chang, M. A. Clark, Balint Joo, Thorsten Kurth, Christopher Monahan, Amy Nicholson, Kostas Orginos, Enrico Rinaldi, Pavlos Vranas, Andre Walker-Loud

(Submitted on 26 Jan 2017)

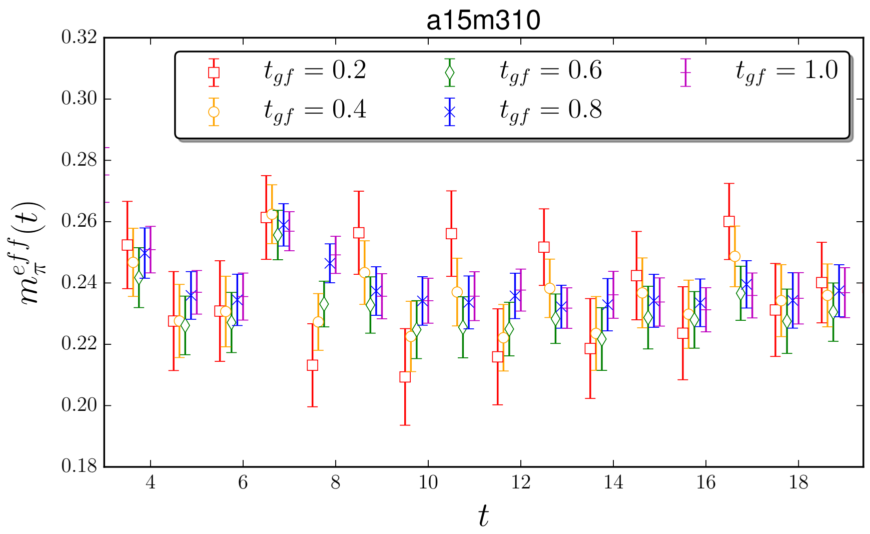
Möbius DWF on HISQ: chiral symmetry in valence sector

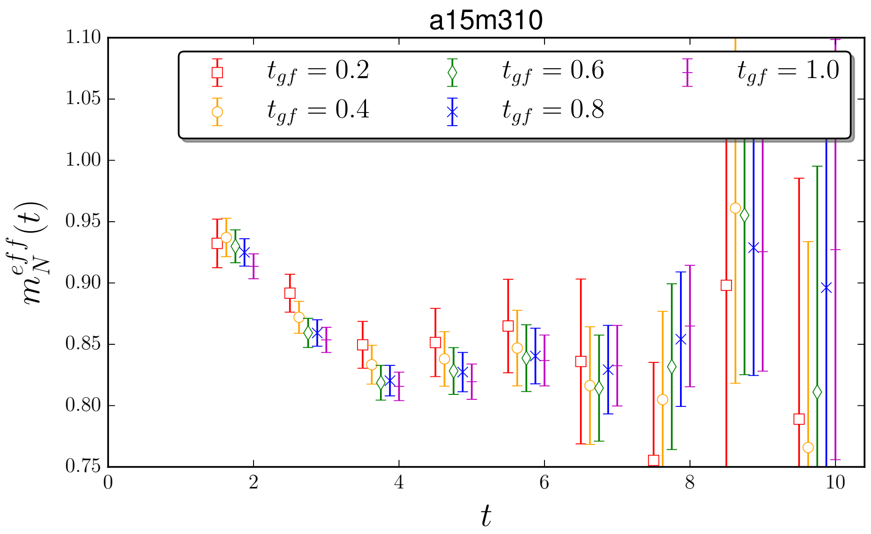
- Gradient flow method for smearing configs
  - $m_{res} < 0.1\ m_l$ for moderate L_5

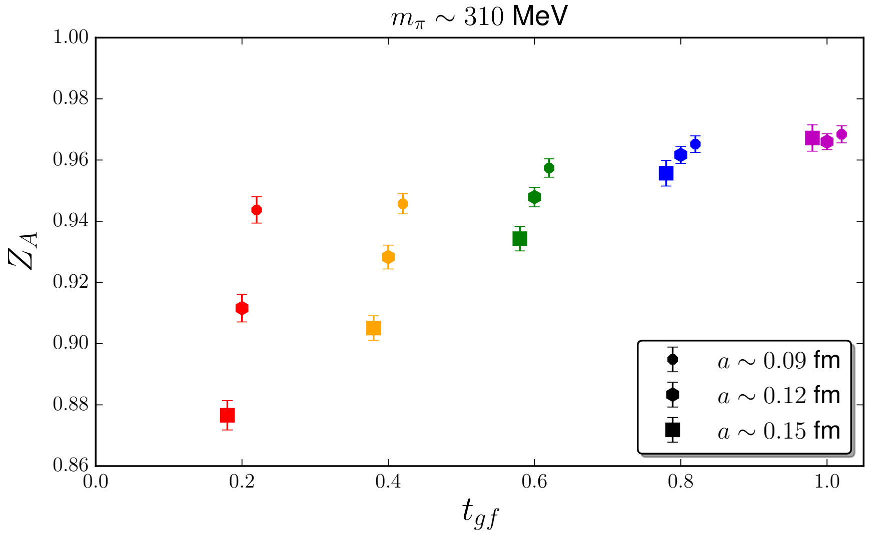
m_π[MeV] a[fm]	400	350	310	220	130
0.15			$16^3 \times 48,\ m_\pi L \sim 3.8$	$24^3 \times 48,\ m_\pi L \sim 4.0$	$32^3 \times 48,\ m_\pi L \sim 3.2$
0.12				$24^3 \times 64,\ m_\pi L \sim 3.2$	
0.12	$24^3 \times 64,\ m_\pi L \sim 5.8$	$24^3 \times 64,\ m_\pi L \sim 5.1$	$24^3 \times 64,\ m_\pi L \sim 4.5$	$32^3 \times 64,\ m_\pi L \sim 4.3$	$48^3 \times 64,\ m_\pi L \sim 3.9$
0.12				$40^3 \times 64,\ m_\pi L \sim 5.4$	
0.09			$32^3 \times 96,\ m_\pi L \sim 4.5$	$48^3 \times 96,\ m_\pi L \sim 4.7$	

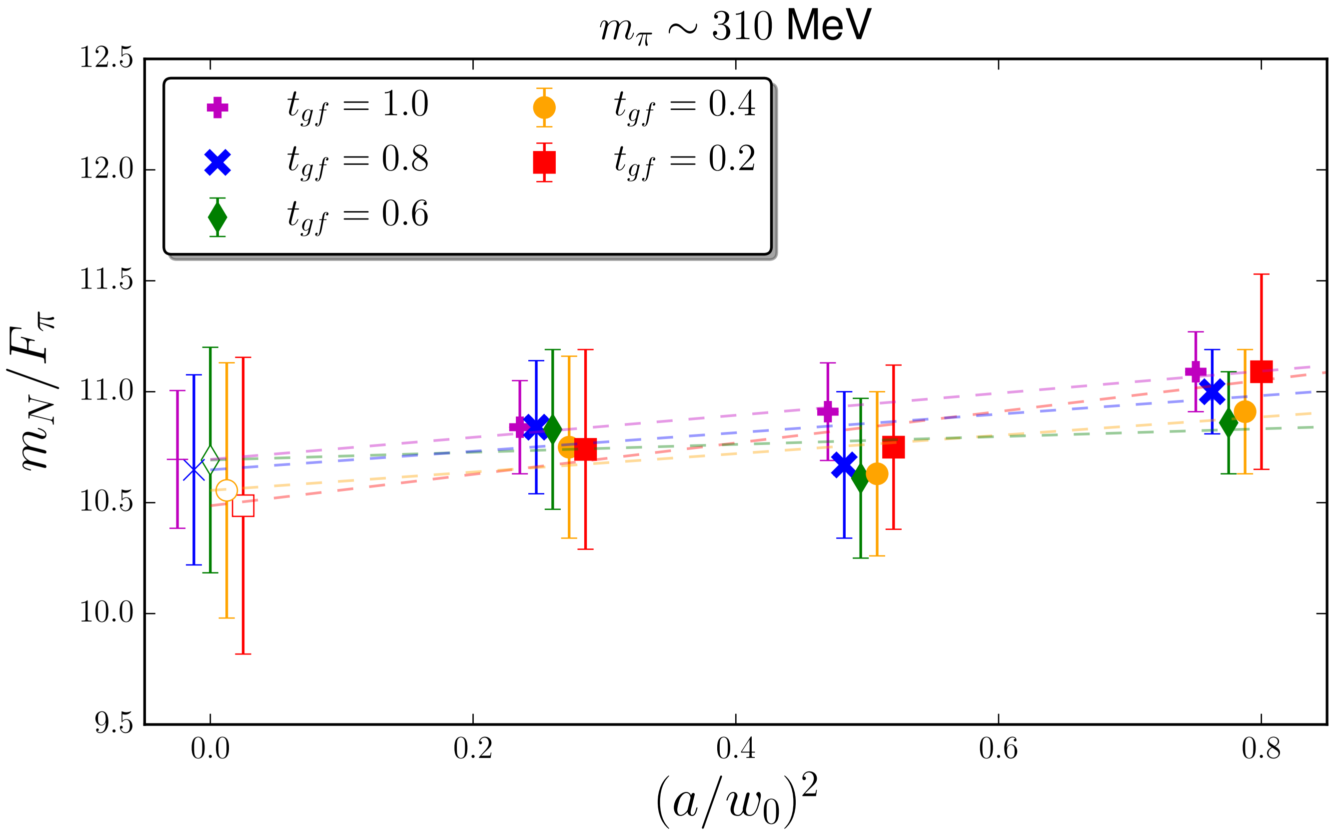
Some Lattice QCD Details

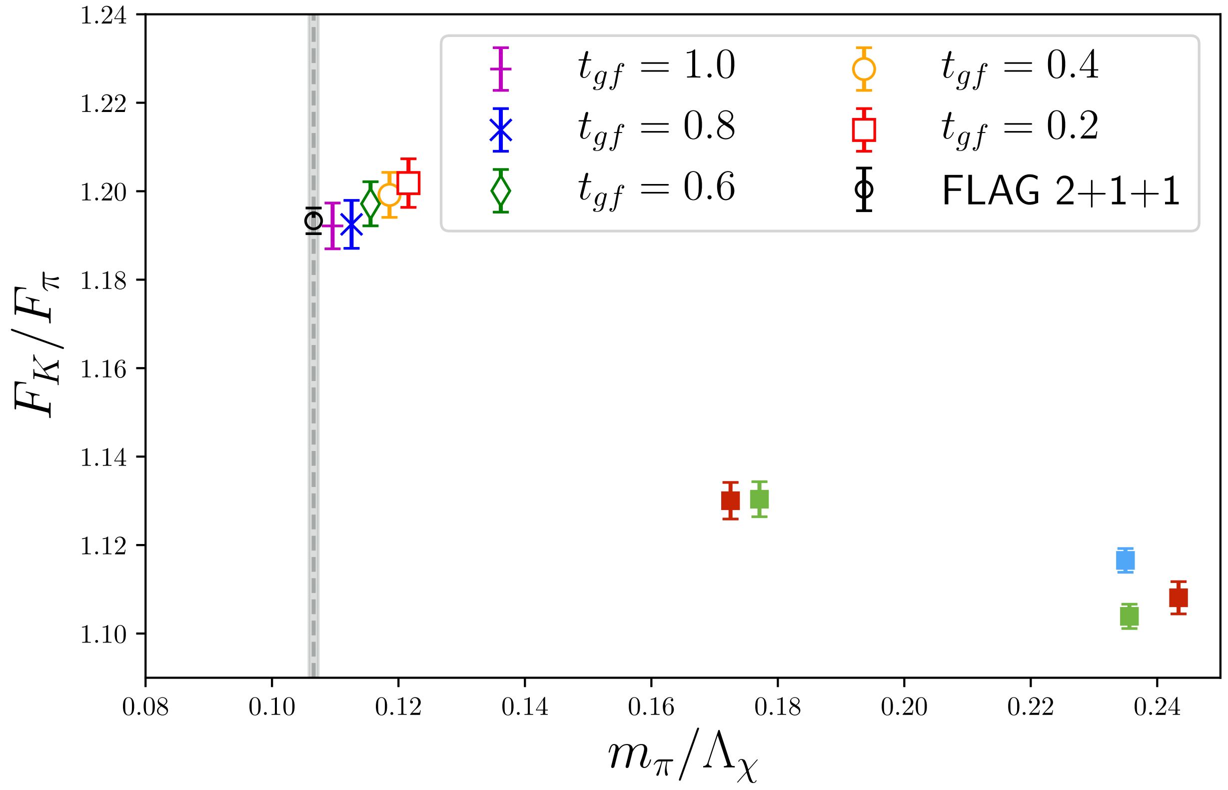
- Gradient-Flow reduces stochastic error, drives Z_A towards 1
- We hold flow time $t_{gf} = 1.0\,a^2$
- We verify flow time independence in continuum limit





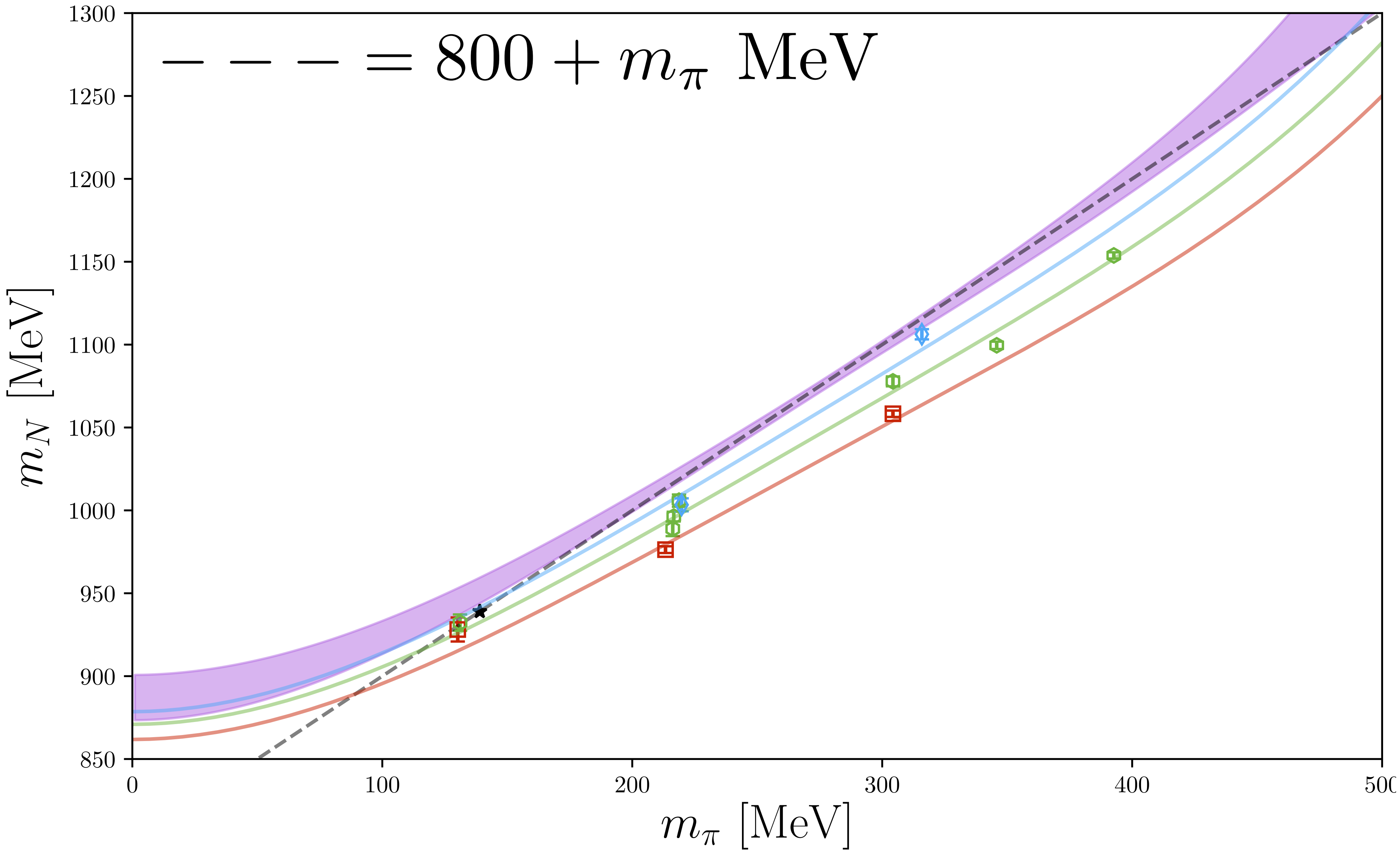






$\mathrm{M_N}$

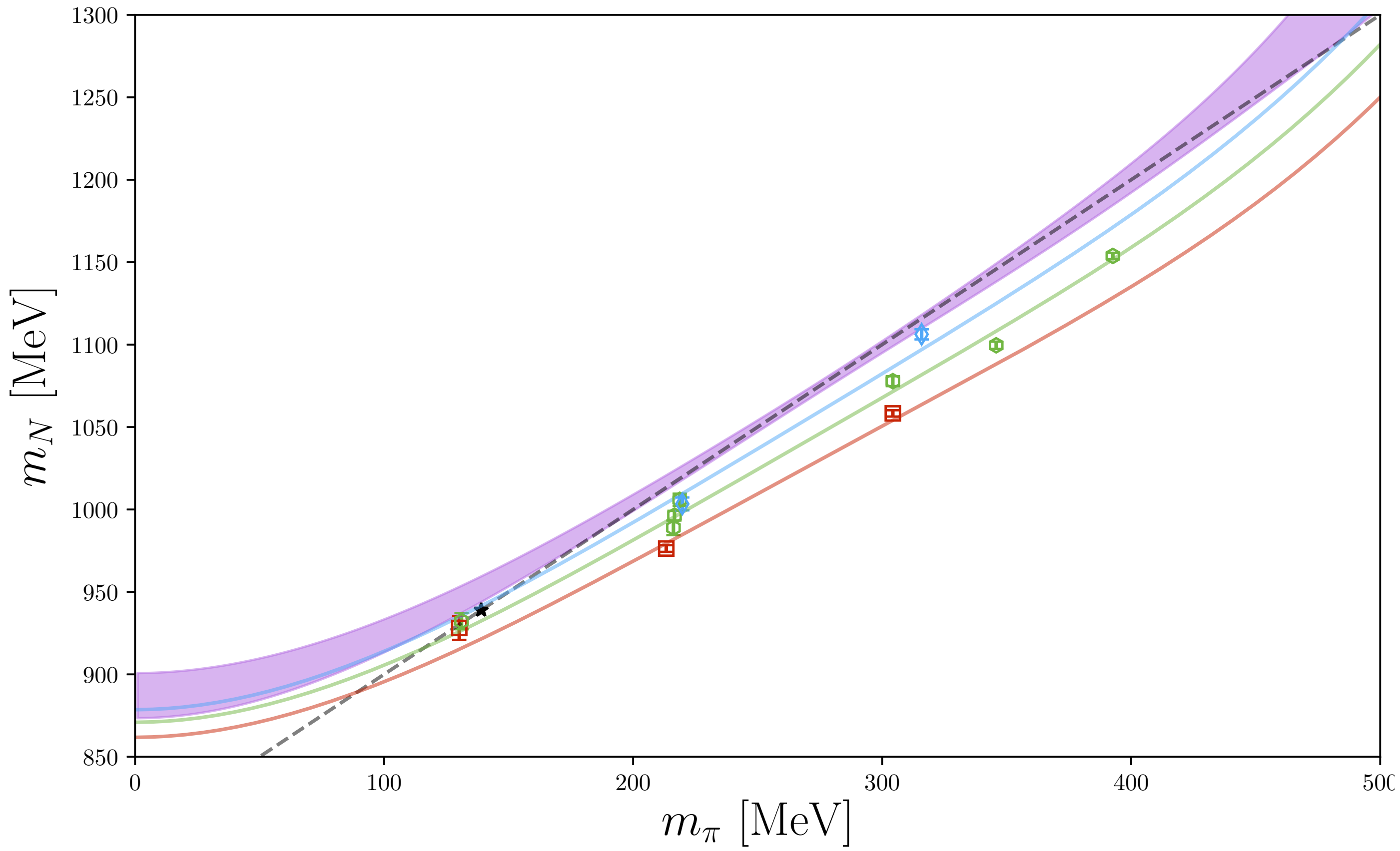
work in progress



- NNLO fit $\mathrm{O}(m_\pi^4) + a^2/\omega_0^2 + a^2/\omega_0^2 \times m_\pi^2/(4\pi F)$
- gA taken from our calculation (see Chia Cheng Chang's talk Tues)
- continuum, infinite volume extrapolated mass is consistent with RULER

M_N

work in progress



- The approximate linear dependence of M_N on $m\pi$ seems to be robust - is there some well founded theoretical reason?
- Future work
  - Work to NNNLO
  - Do our own scale setting
  - include NNLO FV corrections
  - simultaneous fit with g_A and $F\pi$
- Our correlation functions and analysis will be made available to interested parties with the publication of the work

Thank You