Color screening in 2+1 flavor QCD

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in collaboration with
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(**TUMQCD** collaboration)

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TUM-EFT 81/16; PRD 93 114502 (2016); arXiv:1601:08001 (2016)

Color screening in 2+1 flavor QCD

- Overview & Introduction
- Correlators of Polyakov loops
- Comparison to weak coupling
- Summary

Polyakov loops and free energies of static quark states

- The *Polyakov loop L* is the gauge-invariant expectation value of the traced propagator of a static quark (P) and related to its **free energy**: $L(T) = \langle P \rangle_T = \langle \mathrm{Tr}\, S_Q(x,x) \rangle_T = e^{-F_Q^{\mathrm{b}}/T}$. L needs renormalization.

  A. M. Polyakov, **PL 72B** (1978); L. McLerran, B. Svetitsky, **PRD 24** (1981)

- The *Polyakov loop correlator* is related to *singlet* & *octet free energies*

  $$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \tfrac{1}{9} e^{-F_S^{\mathrm{b}}/T} + \tfrac{8}{9} e^{-F_A^{\mathrm{b}}/T} = \tfrac{1}{9} C_S(r,T) + \tfrac{8}{9} C_A(r,T).$$

  S. Nadkarni, **PRD 33, 34** (1986)

- Meaning of **gauge dependent** *singlet* & *octet free energies* is unclear.
- The *Polyakov loop correlator* is related to the **potentials** of **pNRQCD**

  $$C_P(r,T) = e^{-F^{\mathrm{b}}_{Q\bar{Q}}(r,T)} = \tfrac{1}{9} e^{-V_S^{\mathrm{b}}/T} + \tfrac{8}{9} L_A^{\mathrm{b}}\, e^{-V_A^{\mathrm{b}}/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$

  N. Brambilla et al., **PRD 82** (2010)

Renormalization of free energies

- **Singlet free energy** and **potential** appear to be related:

$$F_S(r,T) = -C_F\alpha_s\left[\frac{e^{-r\,m_D}}{r} + m_D\right] + \mathcal{O}(g^4) = V_S(r) + \mathcal{O}(g^3).$$

N. Brambilla et al., **PRD 82** (2010)

$\Rightarrow$ F_S and V_S share the same renormalization $2C_Q$, which depends on T only through the lattice spacing: $V_S = V_S^b + 2C_Q \Rightarrow F_S = F_S^b + 2C_Q$.

- Use V_S at $T=0$: fix r_1 scale & determine $2C_Q$ using **static energy**.

A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]

- Cluster decomposition theorem: $F_{Q\bar{Q}} = F_S = 2F_Q$ for $r \gg 1/T$.

$\Rightarrow$ renormalize as $F_{Q\bar{Q}} = F_{Q\bar{Q}}^b + 2C_Q$ and $F_Q = F_Q^b + C_Q$. → **PRD 93 114502 (2016)**

Beyond $C_Q(\beta)$ from $T=0$ lattices – use **direct renormalization** of F_Q
$\Rightarrow$ Infer unknown $C_Q(\beta)$ from known $C_Q(\beta^{\mathrm{ref}})$ using different N_τ, N_τ^{ref}

$$C_Q(\beta) = \left\{C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau)\right\}$$

→ S. Gupta et al., **PRD 77** 034503 (2008)

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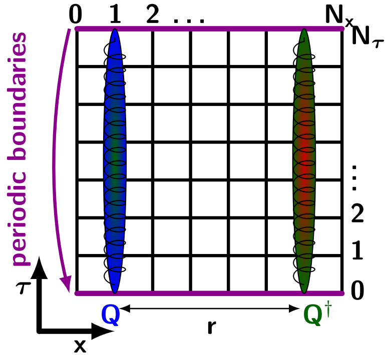
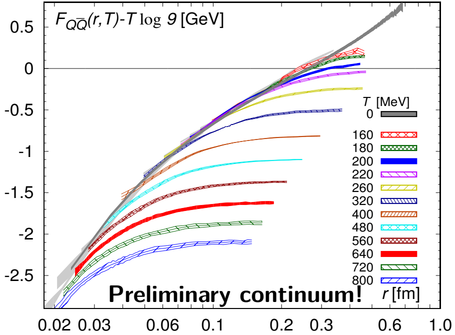
$$C_Q(\beta) = \left\{ C_Q(\beta^{\mathrm{ref}}) + F_Q^{\mathrm{b}}(\beta^{\mathrm{ref}}, N_\tau^{\mathrm{ref}}) - F_Q^{\mathrm{b}}(\beta, N_\tau) + \Delta_{N_\tau, N_\tau^{\mathrm{ref}}} \right\}$$ → **PRD 93 114502 (2016)**

Color screening for a static quark-antiquark pair



Polyakov loop correlator and $Q\bar{Q}$ free energy

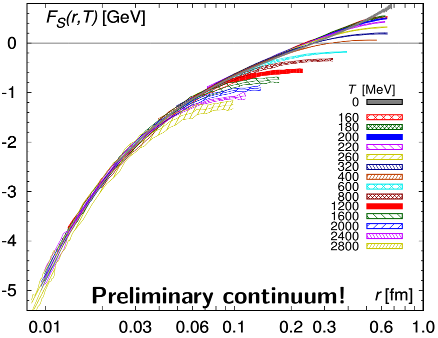


- **Free energy of a $Q\bar{Q}$ pair**, $F_{Q\bar{Q}}$, is also called *color-averaged potential*:

$$C_P^{\rm ren}(r,T) = \langle P(0)P^\dagger(\boldsymbol{r})\rangle_T^{\rm ren} = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9}e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9}e^{-\frac{F_A(r,T)}{T}}.$$

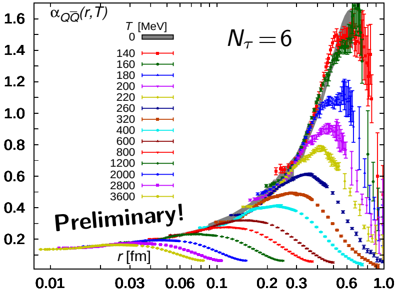
- $F_{Q\bar{Q}} - T\log 9$ is close to the $T=0$ **static energy** V_S for very small rT.
- Plane-plane correlators: $C_{PL}^{\rm ren}(z,T) = \sum_{x_\mu} \langle P(x_1,x_2,0)P^\dagger(x_3,x_4,z)\rangle_T^{\rm ren}$.

Singlet free energy in Coulomb gauge



- **Singlet free energy:** $C_S^{\mathrm{ren}}(r,T) = \frac{1}{3}\left\langle \sum_{a=1}^{3} W_a(0) W_a^\dagger(\boldsymbol{r}) \right\rangle_T^{\mathrm{ren}} = e^{-F_S(r,T)/T}$
- Wilson line correlator requires explicit **gauge fixing** (Coulomb gauge)
- F_S is numerically close to the $T=0$ **static energy** V_S for $rT \lesssim 0.3$.

Effective coupling: vacuum-like and screening regimes

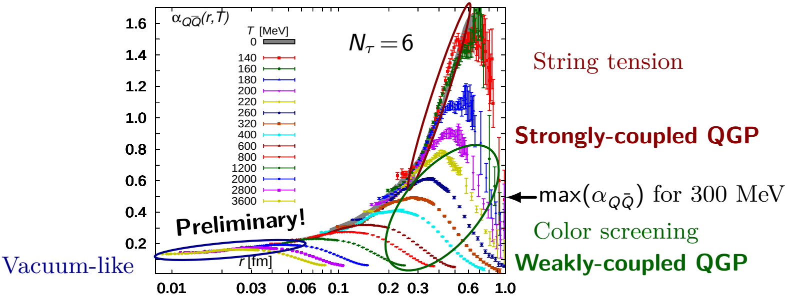


- **Effective coupling** $\alpha_{Q\bar{Q}}(r, T)$ is a proxy for the **force** between Q and $\bar{Q}$.

$$\alpha_{Q\bar{Q}}(r, T) = \frac{r^2}{C_F}\frac{\partial E(r,T)}{\partial r}, \quad E = \{F_S(r, T), V_S(r)\}$$

- $\alpha_{Q\bar{Q}}$ clearly distinguishes different regimes at small and large r.

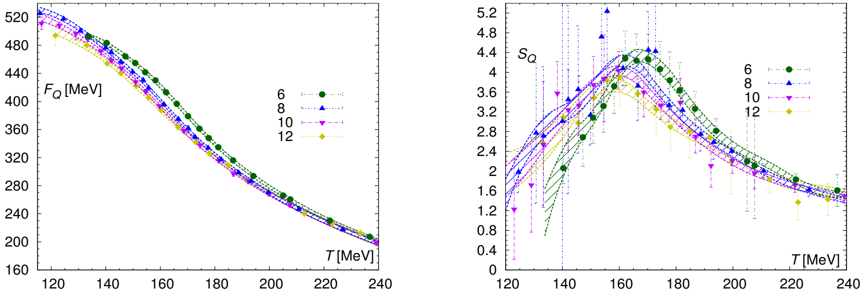
Effective coupling: vacuum-like and screening regimes



Vacuum-like regime	Screening regime	$\max(\alpha_{Q\bar{Q}})$
$rT \lesssim 0.2$	$rT \gtrsim 0.3$	$r_{\max} T \sim 0.4$

- $r_{\max}$ defined through $\max(\alpha_{Q\bar{Q}})$, which is proxy for the **maximal force**.
- For $T \lesssim 300\,\mathrm{MeV}$: $\max(\alpha_{Q\bar{Q}})(T) \gtrsim 0.5$ – **strongly-coupled QGP**.

Critical behavior in renormalized single quark free energy



Single Q free energy: $F_Q = \lim_{r\to\infty} \frac{F_{Q\bar{Q}}}{2}$

Alternative def.: $F_Q = -T \log(L)$,

static **D** mesons: $F_Q(T < T_c) < \infty$,

definitions agree in limit $V \to \infty$.

The temperature derivative,

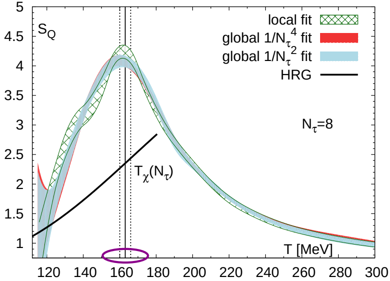
$S_Q = -\frac{dF_Q}{dT}$ peaks at $T_S \sim 160$ MeV

measurable deconfinement observable

(scheme independent in cont. limit).

MPL A31 no.35, 1630040 (2016)

T_χ from chiral observables vs T_S from the peak of the entropy



Bazavov et al. [TUMQCD]
PRD 93 114502 (2016)

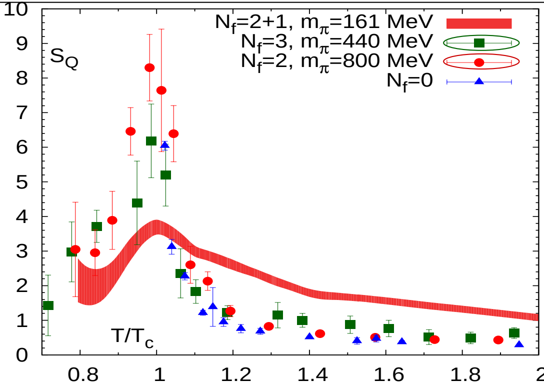
- T_χ defined via $O(2)$ scaling of $\chi_{m,l}$ ($O(4)$: $1-3.5\,\mathrm{MeV}$ lower T_χ)

  A. Bazavov et al., **PRD 85** 054503 (2012) [HotQCD]
- $T_S(N_\tau) \simeq T_\chi(N_\tau)$ for any N_τ Bazavov et al., PRD 93 114502 (2016) [TUMQCD], suggests a **tight link between chiral symmetry and deconfinement**.

  e.g. as in glueball-sigma mixing scenarios, Y. Hatta, K. Fukushima **PRD 69** 097502 (2004).
- *Hadron resonance gas* (HRG) limited to only below $T \sim 125\,\mathrm{MeV}$.

  static HRG results from: A. Bazavov, P. Petreczky, **PRD 87**, 094505 (2013)

Critical behavior of the entropy



O. Kaczmarek, F. Zantow, **hep-lat/0506019 (2005)**

P. Petreczky, K. Petrov **PRD 70** 054503 (2004)

Bazavov et al. [TUMQCD] **PRD 93** 114502 (2016)

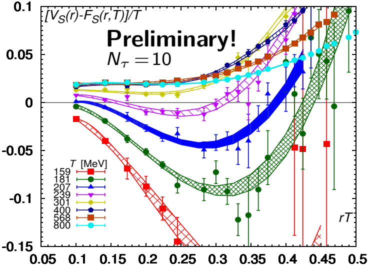
- The **peak decreases for lower quark masses** and for finer lattices.

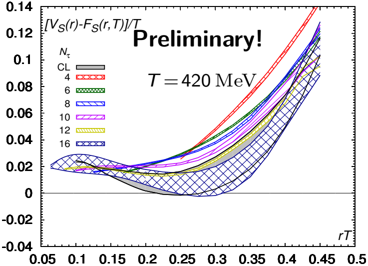
$\rightarrow$ interpret critical behavior as **melting of the static-light mesons**.

- The entropy peaks at $T_S = 153^{+6.5}_{-5}$ MeV in the continuum limit.

Bazavov et al., **PRD 93** 114502 (2016) [TUMQCD]

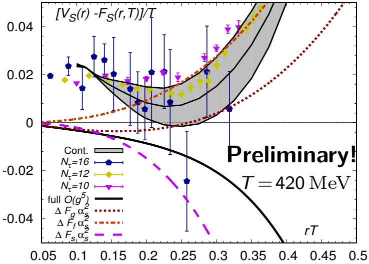
Static energy and singlet free energy (I) - discretization effects

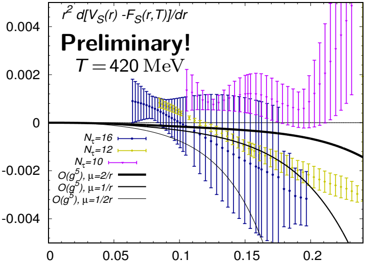




- *pNRQCD*: $V_S(T=0) - F_S(T>0)$ up to $\mathcal{O}(g^6)$ M. Berwein et al., arXiv:1704.07266
- **Smooth r dependence** due to cancellations in $V_S - F_S$ for $r/a < 3$.
- **Strong N_τ dependence** for $rT > 0.15$, but $N_\tau \geq 12 \sim$ continuum limit.
- $V_S - F_S \sim 0.02T$ for $rT \lesssim 0.1$ & $T > 300\,\mathrm{MeV}$, mild N_τ dependence.

Static energy and singlet free energy (II) – weak coupling





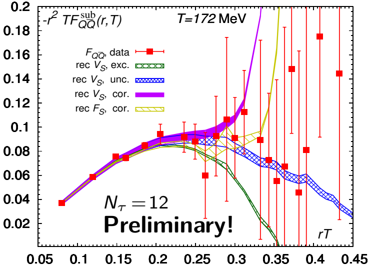
- $pNRQCD$: $V_S(T=0)-F_S(T>0)$ up to $\mathcal{O}(g^6)$ M. Berwein et al., arXiv:1704.07266

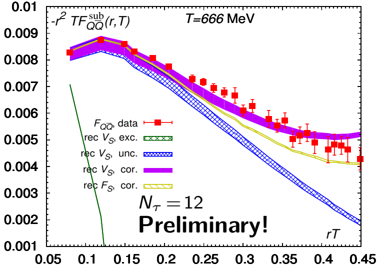
  $$V_S(r)-F_S(r,T)=(\Delta F_g+\Delta F_f+\Delta F_s)\,\alpha_s^2 T+\mathcal{O}(g^6) \text{ for } x=\frac{\pi}{3}rT\ll 1,$$

  $$\underbrace{\Delta F_g = N_c C_F\left[-\frac{1}{3}x+2x^2-\frac{22}{25}x^3\right]}_{\text{gluons}},\quad \underbrace{\Delta F_f = N_f C_F\left[\frac{3}{2}x^2-\frac{7}{10}x^3\right]}_{\text{fermions}},\quad \underbrace{\Delta F_s = -C_F\sqrt{\frac{2\alpha_s}{3\pi}\left[2N_c+N_f\right]^3}\,x^2}_{\text{Debye screening}}$$

- $\alpha_{Q\bar{Q}}$ scheme – **strong N_τ dependence**, but $x\ll 1$ finally under control?
- Thermal/screening effects for $x\lesssim 0.2$ are at most at **sub-percent level**.

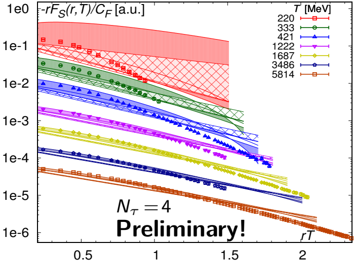
Color octet contribution in the Polyakov loop correlator





- *pNRQCD*: C_P is given in terms of **potentials** V_S **and** V_A **at** $T=0$ and of the *adjoint Polyakov loop* L_A *at* $T>0$ N. Brambilla et al., **PRD 82** (2010)
  $$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{9} e^{-V_S/T} + \frac{8}{9} L_A\, e^{-V_A/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$
- We reconstruct V_A from V_S and L_A from L via **Casimir scaling** and include the **Casimir scaling violation**: $8V_A + V_S = 3\frac{\alpha_s^3}{r}\left[\frac{\pi^2}{4} - 3\right] + \mathcal{O}(\alpha_s^4)$.
- Sensitivity of **CSV** for $T \gg T_c$: *pNRQCD* describes C_P correctly.

Confronting weak-coupling predictions in the screening regime (I)



Hashed bands: LO
Solid bands: NLO

Scale uncertainty $\mu = (1-4)\pi T$ due to resummation smaller for larger T

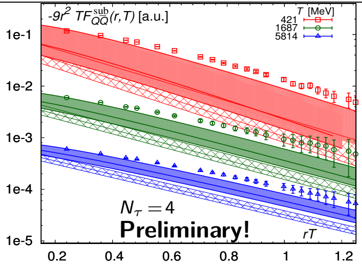
$\Lambda_{\rm QCD} = 320\,\rm MeV$

- Do we see an **electric screening regime** for $rm_D \sim 1 \Leftrightarrow 0.3 \lesssim rT \lesssim 0.6$?
- $F_S(r,T)$ known for $rm_D \sim 1$ at NLO — M. Laine et al., **JHEP 0703** 054 (2007)
- We compute $F_S^{\rm sub}(r,T) = F_S(r,T) - 2F_Q(T)$ on the lattice.
- F_S **is compatible with NLO** up to $rT \sim 0.8$, if the **running coupling in the LO** term is used.

$\Rightarrow$ **Weak coupling** is correct in the electric screening regime of $F_S(r,T)$.

- For $rT > 0.8$: asymptotic screening is inherently non-perturbative.

Confronting weak-coupling predictions in the screening regime (II)



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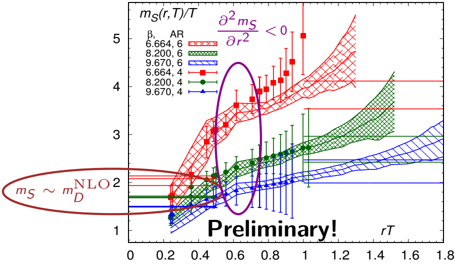
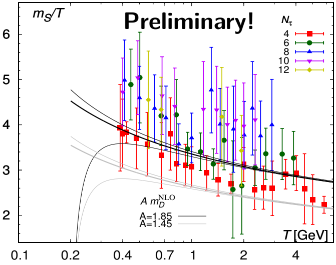
$\Lambda_{\rm QCD} = 320\,\mathrm{MeV}$

- Do we see an **electric screening regime** for $rm_D \sim 1 \Leftrightarrow 0.3 \lesssim rT \lesssim 0.6$?
- *Singlet and octet cancel* at LO in $F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2}{9}\frac{e^{-2rm_D}}{r^2} + C_F\alpha_s m_D$.
- The **perturbation series of $F_{Q\bar{Q}}$ breaks down**! S. Nadkarni, **PRD 33** (1986)
- We compute $F_{Q\bar{Q}}(r,T) = F_{Q\bar{Q}}(r,T) - 2F_Q(T)$ on the lattice.
- $F_{Q\bar{Q}}$ is **almost compatible $r \sim 1/m_D$ ($\sim 10\%$ level) with NLO**, if the **running coupling in the LO** term is used.

$\Rightarrow$ **Weak coupling** is good in the electric screening regime of $F_{Q\bar{Q}}(r,T)$.

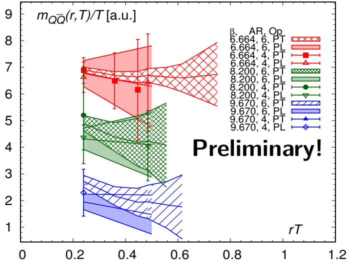
Asymptotic singlet screening mass

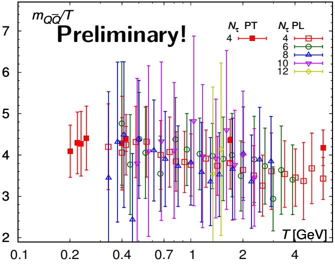




- Color screening becomes stronger for larger rT. Both $F_{Q\bar{Q}}$ & F_S must reach **asymptotic screening**: $F = -a\frac{e^{-rm}}{r} + c$, $c = 2F_Q$ for $N_\sigma \to \infty$.
- The asymptotic **singlet screening mass** m_S exceeds the NLO Debye mass (electric mass in Electrostatic QCD). E. Braaten, A. Nieto, **PRD 53** (1996).
- Triple lines: scale uncertainty $\mu = (1-4)\pi T$ due to resummation.
- Similar T dependence: m_S & rescaled $m_D^{\rm NLO}$. O. Kaczmarek, **PoS CPOD07** (2007).

Asymptotic screening mass of $F_{Q\bar{Q}}$



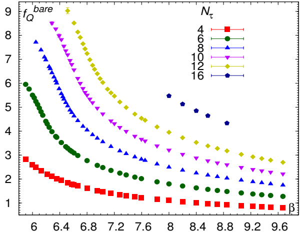


- Screening mass $m_{Q\bar{Q}}$ from point-point and plane-plane correlator.
- The screening mass $m_{Q\bar{Q}}$ seems **already at $rT \sim 0.45$ asymptotic**.
- $m_{Q\bar{Q}}/T$ is **at most mildly temperature dependent** for $T > 200$ MeV.
- $m_{Q\bar{Q}}$ is compatible with the *magnetic mass m_M* from smeared Polyakov loop correlators and with the ground state of massless $N_f = 3$ EQCD.

S. Borsányi et al., **JHEP 1504** 138 (2015) [BW coll.]; A. Hart et al., **NPB 586** (2000)

- We study color screening and deconfinement using the renormalized Polyakov loop correlator and related observables.
- We identify in the entropy $S_Q = -\frac{dF_Q}{dT}$ crossover behavior at $T \sim T_c$.
- We extract $T_S = 153^{+6.5}_{-5}$ MeV from the entropy, in agreement with $T_\chi = 160(6)$ MeV (chiral susceptibilities, O(2) scaling fits, $\frac{m_l}{m_s} = \frac{1}{20}$).
- Continuum limit of static quark correlators in $N_f = 2{+}1$ QCD up to $T \sim 1.9$ GeV and down to $r \sim 0.01$ fm.
- **Color-singlet correlators** are **vacuum-like up to** $rT \ll 0.3$, exhibit *color-electric screening for* $rm_D \sim 1 \Leftrightarrow 0.3 \lesssim rT \lesssim 0.6$ *compatible with weak coupling* and change to *asymptotic screening for* $rT \gg 0.7$.
- The *Polyakov loop correlator* C_P has a significant *color adjoint contribution* for $rT \gtrsim 0.2$. Weak coupling (pNRQCD) describes C_P quite well in terms of $T = 0$ **potentials** and the *adjoint Polyakov loop* L_A.
- C_P has a *color-electric screening regime* $rm_D \sim 1$.
- The screening mass of $m_{Q\bar{Q}}$ is consistent with **EQCD predictions for the lowest scalar glueball** and has a trivial temperature dependence.

Details of the ensembles

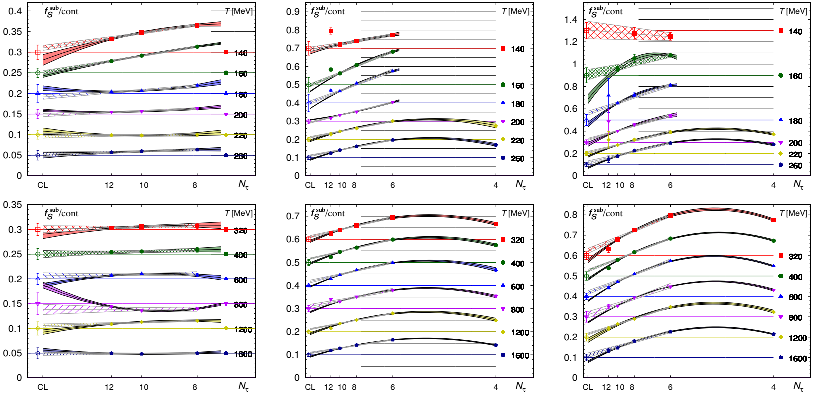


TUMQCD collaboration, **TUM-EFT 81/16**

- $N_\tau < 16$: ∼30 ensembles each, $5.9 \leq \beta \leq 9.67$, $a = 0.0085 - 0.25$ fm.
- **HISQ/Tree** action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$; taste-breaking much reduced.
- Ensembles: $\frac{N_\sigma}{N_\tau} = 4$, $m_l = \frac{m_s}{20} \Leftrightarrow m_\pi = 161$ MeV; $\beta \leq 7.825$, $a \geq 0.04$ fm most from A. Bazavov et al., **PRD 85** 054503 (2012), **PRD 90** 094503 (2014) [HotQCD]
- 3 ensembles: also $\frac{N_\sigma}{N_\tau} = 6$ with $N_\tau = 4$: $2 - 6 \times 10^5$ TU each.
- All N_τ, $m_l = \frac{m_s}{5}$: $3 - 5$ ensembles each, $1 - 10 \times 10^4$ TU each, $7.03 \leq \beta \leq 8.4$, $a = 0.025 - 0.083$ fm; $T = 0$ lattices available.

Continuum extrapolations: singlet free energy

$rT = 0.15$ $rT = 0.30$ $rT = 0.45$



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