“Multirep” SU(4) Gauge Theory

Spectroscopy of SU(4) gauge theory with simultaneous dynamical fermions in multiple representations

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With the Tel Aviv-Colorado (TACo) Collaboration

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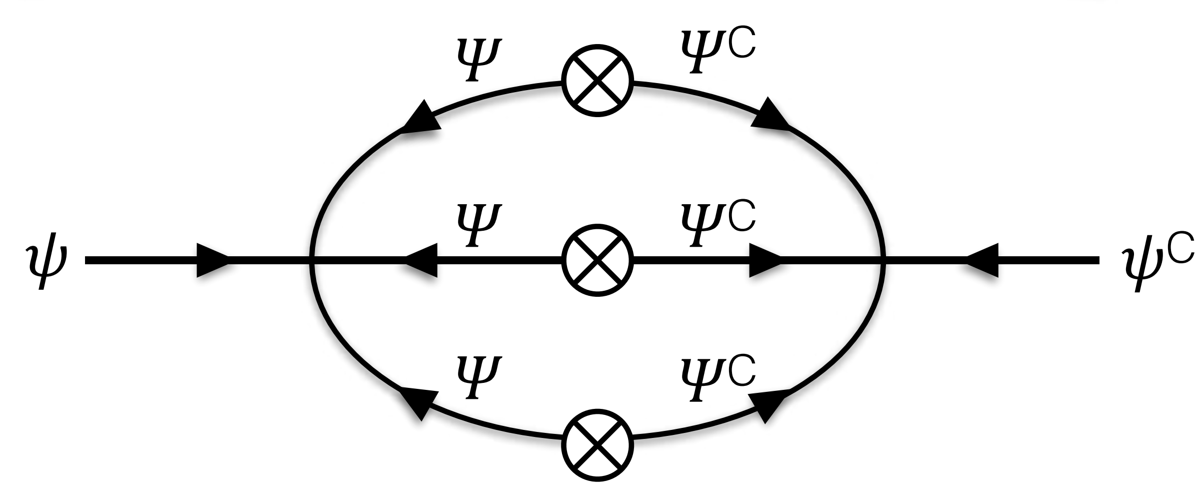
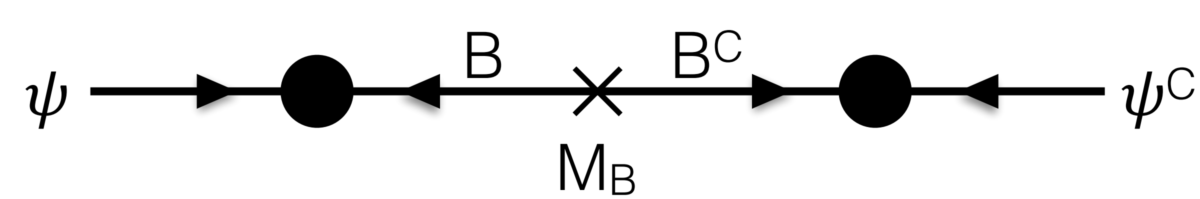
Mass Generation in strongly coupled BSM models

- Classic "**extended technicolor**"
  - Chiral condensate breaks $SU(2)_L$
  - The Higgs emerges from dynamics (dilaton?)
- **Composite Higgs**—Limited lattice investigation to date(!)
  - Chiral condensate preserves $SU(2)_L$
  - Higgs arises from SSB as an exact Goldstone boson
  - SM loops generate a potential for the Higgs
- 4-fermion interactions generate fermion masses
  - Often, fermions couple quadratically to UV operators
  - Partial compositeness = linear coupling to "baryons"
  - Idea: Kaplan D.B., Nucl Phys B365 (1991) 259-278

$$\bar{\psi}\psi\bar{\Psi}\Psi \sim \bar{\psi}\psi\mathcal{O}_{\mathrm{ETC}}$$

$$\bar{\psi}\Psi\Psi\Psi \sim \bar{\psi}\mathcal{O}_{\mathrm{PC}}$$



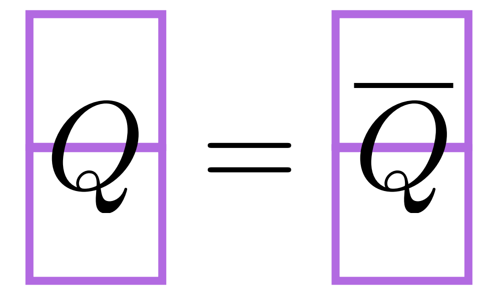


Ferretti's Model (1404.7137)

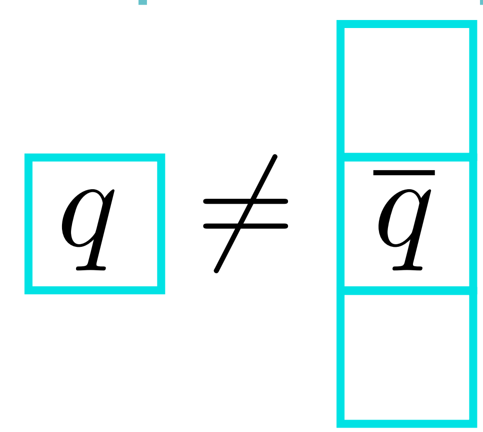
A specific UV continuum theory of partial compositeness

- SU(4) gauge theory with “multirep" matter content
  - 5 sextet Majorana fermions
    - Equivalent DOF: “2.5 sextet Dirac fermions”
    - Sextet SU(4) is a real representation ≘ to SO(6)
  - 3 fundamental Dirac fermions
- Symmetry breaking: SU(5)/SO(5) in the IR (for sextets)
  - Symmetry breaking pattern *different from QCD*
- New territory for lattice simulations

real irrep



complex irrep



Our lattice deformation

(What we actually simulate)

- Still SU(4) gauge theory, but modified matter content
  - $2.5 \longmapsto$ 2 Dirac sextet SU(4) fermions
  - $3 \longmapsto$ 2 Dirac fundamental SU(4) fermions
- Symmetry breaking: SU(4)/SO(4) in the IR (for sextets)
  - Still a rich system for lattice investigation
  - Expected to capture the important qualitative features of Ferretti's model

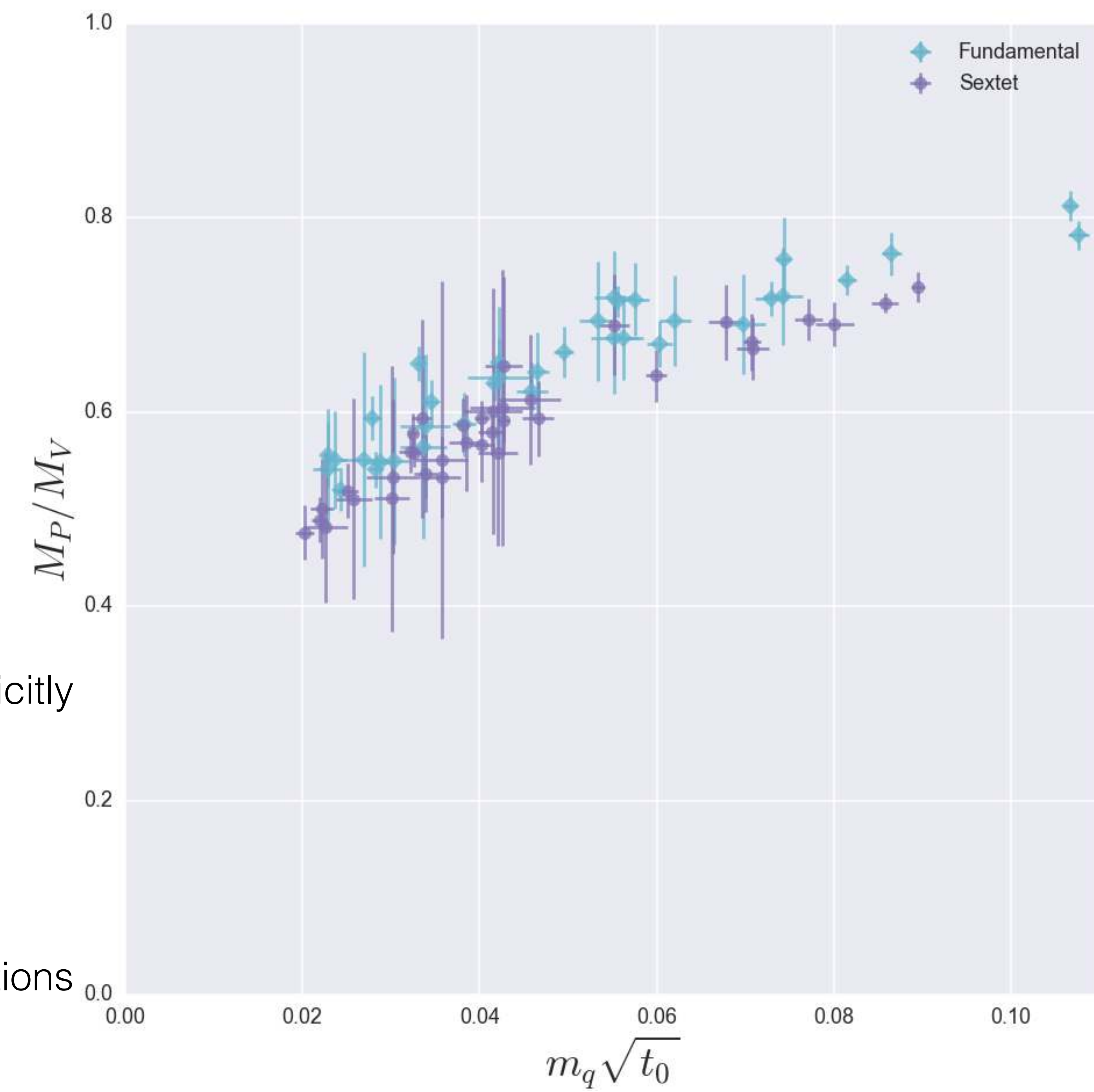
Technical specifications

- Multirep MILC code - Y. Shamir
- NDS Action - T. Degrand, Y. Shamir, and B. Svetitsky (1407.4201)
- Clover-improved Wilson fermions
- Gauge generation with hybrid Monte Carlo
- Today in this talk:
  - First-ever simulations with simultaneous dynamical fermions in multiple representations
  - Preliminary “zero-temperature” meson spectroscopy across dozens of ensembles

Overview of Ensembles

Pseudoscalar-to-vector mass ratio: M_P/M_V

- ~40 total ensembles
- Volumes 16^3 x 32, 16^3 x 18
- Fermion masses m_q from the axial Ward identity
- Meson masses from 2-point functions
- $0.5 \lesssim M_P/M_V \lesssim 0.8$
    - QCD language: "$M_\pi \gtrsim 400$ MeV"
- Finite volume under control: $M_P L > 4$
    - In progress: check volume dependence explicitly
    - In progress: simulations on 24^3 x 48
- Safe flow scales ~1.0 < t_0 /a^2 < ~3.0
- Comparable behavior in both fermion representations

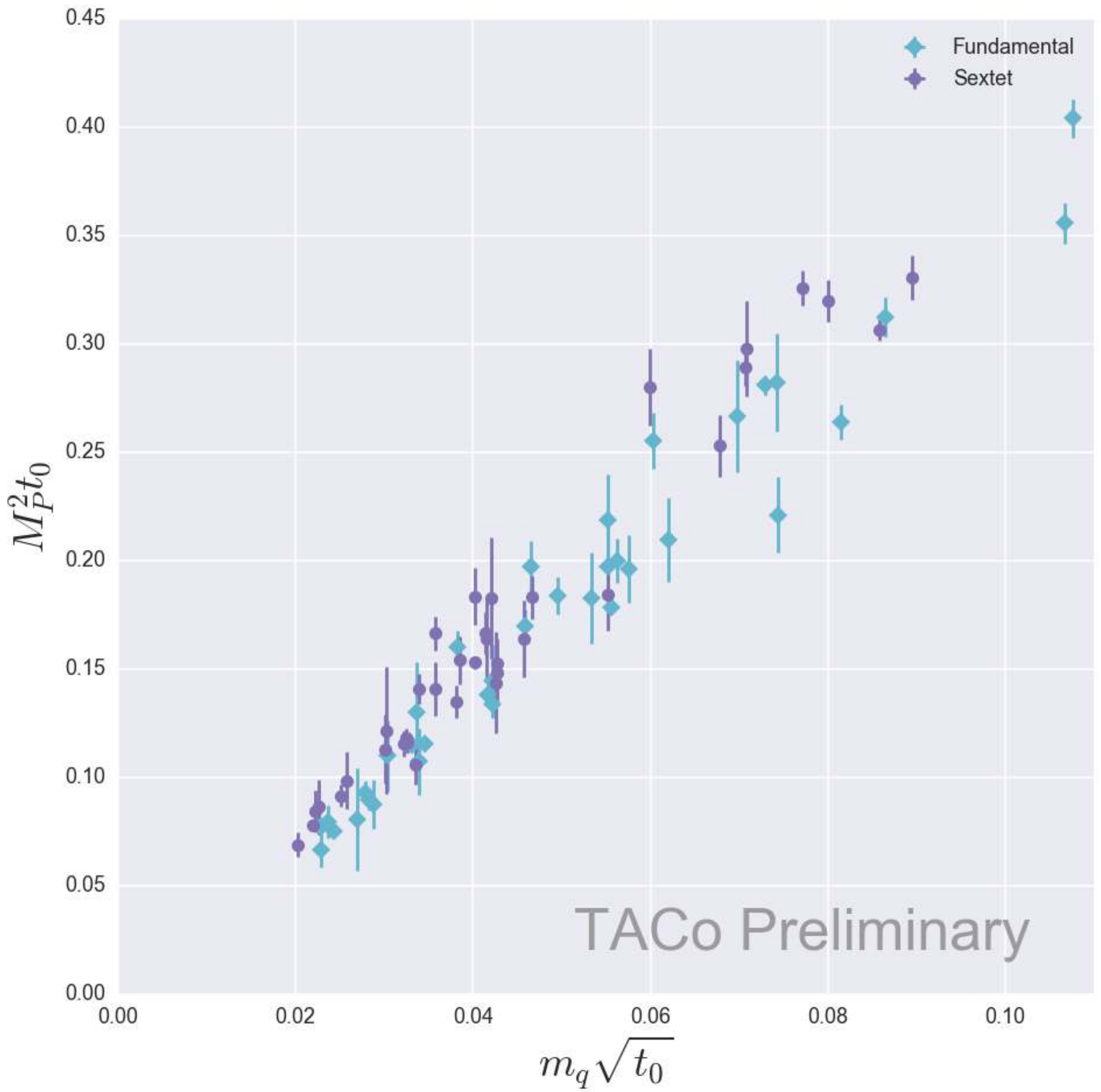


Pseudoscalar masses

- Cancel lattice spacing with dimensionless "ratios" using t_0
- Leading-order ChiPT says:

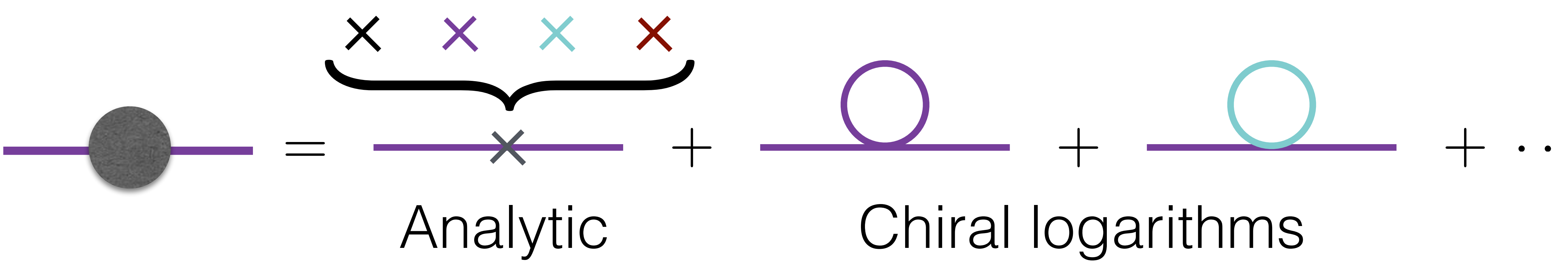
$$M_P^2 \sim m$$

- (Plausibly) linear behavior
- Lattice artifacts?
    - (Clover-improved) Wilson fermions are not chiral
    - Some remnant additive renormalization?
- Model with NLO ChiPT



ChiPT predictions

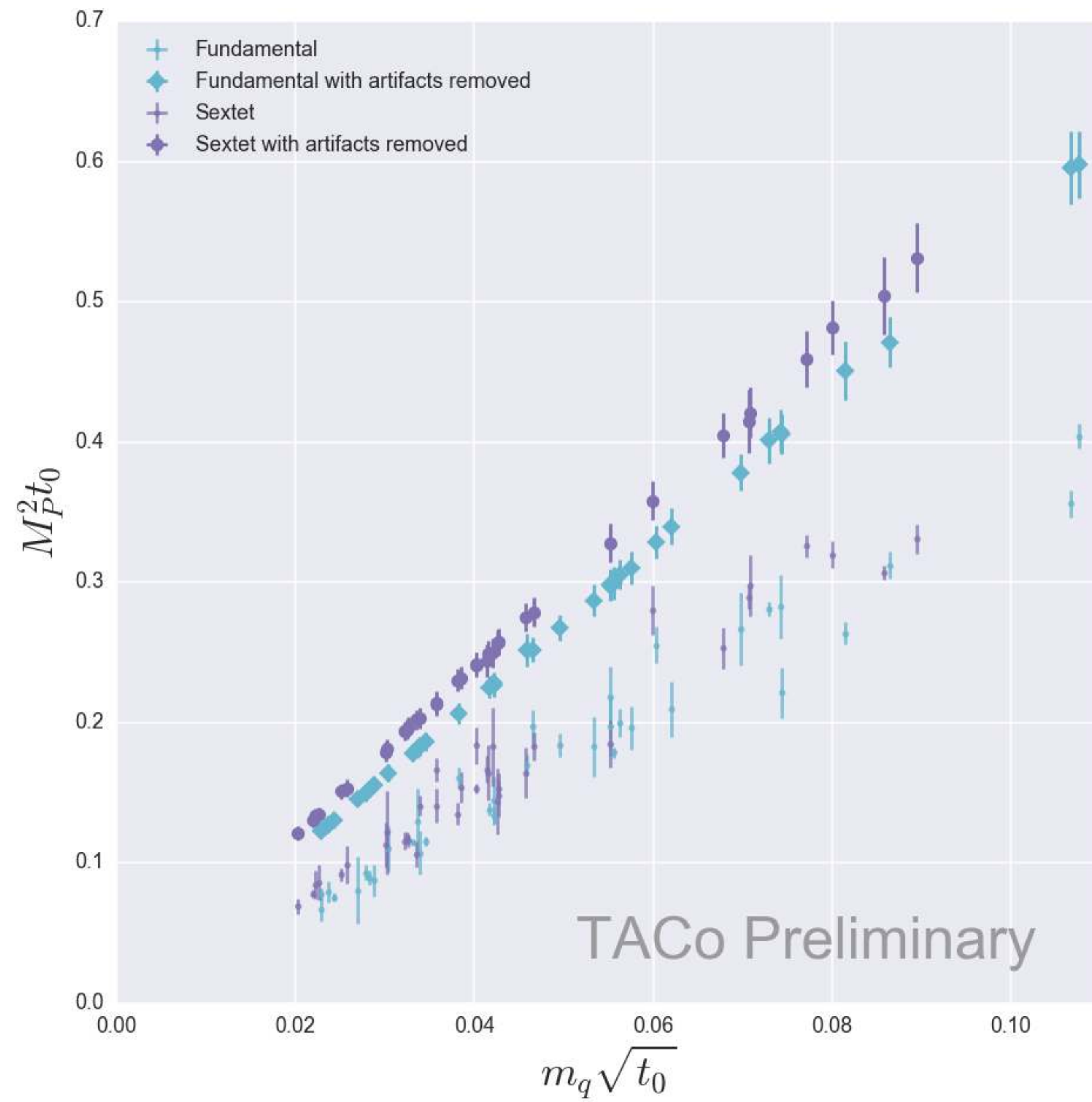
- Multirep ChiPT worked out to NLO by DeGrand, Golterman, Neil, and Shamir in 1605.07738
    - Schematically similar to single-rep ChiPT with analytic terms and chiral logarithms
- Wilson ChiPT at NLO suggests $(M_P^2 t_0)$ and $(F_P\sqrt{t_0})$ also depend explicitly on (ma) and $(a/\sqrt{t_0})$
    - Coefficients of (ma) and $(a/\sqrt{t_0})$ are lattice artifacts



Pseudoscalar masses

Subtraction of lattice artifacts

- Leading-order ChiPT: $M_P^2 \sim m$
- Linear behavior now consistent with estimated uncertainties
- Good proof-of-concept for the modeling, since we know how Goldstone bosons behave
- Allows us to subtract lattice artifacts and take a chiral limit
- The sextet has a larger slope, related to the condensate



More about ChiPT results

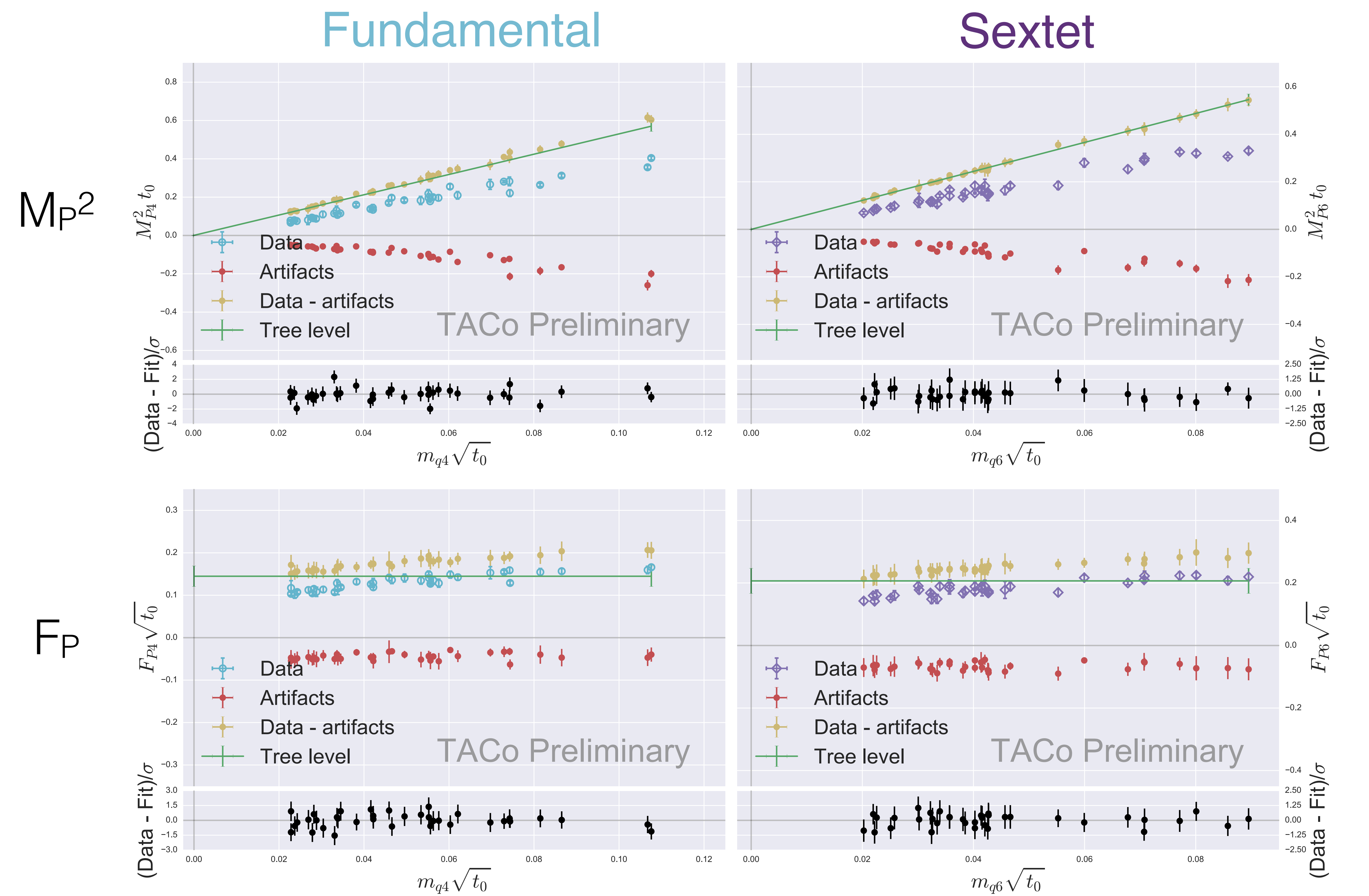
- The masses M_{P4} and M_{P6} and decay constants F_{P4} and F_{P6} depend on a shared set of low-energy constants
- A simultaneous correlated fit to all four quantities captures the full statistical information in our measurements

ChiPT fits at NLO

Fundamental | Sextet

M_P^2

F_P



More about ChiPT results

- The masses M_{P4} and M_{P6} and decay constants F_{P4} and F_{P6} depend on a shared set of low-energy constants
- A simultaneous correlated fit to all four quantities captures the full statistical information in our measurements
- The NLO fit is both
  - Good (χ^2/DOF [DOF] = 0.5 [144]) and
  - Significant (Q~1)
- The NLO LECs are not constrained by the current analysis, i.e., consistent with zero at (1-2)σ
- For the vector quantities, we use empirical fits inspired by ChiPT to model and subtract lattice artifacts

Decay constants

F_V/F_P in a fixed representation

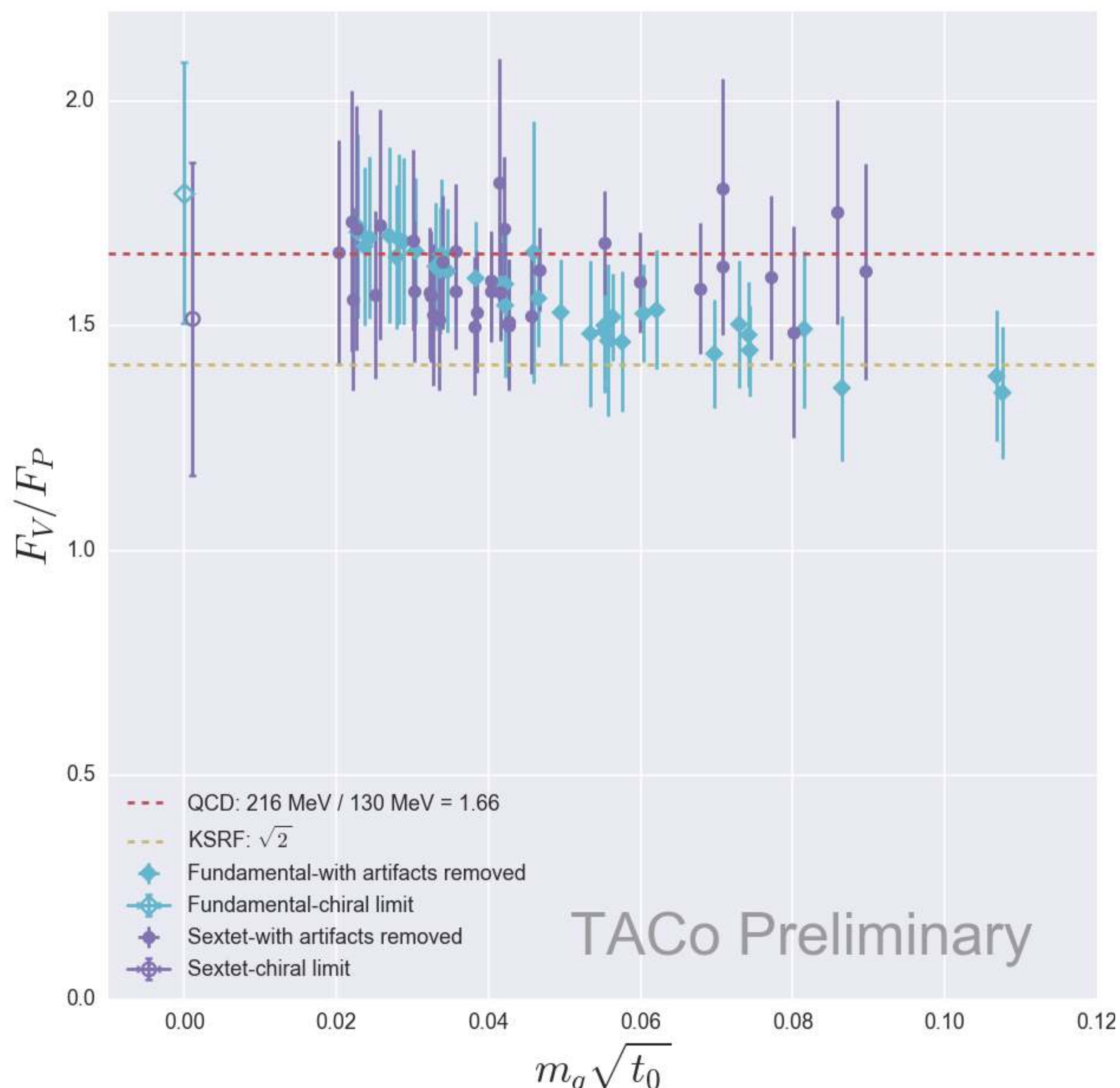
- A priori, F_V and F_P are unrelated
- KSRF (1966) related F_V and F_P using current algebra and vector meson dominance:

$$F_V = \sqrt{2} F_P$$

- Vector meson dominance is an uncontrolled but enlightening and physically motivated approximation
- QCD experiment:

F_V/F_P ~ 216 MeV/130 MeV ~1.66

- Success is comparable to that of QCD
- Both representations are comparable



K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. 16, 255 (1966)

Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966)

Decay widths via KSRF

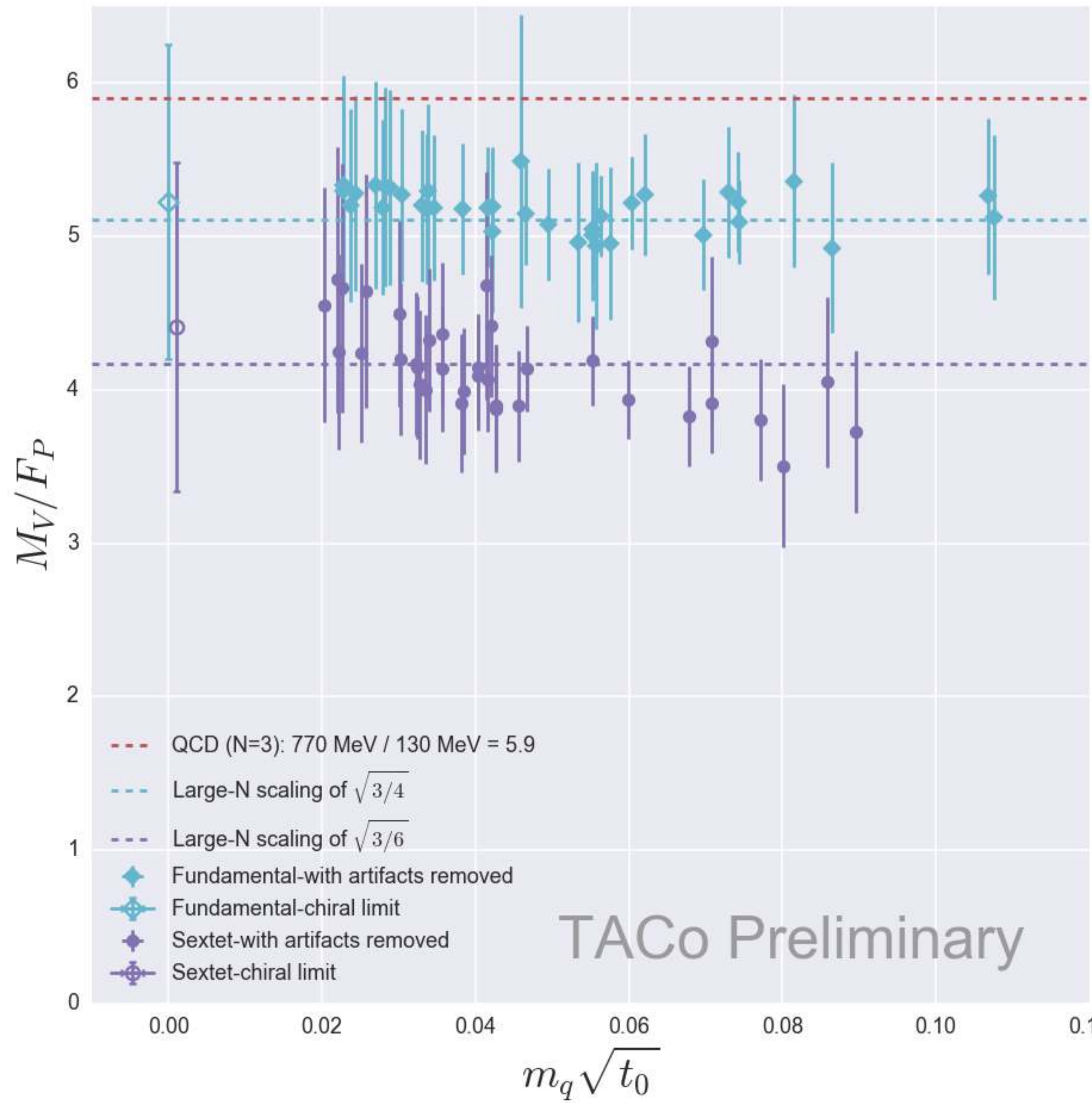
- KSRF (1966) also predict the coupling strength:

$$g_{VPP} = \frac{M_V}{F_P}$$

- This coupling allows for tree-level estimation of the vector width:

$$\Gamma_V \simeq \frac{g_{VPP}^2 M_V}{48\pi}$$

← {Polarization average + Phase space}



Decay widths via KSRF

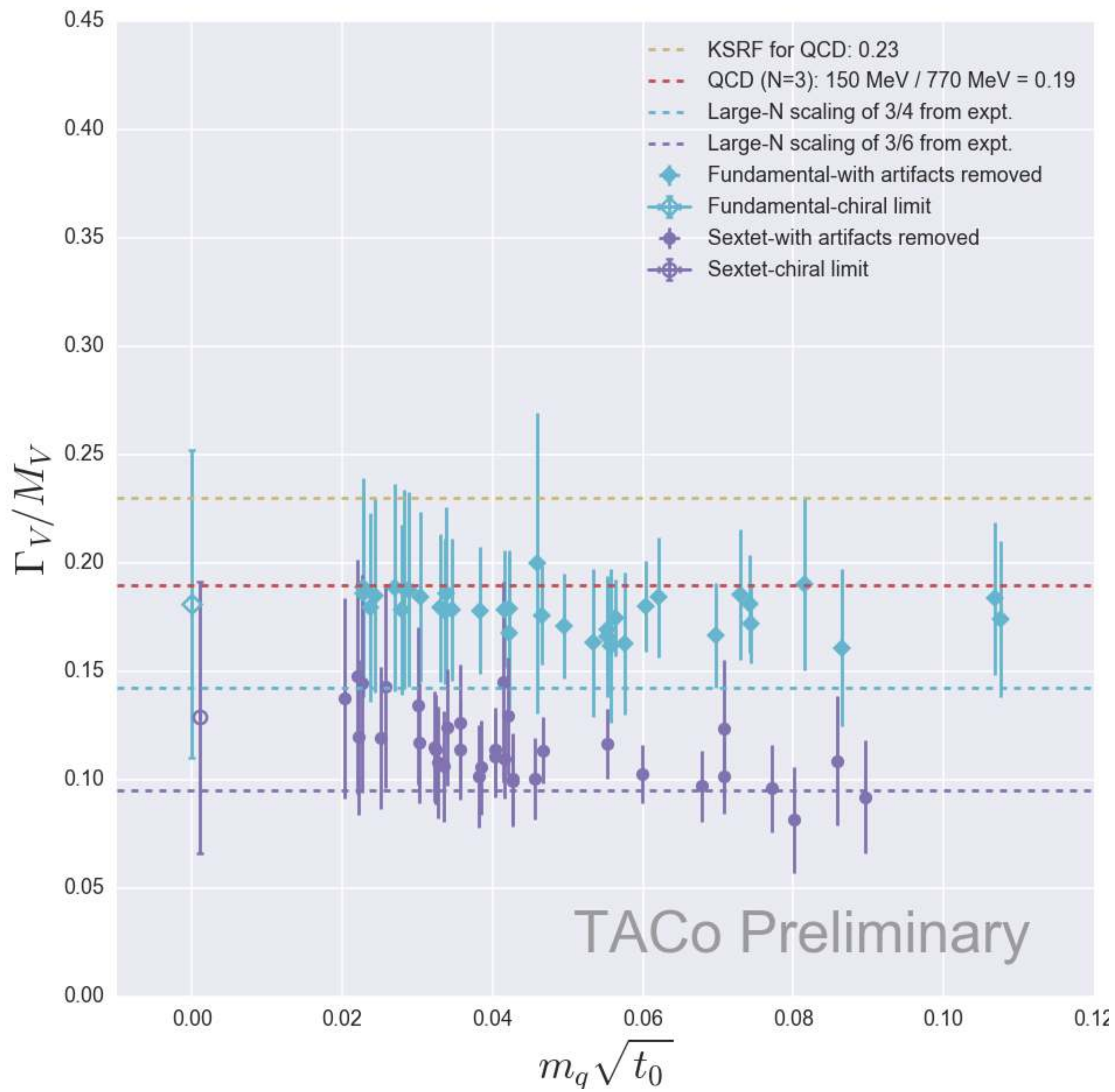
- KSRF prediction:

$$\frac{\Gamma_V}{M_V} \simeq \frac{M_V^2}{48\pi F_V^2}$$

- Broad states, although probably narrower than QCD (for $M_P \ll M_V$):

    - Γ_{V4}/M_{V4} ~0.13

    - Γ_{V6}/M_{V6} ~0.18

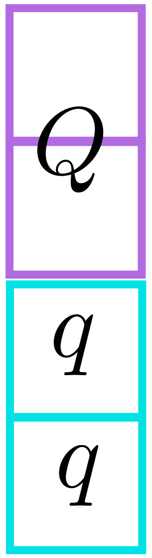


Take-home points

- You heard about preliminary results from simulations of SU(4) gauge theory with dynamical fermions in the fundamental and sextet representations
  - First-ever simulations with dynamical fermions in multiple representations
- Data are consistent with ChiPT for pseudoscalar observables
  - Wilson ChiPT yielded significant lattice artifacts, which can be subtracted
  - Fits suggest the presence of numerically small coupling between representations
  - Current fits do not constrain NLO low-energy constants
- KSRF relations furnish estimates of the vector widths, which are broad yet narrower than QCD
- The data also seem consistent with a version of Large-N — stay tuned for details in the paper

Other directions

- Thermodynamics of SU(4) gauge theory with fundamental and sextet fermions — See the talks and slides of Daniel Hackett and Venkitesh Ayyar
- Baryon spectroscopy and chimera states (in progress)
  - Top partners
  - cf. Lattice 2016 Proceedings



- The Higgs potential from
  - Electroweak gauge boson contribution Π_{LR}, see TACo 1606.02695 for a preliminary lattice study
  - Top quark contribution: hard (baryon 4-point function…), but simplifies in large-N. See Golterman and Shamir 1502.00390

Back-up slides

The Wilson Flow

- Take observables built from the flowed field strength

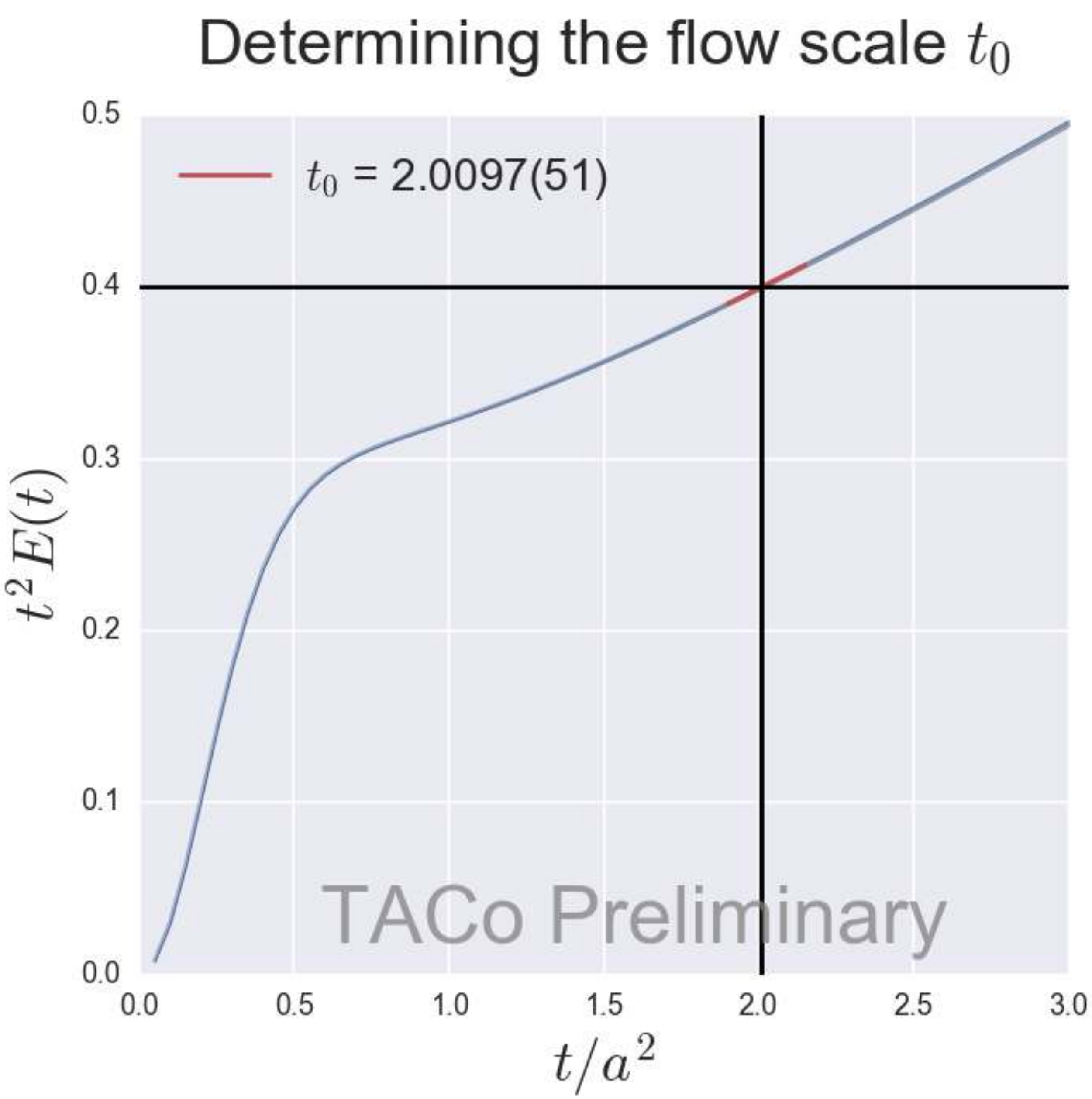
$$\langle E(t) \rangle \sim \langle G^2(t) \rangle$$

$$t_0^2 \langle E(t_0) \rangle \stackrel{!}{=} M(N_c)$$

- In QCD, $\sqrt{t0}$ = 0.14 fm with $M(N_c{=}3) = 0.3$
- Large-N: $t_0 \sim N_c$, so take $M(N_c{=}4) = 0.4$
- DeGrand (1701.00793) gives details, compares to other scale setting schemes, and provides more careful connection to large-N
- Also: talk to R. Hudspith (York) for a thermodynamic perspective varying N
- "Safe" values of the flow scale?
  - $1 < t_0 / a^2 < 3$
  - "QCD" analogy: 0.08 fm $\lesssim$ a $\lesssim$ 0.13 fm

Worry about finite volume

Worry about a large cutoff



ChiPT at NLO — M_P^2

$$M_{P4}^2 t_0 = \left(2m_{q4}\sqrt{t_0}B_4\right) \times \left[1 + 8L_{44}\frac{(2m_{q4}\sqrt{t_0}B_4)}{F_4^2} + 8L_{46}n_6\frac{(2m_{q6}\sqrt{t_0}B_6)}{F_4^2} + \frac{1}{2}\Delta_4 + \frac{4}{5}\Delta_\zeta\right] + A_{\rm art}^{M4}(m_{q4}a) + B_{\rm art}^{M4}\frac{a}{\sqrt{t_0}} + C_{\rm art}^{M4}(m_{q6}a)$$

$$M_{P6}^2 t_0 = \left(2m_{q6}\sqrt{t_0}B_6\right) \times \left[1 + 8L_{66}\frac{(2m_{q6}\sqrt{t_0}B_6)}{F_6^2} + 8L_{64}n_4\frac{(2m_{q4}\sqrt{t_0}B_4)}{F_6^2} - \frac{1}{4}\Delta_6 + \frac{1}{5}\Delta_\zeta\right] + A_{\rm art}^{M6}(m_{q4}a) + B_{\rm art}^{M6}\frac{a}{\sqrt{t_0}} + C_{\rm art}^{M6}(m_{q6}a)$$

ChiPT at NLO — F_P

$$F_{P4}\sqrt{t_0} = F_4\left[1 + 4C_{44}\frac{(2m_{q4}\sqrt{t_0}B_4)}{F_4^2} + 4C_{46}n_6\frac{(2m_{q6}\sqrt{t_0}B_6)}{F_4^2} - \Delta_4\right]$$
$$+A_{\mathrm{art}}^{F4}(m_{q4}a) + B_{\mathrm{art}}^{F4}\frac{a}{\sqrt{t_0}} + C_{\mathrm{art}}^{F4}(m_{q6}a)$$

$$F_{P6}\sqrt{t_0} = F_6\left[1 + 4C_{66}\frac{(2m_{q6}\sqrt{t_0}B_6)}{F_6^2} + 4C_{64}n_4\frac{(2m_{q4}\sqrt{t_0}B_4)}{F_6^2} - \Delta_6\right]$$
$$+A_{\mathrm{art}}^{F6}(m_{q6}a) + B_{\mathrm{art}}^{F6}\frac{a}{\sqrt{t_0}} + C_{\mathrm{art}}^{F6}(m_{q4}a)$$

Empirical fits: M_V and F_V

$$
\begin{aligned}
M_{V4} = {} & p_0 + (m_{q4}\sqrt{t_0})\left[p_1 + (m_{q4}\sqrt{t_0})p_2 + (m_{q6}\sqrt{t_0})p_3\right] \\
& + A^{V4}_{\mathrm{art}}(m_{q4}a) + B^{V4}_{\mathrm{art}}(a/\sqrt{t_0}) + C^{V4}_{\mathrm{art}}(m_{q6}a)
\end{aligned}
$$

$$
\begin{aligned}
M_{V6} = {} & p_0 + (m_{q6}\sqrt{t_0})\left[p_1 + (m_{q6}\sqrt{t_0})p_2 + (m_{q4}\sqrt{t_0})p_3\right] \\
& + A^{V6}_{\mathrm{art}}(m_{q6}a) + B^{V6}_{\mathrm{art}}(a/\sqrt{t_0}) + C^{V6}_{\mathrm{art}}(m_{q4}a)
\end{aligned}
$$

- Analogous model functions for F_{V4} and F_{V6}…
- The parameters p_i are unrelated among the four quantities M_{V4}, M_{V6}, F_{V4}, and F_{V6}

Slide credit: Ethan Neil

Software: Multirep MILC

- Based on a branch of the MILCv7 code, focused on Wilson fermions
- Dynamical code generation using Perl: N_c and representation(s) are fixed in code generation, allowing the C compiler to optimize matrix operations
- Bells and whistles: clover term, nHYP smearing, Hasenbusch preconditioning, multi-level integrator, NDS action, …
- We use all of the above in our simulations. The clover term c_{SW} is set equal to unity (shown to work well with smearing)

The NDS Action
nHYP Dislocation Suppressing Action

- nHYP is a smearing scheme invented and optimized by Hasenfratz and Knechtli
    - The usual gauge links U are “thin” links. The fat link V is “smeared” link — a sum of products of gauge links connecting points on the lattice.
    - Smearing provides a smoother background for fermion propagation. Smoothing is known to reduce lattice artifacts.
- Dislocation suppression refers to taming large spikes in the fermion force during HMC evolution.
    - Enacted by extra marginal gauge terms
    - Creates a “repulsive potential” to cancel out the offending large spikes in the fermion force.

Lattice Spectroscopy

- Two-point functions encode spectral information, as usual
  - The axial Ward identity yields the quark mass

$$\partial_\mu \langle 0|A^\mu(x)\mathcal{O}|0\rangle = 2m_q\langle 0|(\overline{\psi}\gamma^5\psi)_x\mathcal{O}|0\rangle$$

  - Decay constants use "130 MeV" conventions (and its natural generalization)

$$\langle 0|\overline{u}\gamma^\mu\gamma^5 d|\pi(p)\rangle = iF_\pi p^\mu$$

- Many possible ways to set the scale
  - Sommer parameters r_0, r_1
  - The flow scale t_0
  - Mass of the Ω-baryon, decay constants $F\pi$ or $F_{K,\ etc\ldots}$

Decay Constants
Normalization and Conventions

- Decay constants with Wilson fermions involve a rescaling factor which depends on the critical value of the hopping parameter κ_{critical}.
  - For these ensembles, $\kappa \sim \kappa_{\text{critical}}$.
  - The Wilson normalization term does not vary much across the ensembles
- Decay constants also involve a (perturbative) matching factor, Z
  - For these ensembles, the Z-factors were approximately unity

$$F_P \sim (\text{Z-factor}) \times (\text{Wilson-}\kappa_{\text{critical}}\text{ factor}) \times F_{P,\text{raw}}$$

ChiPT fits at NLO — Normalized

Fundamental | Sextet

M_P^2

F_P