Improving the theoretical prediction for the $B_s - \bar{B}_s$ width difference: matrix elements of NLO ΔB=2 operators

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Work done in collaboration with
CTH Davies, JGIH Harrison, CJ Monahan, GP Lepage, J Shigemitsu
(HPQCD)

Outline

- B-mixing description
- Heavy Quark Expansion (HQE)
- Status of Standard Model calculation
- Our calculation

B mixing

Wigner-Weisskopf approximation

$$i\frac{d}{dt}\begin{pmatrix} a(t)|B^0\rangle \\ b(t)|\bar{B}^0\rangle \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a(t)|B^0\rangle \\ b(t)|\bar{B}^0\rangle \end{pmatrix}$$

Flavour basis ≠ mass basis ⇒ M & Γ non-diagonal

Mixing governed by 3 parameters:

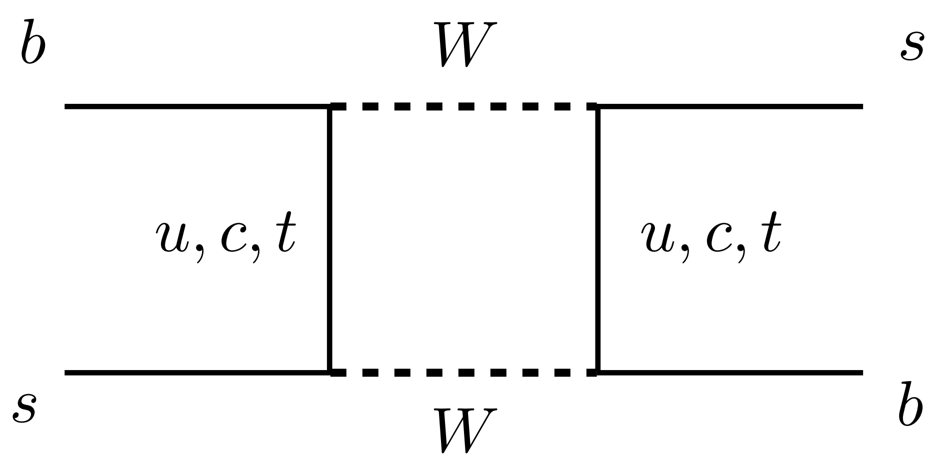
$$|M_{12}| \qquad |\Gamma_{12}| \qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

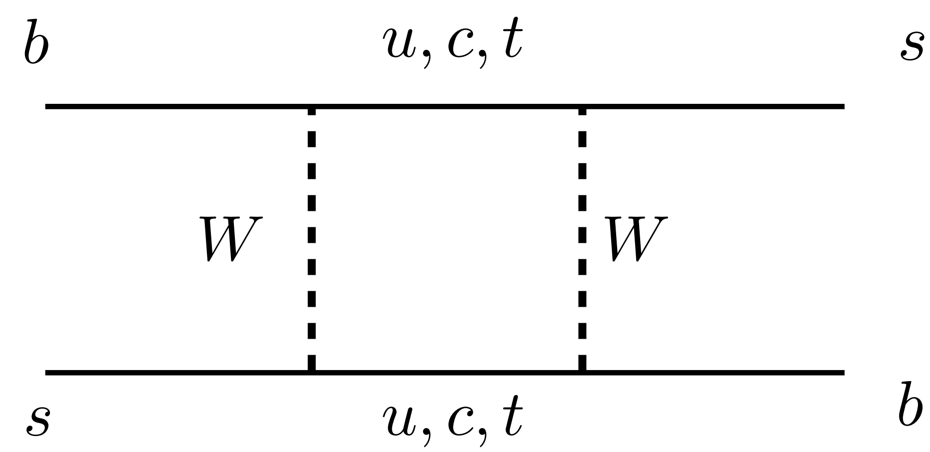
Observables

$$\Delta M = 2|M_{12}| \qquad \Delta\Gamma = 2|\Gamma_{12}|\cos\phi \qquad a_{\rm fs} = \frac{\Delta\Gamma}{\Delta M}\tan\phi$$

[up to corrections O(m_b^2/m_W^2)]

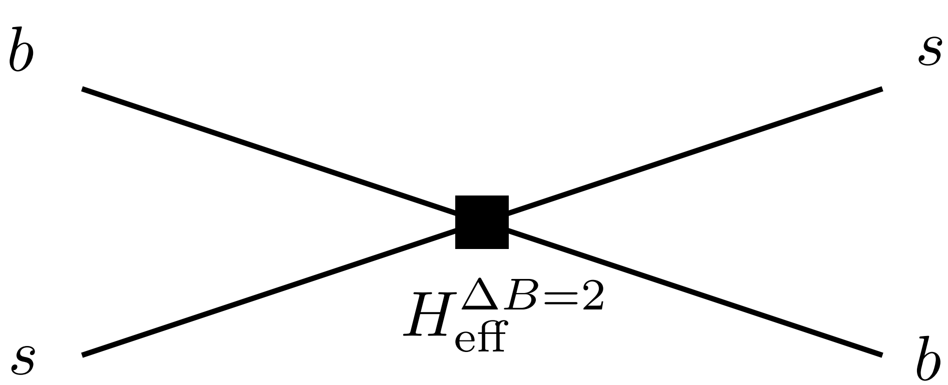
SM mixing

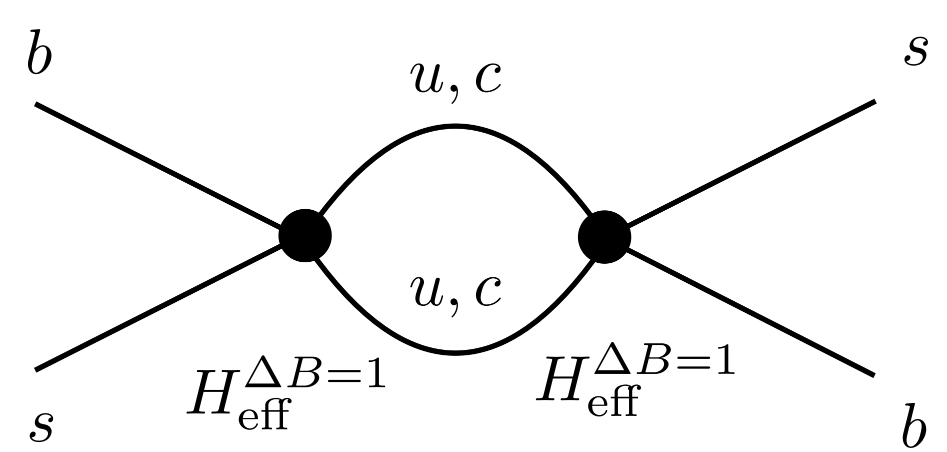




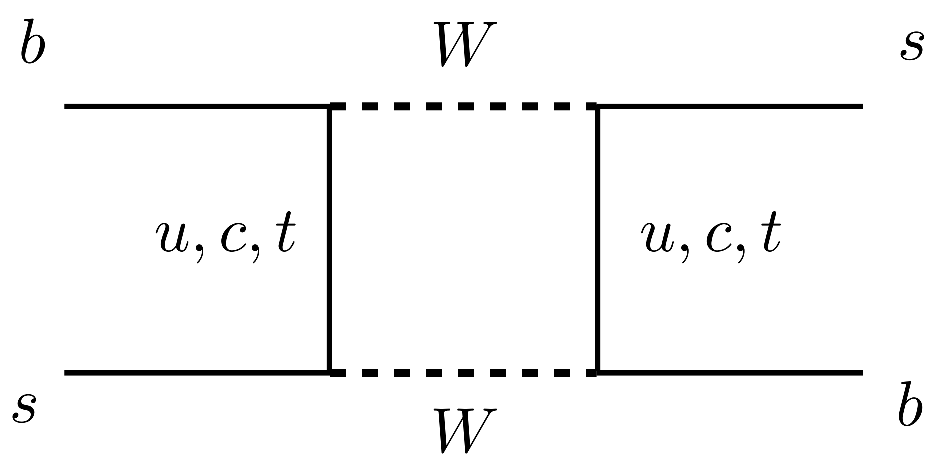
Integrate out *W* and *t*

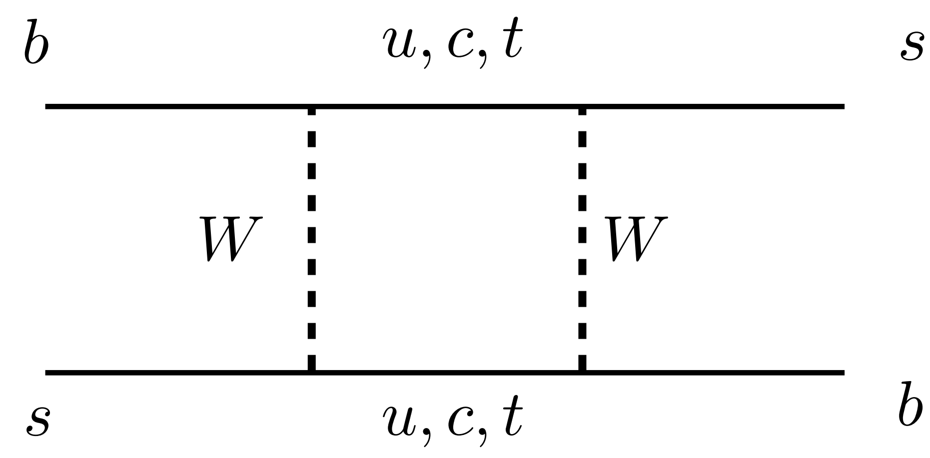
$\Downarrow$





SM mixing





Integrate out *W* and *t*

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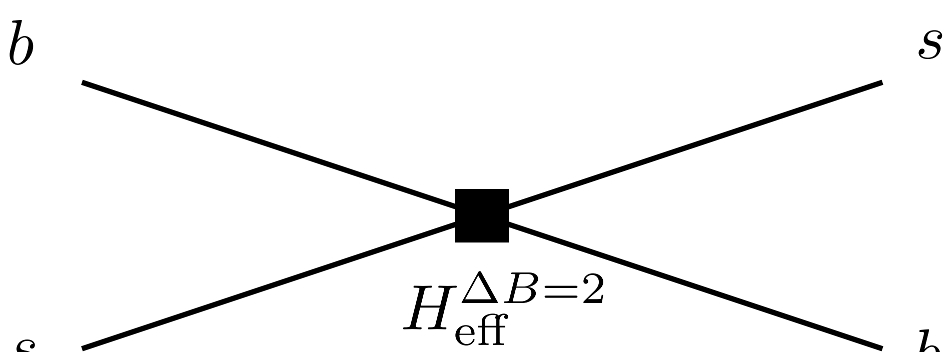
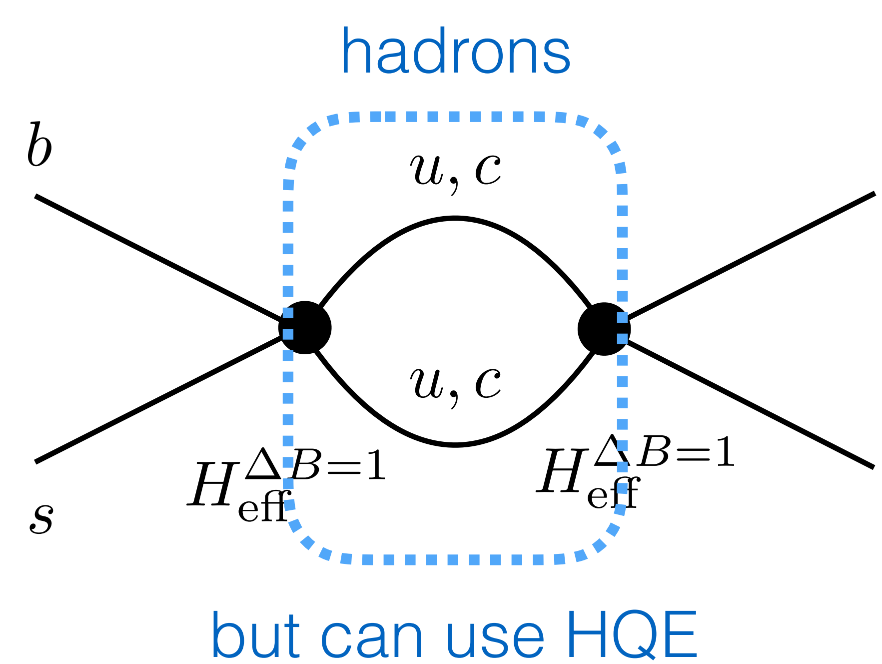




Mass difference

Oscillations governed by Hermitian part: dominated by local $\Delta B = 2$ matrix el.

$$\Delta M_s = \frac{1}{2m_{B_s}} \langle \bar{B}_s | H_{\text{eff}}^{\Delta B=2} | B_s \rangle$$

$$H_{\text{eff}}^{\Delta B=2} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{ts}^* V_{tb})^2 \sum_{i=1}^{5} C_i Q_i$$

$$Q_1 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$Q_2 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

$$Q_3 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1-\gamma^5) s^\alpha)$$

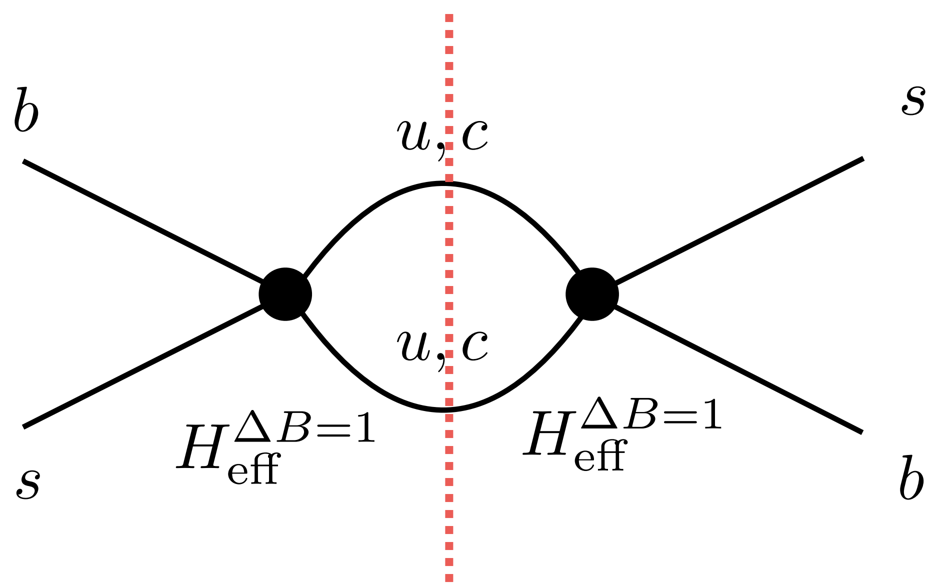
$$Q_4 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta)$$

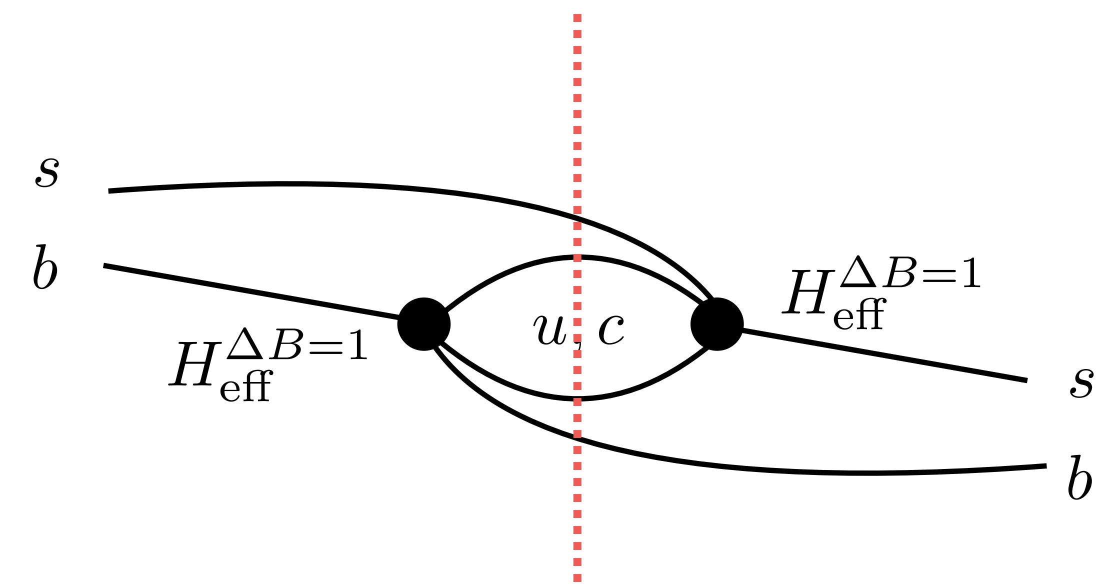
$$Q_5 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1+\gamma^5) s^\alpha)$$

In the Standard Model only Q_1 enters ΔM_s.

Recent work: HPQCD (prelim) arXiv:1411.6989, FNAL/MILC arXiv:1602.03560v2, HPQCD (in progress)

Lifetime difference & HQE





- Γ_{12} from imaginary part (optical theorem)
- Large momentum through loop
- Operator product expansion: Heavy Quark Expansion (HQE), quark-hadron duality
- Nonlocal operator related to local ΔB = 2 operators

$$\Gamma_{21} = \frac{1}{2m_{B_s}} \langle \bar{B}_s | \mathscr{T} | B_s \rangle$$

$$\mathscr{T} = \mathrm{Im}\, i \int d^4x\, \mathcal{T}\, H_{\rm eff}^{\Delta B=1}(x)\, H_{\rm eff}^{\Delta B=1}(0)$$

$\Downarrow$ HQE @ LO

$$\mathscr{T} = -\frac{G_F^2 m_b^2}{12\pi} (V_{cb}^* V_{cs})^2 [F(z,\mu_2) Q_1(\mu_2) + F_S(z,\mu_2) Q_2(\mu_2)]$$

HQE expressions

Dominant

$$\Gamma_{12}^{s} = -\left[\, \lambda_c^2\, \Gamma_{12}^{cc} \;+\; 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} \;+\; \lambda_u^2\,\Gamma_{12}^{uu}\,\right]$$ with $\lambda_i = V_{is}^* V_{ib}$

Leading order:

$$\Gamma_{12}^{cc} = \frac{G_F^2 m_b^2}{24\pi m_{B_s}} \left[(G + \frac{1}{2}\alpha_2 G_S)\langle \bar{B}_s | Q_1 | B_s \rangle + \alpha_1 G_S \langle \bar{B}_s | Q_3 | B_s \rangle \right] \;+\; \tilde{\Gamma}_{12,1/m_b}^{cc}$$

where lattice QCD gives the matrix elements of Q_1 and Q_3.

G's depend on α_s, m_b, m_c/m_b, μ_1, μ_2 Beneke, et al. PLB459, hep-ph/9808385

Expressions from Lenz & Nierste, JHEP 06 (2007), hep-ph/0612167

NLO

$$\tilde{\Gamma}^{cc}_{12,1/m_b} = \frac{G_F^2 m_b^2}{24\pi m_{B_s}} \left\{ g_0^{cc} \langle \bar{B}_s | R_0 | B_s \rangle + \sum_{j=1}^{3} \left[g_j^{cc} \langle \bar{B}_s | R_j | B_s \rangle + \tilde{g}_j^{cc} \langle \bar{B}_s | \tilde{R}_j | B_s \rangle \right] \right\}$$

g^{cc}'s depend on m_c/m_b

$$R_0 = Q_2 + \alpha_1 Q_3 + \frac{1}{2}\alpha_2 Q_1$$

$$R_1 = \frac{m_s}{m_b} (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta) = \frac{m_s}{m_b} Q_4$$

$$R_2 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$R_3 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

+ Tilde'd operators corresponding to mixing color indices.

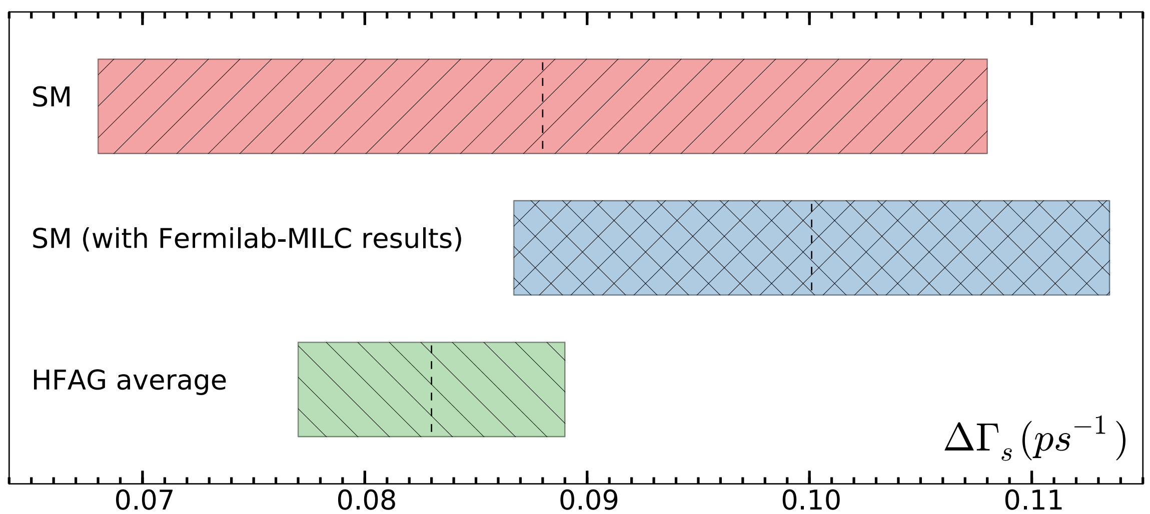
Up to further $1/m_B$ corrections:

$$\langle \tilde{R}_2 \rangle = -\langle R_2 \rangle \qquad \langle \tilde{R}_3 \rangle = \langle R_3 \rangle + \tfrac{1}{2} \langle R_2 \rangle$$

Status

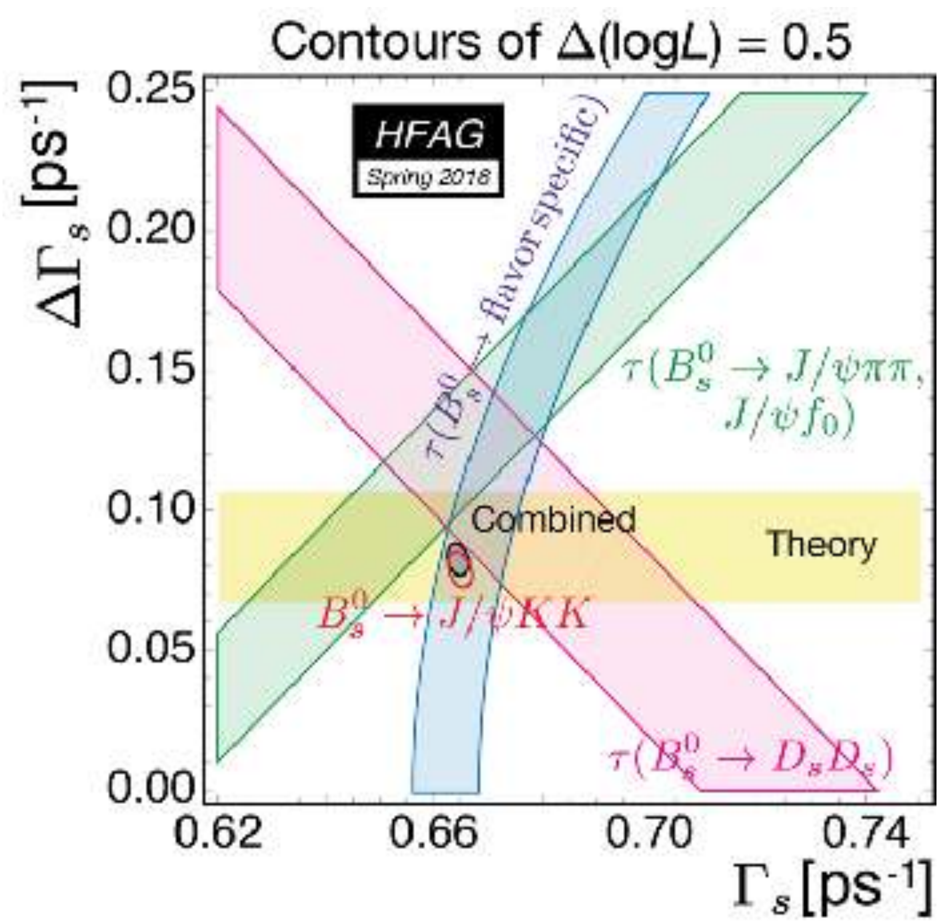
Artuso, Borissov, Lenz, arXiv:1511.09466v1

$$\Delta\Gamma_s^{\mathrm{SM},2015} = 0.088(20)\,\mathrm{ps}^{-1}$$



Plot and updated SM prediction from
MJ Kirk, Lattice 2016 poster

Heavy Flavour Averaging Group



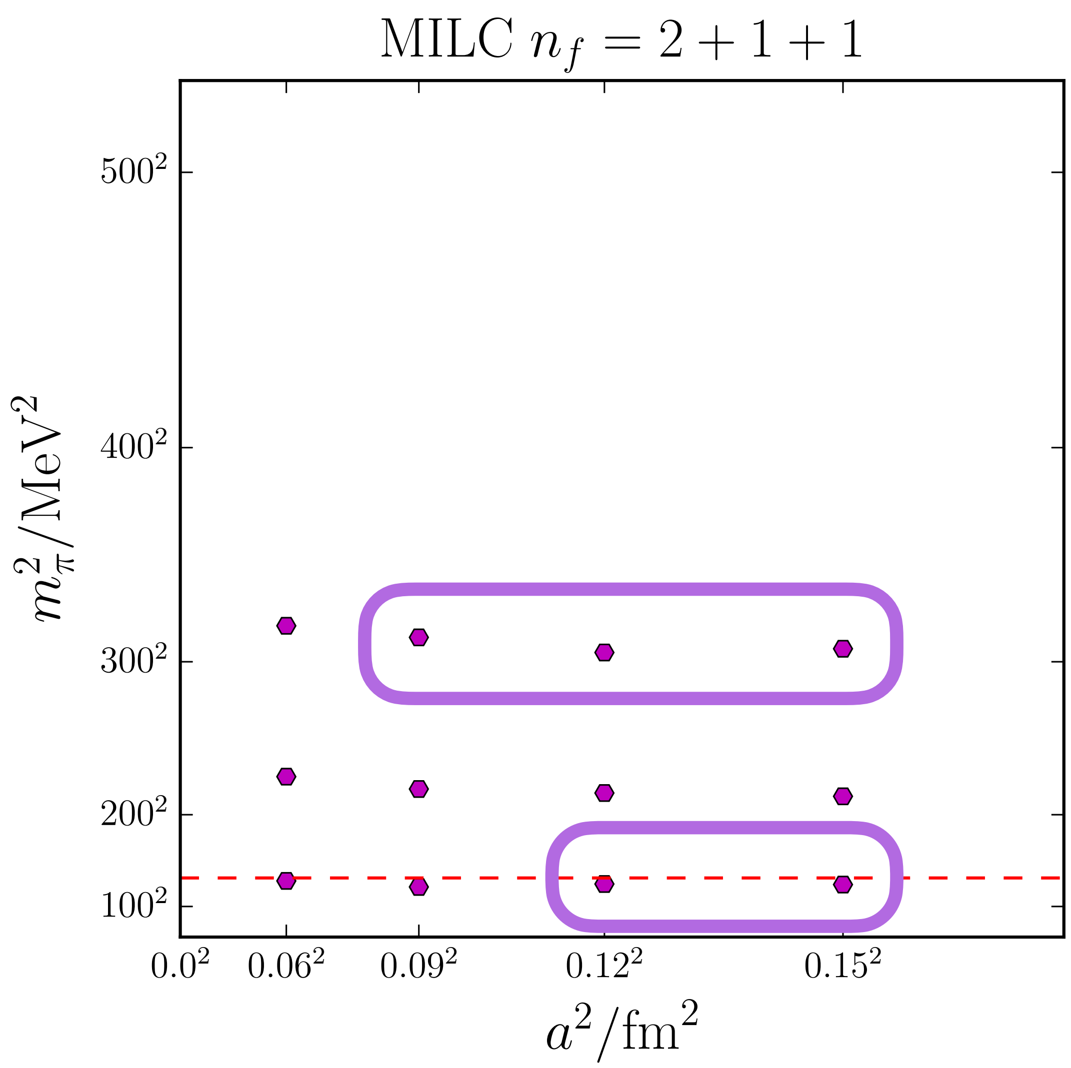
Dominent SM uncertainties:

- 15% due to matrix element of R_2 (bag factor = 1.0 ± 0.5, for one definition of m_b)
- 14% due to matrix element of Q_1 (FLAG, but see new FNAL/MILC)
- 8% due to renormalization scale

This work

HPQCD calculation

- Extends ongoing HPQCD calculation of matrix elements of dimension-6 ΔB=2 operators ($Q_1 \ldots Q_5$)
- MILC highly improved staggered quark (HISQ) gauge field configurations (2+1+1 sea quarks)
- Nonrelativistic bottom quark, HISQ strange quark



Perturbative matching

Match continuum and lattice at $O(\alpha_s)$

$$\langle Q_i \rangle_{\overline{\rm MS}} = \langle \hat{Q}i \rangle + \alpha_s \rho_{ij} \langle \hat{Q}j \rangle + \langle \hat{Q}i1 \rangle^{\rm sub}$$

taking into account power-law "mixing down" at $O\left(\frac{\alpha_s}{aM}\right)$

$$\langle \hat{Q}i1 \rangle^{\rm sub} = \langle \hat{Q}i1 \rangle - \alpha_s \zeta_{ij} \langle \hat{Q}j \rangle$$

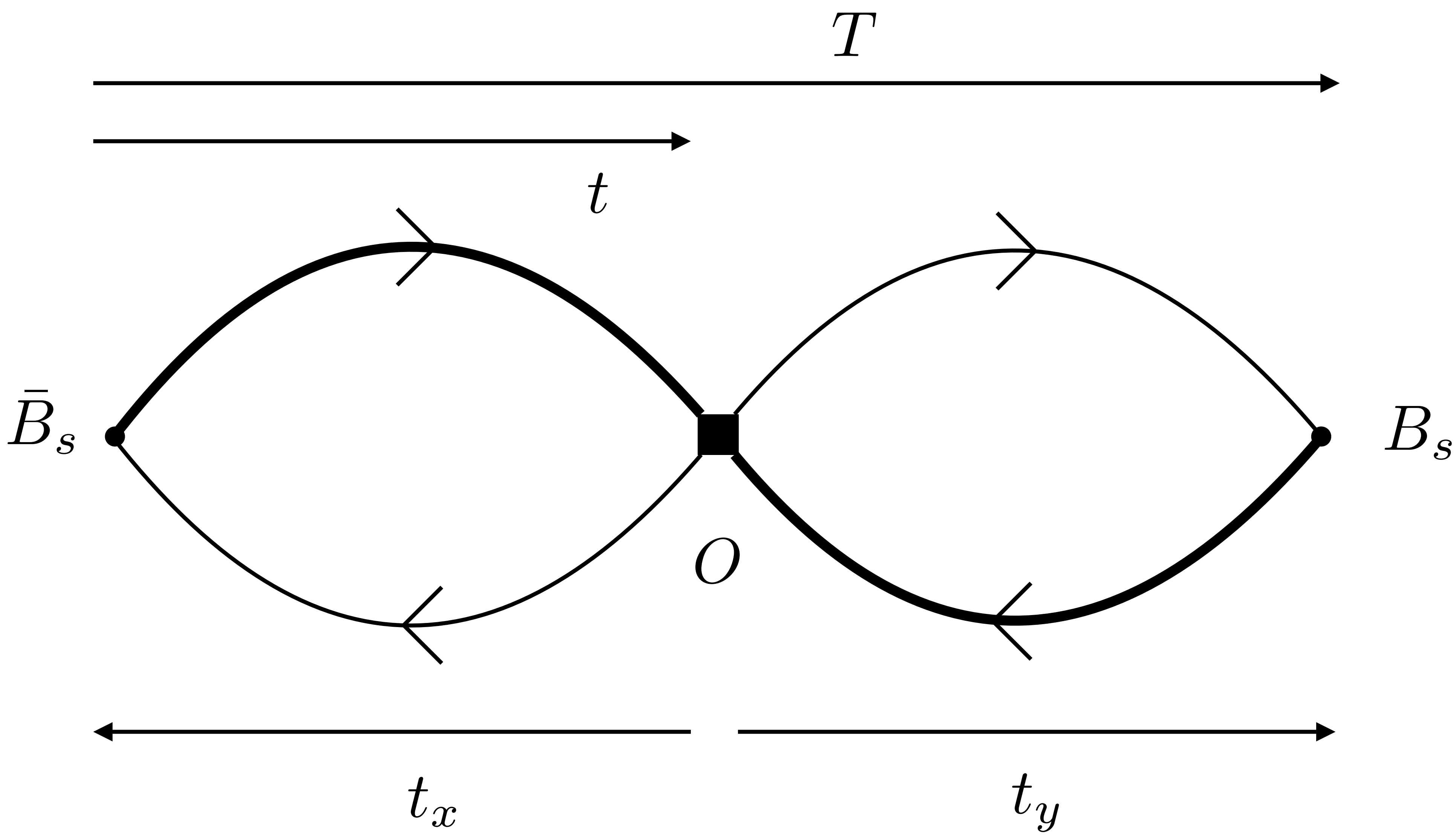
Monahan, Gámiz, Horgan, Shigemitsu, PRD90 (2014), arXiv:1407.4040

Similarly we have now computed coefficients in

$$\langle \hat{R}_i \rangle^{\rm sub} = \langle \hat{R}_i \rangle - \alpha_s \xi_{ij} \langle \hat{Q}j \rangle$$

In fact, $|\rho_{ij}|, |\zeta_{ij}|, |\xi_{ij}| < 1$ for lattices in use here.

Correlation functions



Strange quark "source" at operator *O.*
Derivative source (finite difference) for *R* operators

Dimension-7

Derivative part of R_2 and R_3

$$\frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho \Gamma D^\rho s^\alpha) = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_0 \Gamma D^0 s^\alpha) + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

Using EOM

$$\bar{b}\overleftarrow{D}_0 = \pm m_b \bar{b}\,\gamma_0 \quad \text{and} \quad i\gamma_0 D^0 s = -i(\vec{\gamma}_M \cdot \vec{D})s = (\vec{\gamma}_E \cdot \vec{D})s$$

we have

$$R_{2,3} = \pm\frac{1}{m_b}(\bar{b}_\alpha \Gamma \gamma_0 (\vec{\gamma}\cdot\vec{D}) s_\alpha)(\bar{b}_\beta \Gamma s_\beta)$$

± correspond to outgoing b quark/incoming anti-b quark

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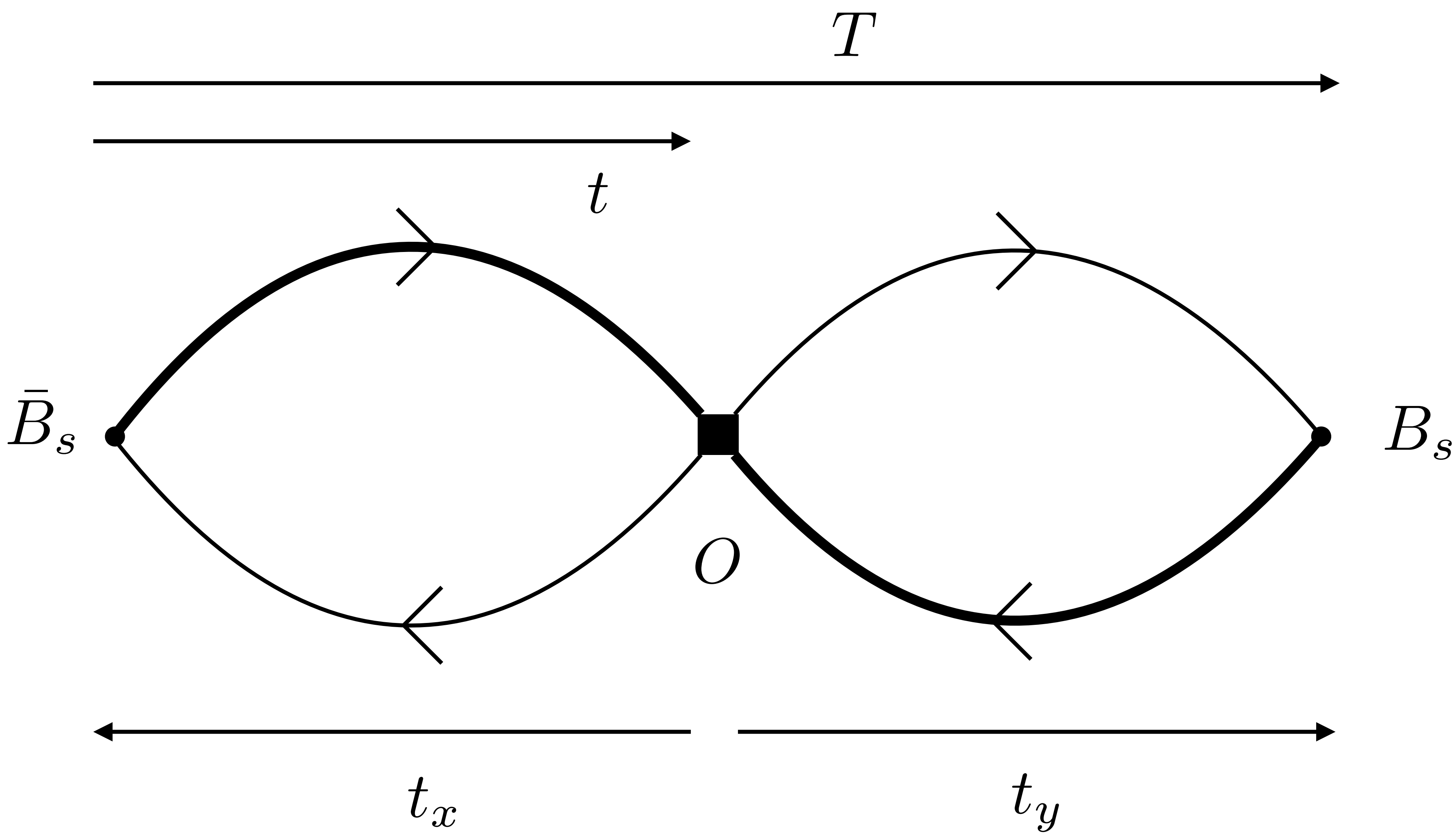
Correlation functions



$$C^{\rm 3pt}_{ab}(t,T) = \sum_{i,j} X_{a,i} V_{nn,ij} X_{b,j} \exp(-E_i t) \exp(-E_j (T-t))$$

Correlation functions

$$C_{ab}^{3\mathrm{pt}}(t,T) = \sum_{i,j} X_{a,i} V_{nn,ij} X_{b,j} \exp(-E_i t) \exp(-E_j (T-t))$$

$$X_{a,0} V_{nn,00} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle \boxed{\langle \bar{B}_s|a^6 O_{comp}|B_s\rangle} \langle B_s|\Phi_b|0\rangle}{(2m_{B_s}a^3)^2}$$

Remove unwanted factors using 2-point functions

$$C_{ab}^{2\mathrm{pt}}(t) = \sum_{i} X_{a,i} X_{b,i} \exp(-E_i t)$$

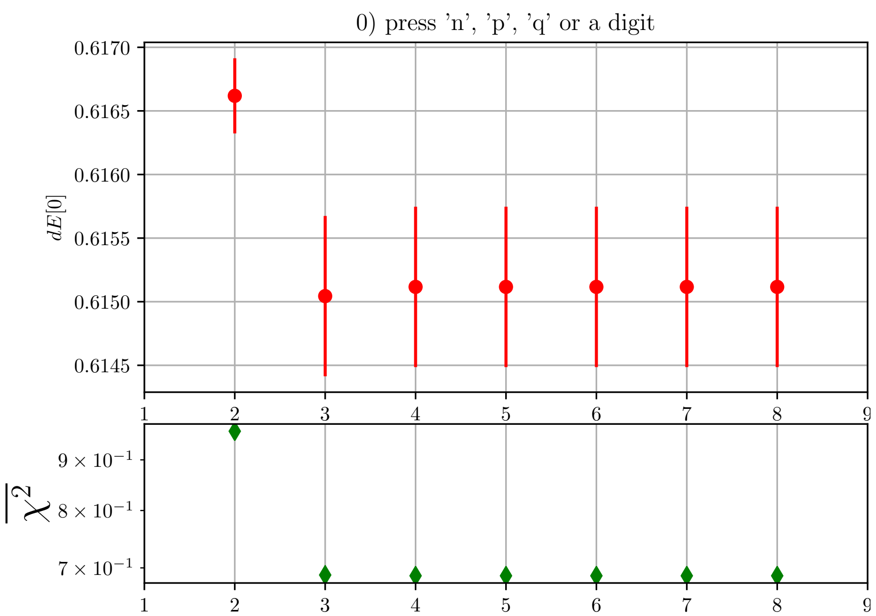
$$X_{a,0} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle \langle B_s|\Phi_b|0\rangle}{2m_{B_s}a^3}$$

Bayesian priors

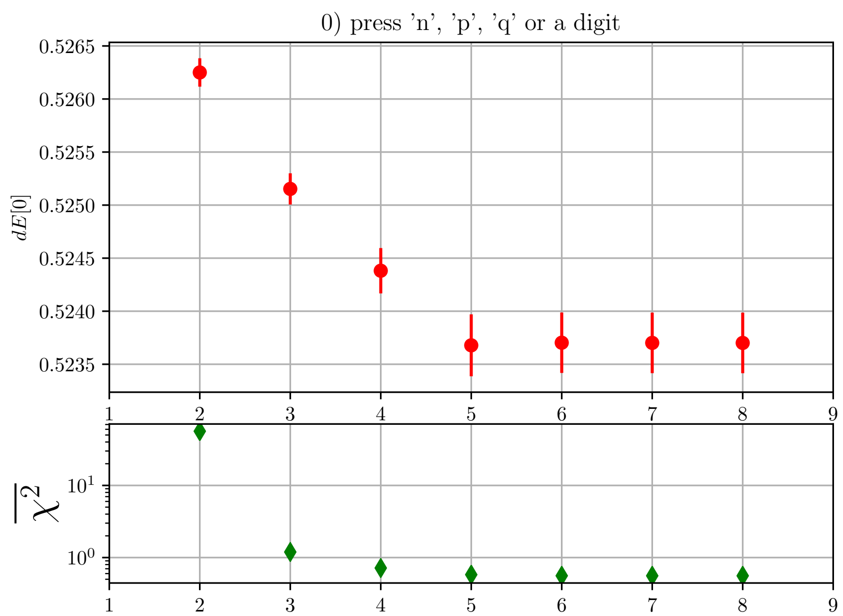
- With wide priors, fit 2-point function using $N_{\text{exp}} = 2+2$, $t_{\text{min}} \approx 1.2$ fm
- Set ground state priors: $\delta E_{prior} = 10{*}\delta E_{fit}$, $\delta X_i = 10{*}\delta X_{fit}$
- Set excited state priors: $\Delta E = a\Lambda_{\text{QCD}}$, $\delta(\Delta E) = 50\%$, $X_i = 0 \pm 1$
- With V's = 0 ± 1, fit 3-point function (2 values of large T) using $N_{\text{exp}} = 3+3$, $t_{\text{min}} \approx 1.0$ fm
- Set ground state V_{00} to be fit result ± 50-100% (100%-400% for oscillating state contributions)
- (Ideally) simultaneous fit to all T, all smearings, t/a=2 or so.

B_s energies vs. N_{exp}

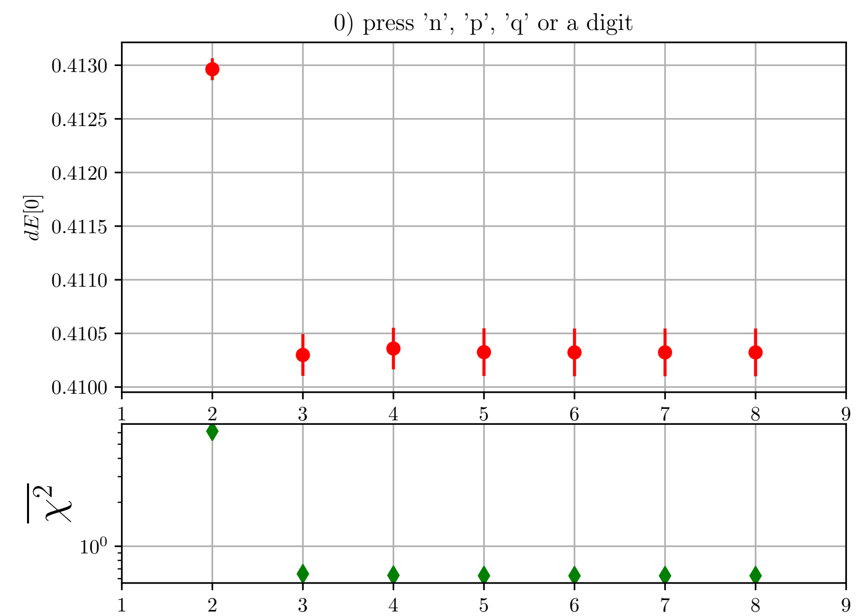
VC5



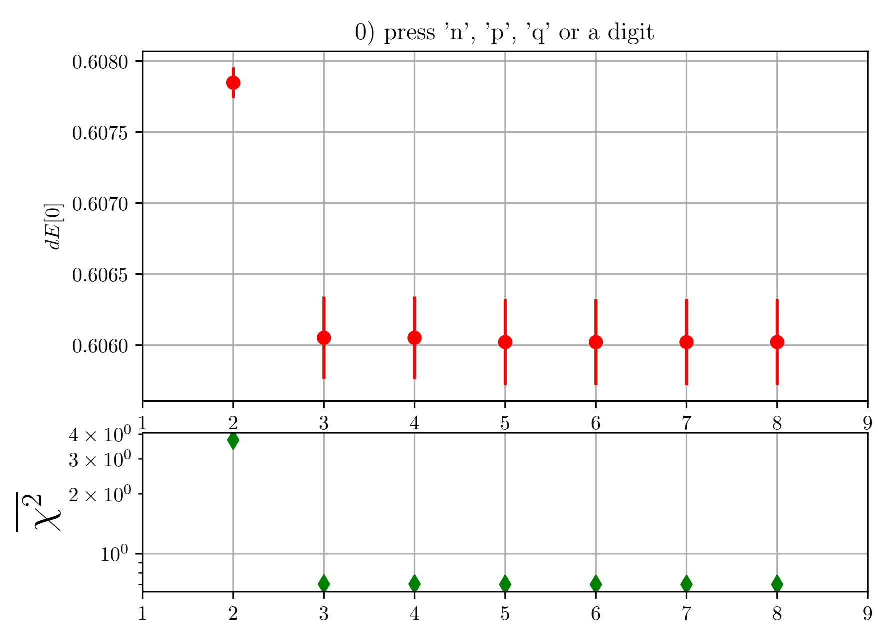
C5



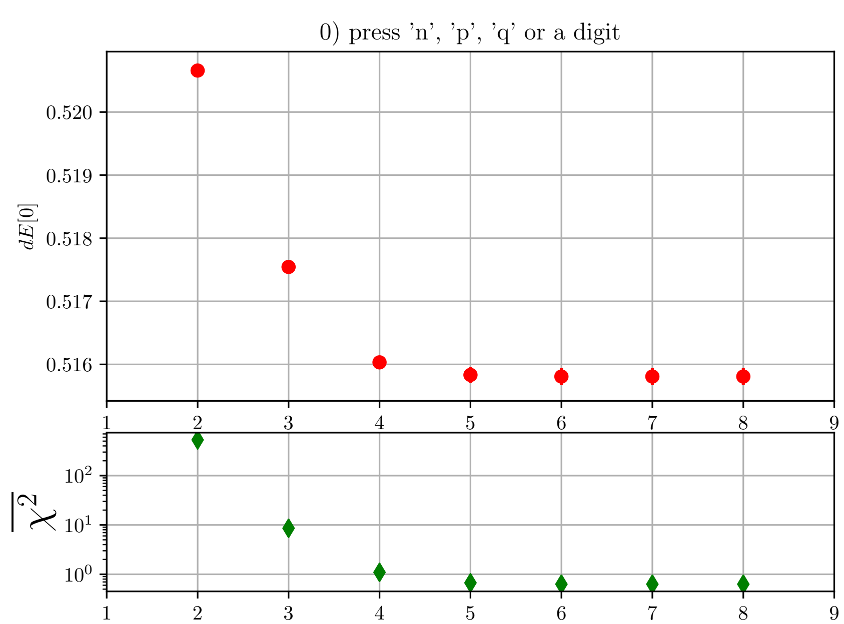
F5



VCp



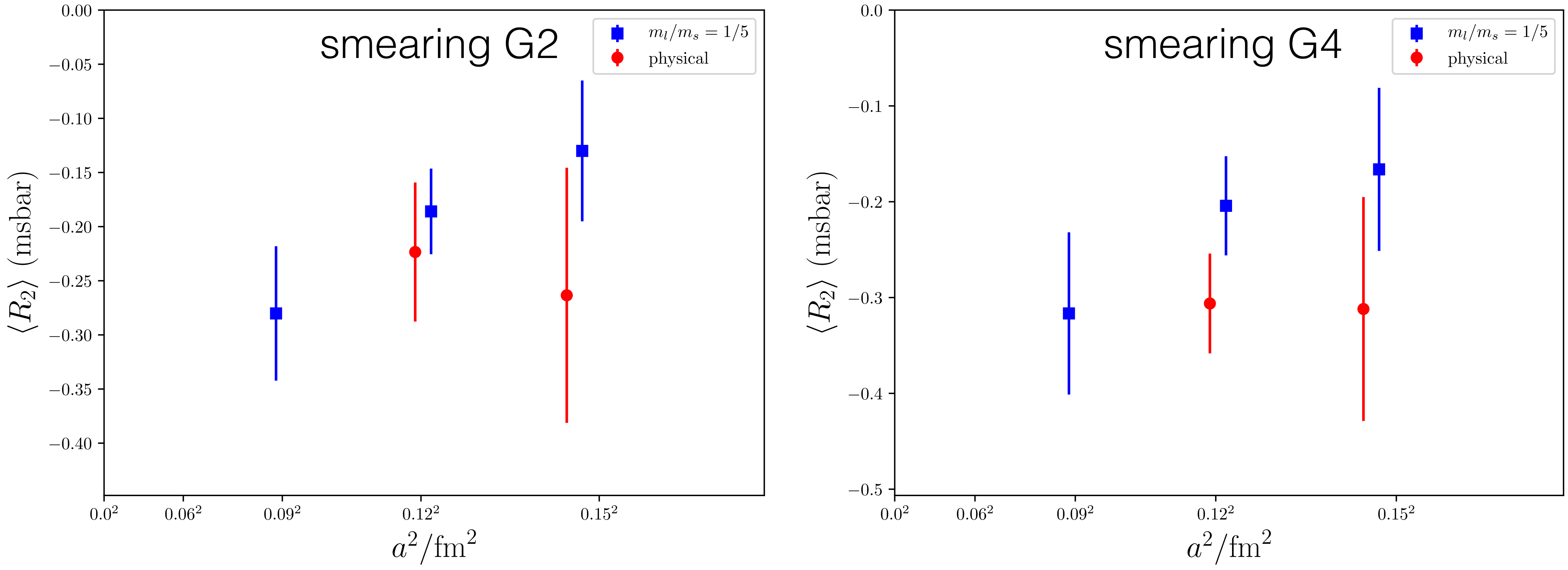
Cp



[N.B. Coarse (C) lattice fits include local source/ sink, VC & F do not.]

Rough fits for R_2

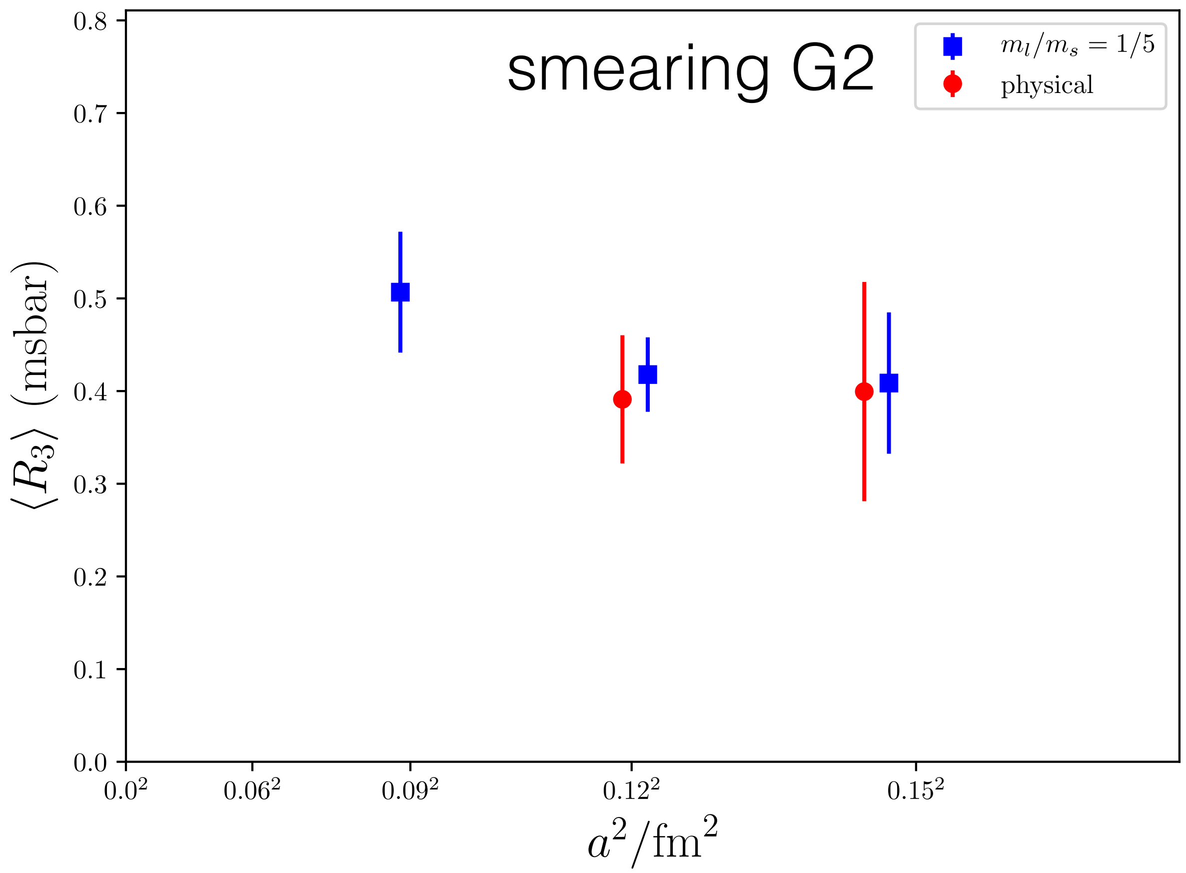
Very preliminary fits, 2-3 T-values, 2 smearings, N_{exp} = 5

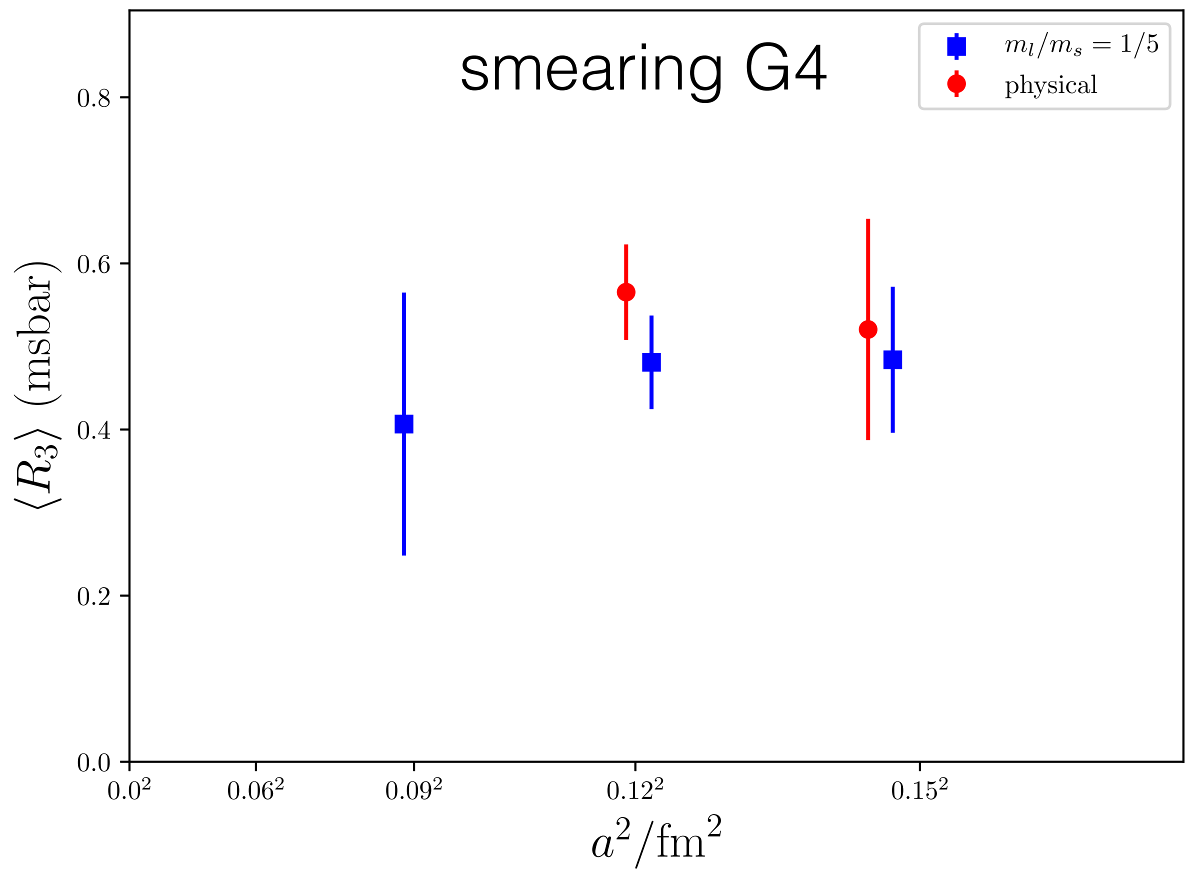


Work remains to assess fitting uncertainties

Rough fits for R_3

Very preliminary fits, 2-3 T-values, 2 smearings, N_{exp} = 5





Work remains to assess fitting uncertainties

Rough numerics

$$\Delta\Gamma_s = \left[0.071(11)\left(\frac{\langle Q_1\rangle}{3\,\mathrm{GeV}^4}\right) + 0.035(6)\left(\frac{\langle Q_3\rangle}{0.8\,\mathrm{GeV}^4}\right) - 0.027(4)\left(\frac{\langle R_2\rangle}{-0.3\,\mathrm{GeV}^4}\right)\right]\ \mathrm{ps}^{-1}$$

Derived from Lenz & Nierste, JHEP 06 (2007), arXiv:hep-ph/0612167

FNAL/MILC arXiv:1602.03560v2: $\langle Q_1\rangle$ @ 6% $\langle Q_3\rangle$ @ 13%

Reducing uncertainty* on $\langle R_2\rangle$ @ 50% $\to$ 25% $\implies$ $\Delta\Gamma_s$ @ 25% $\to$ 18%

[Figures here are rough, e.g. prefactors above may be out of date.]

* 50% estimate from VSA, to be replaced by 25% LQCD calculation

Summary

- First LQCD calculation of dimension-7 operators
- To-do: finalize fits; assess lattice spacing and sea quark mass effects
- Perturbative matching uncertainty O(α_s), but this is good enough!