Nucleon Axial and Electromagnetic Form Factors

Tanmoy Bhattacharya[1], Rajan Gupta[1],
Yong-Chull Jang[1], Huey-Wen Lin[3], Boram Yoon[2]

Theoretical Division T-2[1],
Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Computer, Computational, Statistical Science Division CCS-7[2],
Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Department of Physics and Astronomy, Michigan State University[3],
East Lansing, Michigan 48824, USA

35th International Symposium on Lattice Field Theory,
19–24 June, 2017
Granada, Spain

Overview

- Isovector Form factors, charges, charge radii, magnetic moment

$$\langle N(\vec{p}_f)|A_\mu(\vec{Q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[G_A(Q^2)\gamma_\mu + q_\mu \frac{\widetilde{G}_P(Q^2)}{2M}\right]\gamma_5 u(\vec{p}_i)$$

$$q = p_f - p_i,\ Q^2 = -q^2 = \vec{p}_f^{\,2} - (E-M)^2,\ \vec{p}_i = 0$$

$$\langle r_A^2\rangle = -6\frac{d}{dQ^2}\left(\frac{G_A(Q^2)}{G_A(0)}\right)\bigg|_{Q^2=0},\ G_A(0) \equiv g_A$$

$$\langle N(\vec{p}_f)|V_\mu(\vec{q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[F_1(Q^2)\gamma_\mu + \sigma_{\mu\nu}q_\nu \frac{F_2(Q^2)}{2M}\right]u(\vec{p}_i)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2) \rightarrow \langle r_E^2\rangle,\ G_E(0) \equiv g_V$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \rightarrow \langle r_M^2\rangle$$

$$\frac{G_M(0)}{G_E(0)} = \mu = 1 + \kappa, \quad (\mu = \mu^p - \mu^n, \kappa = \kappa^p - \kappa^n)$$

- PCAC: 2 independent FF

$$\langle N(\vec{p}_f)|A_\mu(\vec{Q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[G_A(Q^2)\gamma_\mu + q_\mu \frac{\widetilde{G}_P(Q^2)}{2M_N}\right]\gamma_5 u(\vec{p}_i)$$

$$q = p_f - p_i,\ Q^2 = -q^2 = \vec{p}_f^{\,2} - (E-M)^2,\ \vec{p}_i = 0$$

$$\langle N(\vec{p}_f)|P_\mu(\vec{q})|N(\vec{p}_i)\rangle = \overline{u}(\vec{p}_f)\left[G_P(Q^2)\gamma_5\right]u(\vec{p}_i)$$

$$2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\widetilde{G}_P(Q^2) \qquad \text{[PCAC]}$$

- Pion pole dominance hypothesis: 1 independent FF

$$\frac{m_\mu}{2M_N g_A}\tilde{G}_P(Q^2) = \frac{m_\mu}{2M_N g_A}\left[\frac{4M_N^2}{Q^2+M_\pi^2}\right]G_A(Q^2)$$

- $g_P^* \equiv \frac{m_\mu}{2M_N}\tilde{G}_P(Q^{*\,2})\,, \quad Q^{*\,2} \equiv 0.88 m_\mu^2 \quad [\mu^- + p \to \nu_\mu + n]$
- $g_{\pi NN} = \lim_{Q^2\to -M_\pi^2} \frac{M_\pi^2 + Q^2}{4M_N F_\pi}\widetilde{G}_P(Q^2)\,, \quad g_{\pi NN} = \frac{M_N g_A}{F_\pi}$ (Goldberger-Treiman)

Lattice Setup

- Clover on $N_f = 2+1+1$ HISQ

Ensemble ID	a (fm)	$M_\pi^{\rm sea}$ (MeV)	$M_\pi^{\rm val}$ (MeV)	$L^3 \times T$	$M_\pi^{\rm val} L$	$t_{\rm sep}/a$	$N_{\rm conf}$	$N_{\rm meas}^{\rm HP}$	$N_{\rm meas}^{\rm AMA}$
a12m310	0.1207(11)	305.3(4)	310.2(2.8)	$24^3 \times 64$	4.55	$\{8, 10, 12\}$	1013	8104	64,832
a12m220L	0.1189(09)	217.0(2)	227.6(1.7)	$40^3 \times 64$	5.49	$\{8, 10, 12, 14\}$	1010	8080	68,680
a09m310	0.0888(08)	312.7(6)	313.0(2.8)	$32^3 \times 96$	4.51	$\{10, 12, 14\}$	881	7048	
a09m220	0.0872(07)	220.3(2)	225.9(1.8)	$48^3 \times 96$	4.79	$\{10, 12, 14\}$	890	7120	
a09m130	0.0871(06)	128.2(1)	138.1(1.0)	$64^3 \times 96$	3.90	$\{10, 12, 14\}$	883	7064	60,044
a06m310	0.0582(04)	319.3(5)	319.6(2.2)	$48^3 \times 144$	4.52	$\{16, 20, 22, 24\}$	1000	8000	64,000
a06m220	0.0578(04)	229.2(4)	235.2(1.7)	$64^3 \times 144$	4.41	$\{16, 20, 22, 24\}$	650	2600	41,600
a06m135	0.0570(01)	135.5(2)	135.6(1.4)	$96^3 \times 192$	3.7	$\{16, 18, 20, 22\}$	322	1610	51,520

- Clover on $N_f = 2+1$ Clover

Ensemble ID	a (fm)	M_π (MeV)	$L^3 \times T$	$M_\pi L$	$t_{\rm sep}/a$	$N_{\rm conf}$	$N_{\rm meas}^{\rm HP}$	$N_{\rm meas}^{\rm AMA}$
a127m285	0.127(2)	285(3)	$32^3 \times 96$	5.85	$\{8, 10, 12, 14\}$	1000	4020	128480
a094m280 $S_5 S_5$	0.094(1)	278(3)	$32^3 \times 64$	4.11	$\{10, 12, 14, 16, 18\}$	1005	3015	96480
a094m280 $S_7 S_7$					$\{10, 12, 14, 16, 18\}$			
a094m280 $S_9 S_9$					$\{8, 10, 12, 14, 16\}$			
a091m170	0.091(1)	166(2)	$48^3 \times 96$	3.7	$\{8, 10, 12, 14, 16\}$	629	2516	80512
a091m170L	0.091(1)	172(6)	$64^3 \times 128$	5.08	$\{8, 10, 12, 14, 16\}$	467	2335	74720

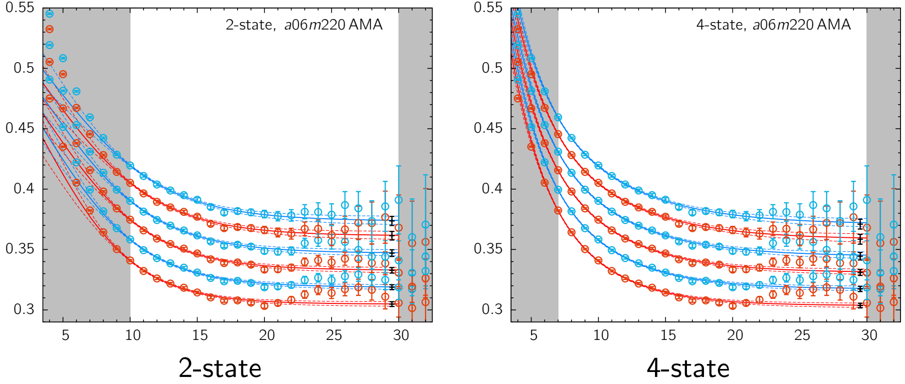
Correlator Fits: 2-pt

- two versus four states fit

$$C^{2\text{pt}}(t,\boldsymbol{p}) = |\mathcal{A}_0|^2 e^{-E_0 t} + |\mathcal{A}_1|^2 e^{-E_1 t} + |\mathcal{A}_2|^2 e^{-E_2 t} + |\mathcal{A}_3|^2 e^{-E_3 t} + \cdots$$

- plot effective mass from fits and data

$$E_{\text{eff}}(t) = \log \frac{C^{2\text{pt}}(t)}{C^{2\text{pt}}(t+1)}$$



We now use 4-state fits.

Correlator Fits: A_μ 3-pt

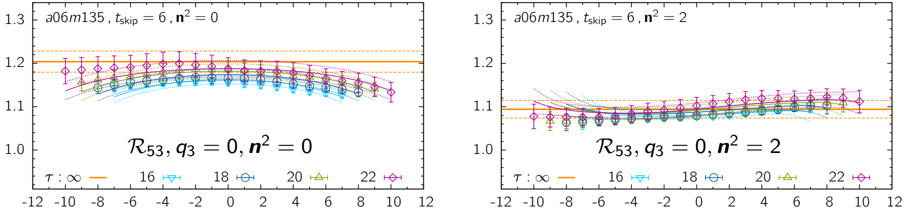
- Results obtained by using following fit.

$$C_\Gamma^{(3pt)}(t;\tau;\boldsymbol{p}',\boldsymbol{p}) = |\mathcal{A}_0'||\mathcal{A}_0|\langle 0'|\mathcal{O}_\Gamma|0\rangle e^{-E_0 t - M_0(\tau - t)} + |\mathcal{A}_1'||\mathcal{A}_1|\langle 1'|\mathcal{O}_\Gamma|1\rangle e^{-E_1 t - M_1(\tau - t)} + |\mathcal{A}_0'||\mathcal{A}_1|\langle 0'|\mathcal{O}_\Gamma|1\rangle e^{-E_0 t - M_1(\tau - t)} + |\mathcal{A}_1'||\mathcal{A}_0|\langle 1'|\mathcal{O}_\Gamma|0\rangle e^{-E_1 t - M_0(\tau - t)}$$

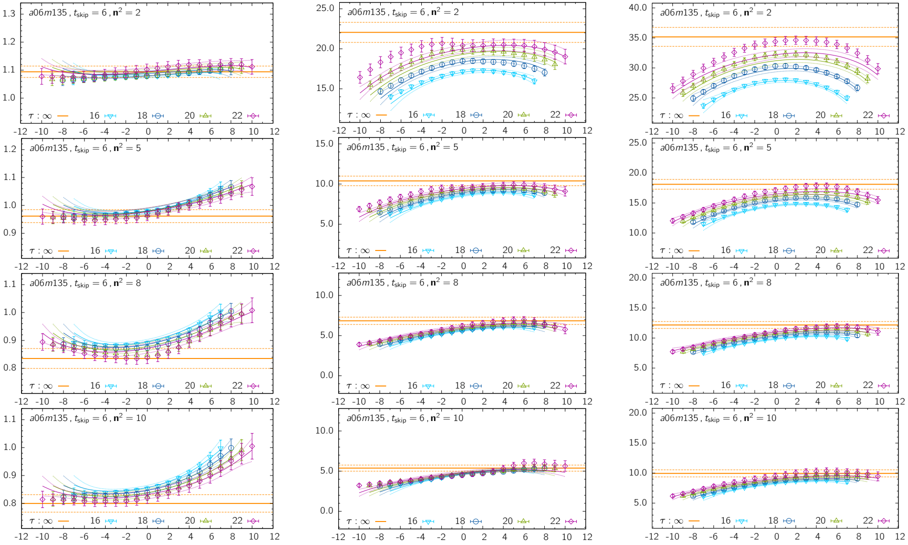
- Data is displayed using the following ratio.

$$\mathcal{R}_\Gamma(t,\tau,\boldsymbol{p}',\boldsymbol{p}) = \frac{C_\Gamma^{(3pt)}(t,\tau;\boldsymbol{p}',\boldsymbol{p})}{C^{(2pt)}(\tau,\boldsymbol{p}')} \times \left[\frac{C^{(2pt)}(t,\boldsymbol{p}')C^{(2pt)}(\tau,\boldsymbol{p}')C^{(2pt)}(\tau - t,\boldsymbol{p})}{C^{(2pt)}(t,\boldsymbol{p})C^{(2pt)}(\tau,\boldsymbol{p})C^{(2pt)}(\tau - t,\boldsymbol{p}')}\right]^{1/2}$$

Γ :	$\gamma_5\gamma_1(\mathrm{Im})$	$\gamma_5\gamma_2(\mathrm{Im})$	$\gamma_5\gamma_3(\mathrm{Im})$	$\gamma_5(\mathrm{Re})$
$\rightarrow$	$q_1 q_3 \widetilde{G}_P$,	$-q_2 q_3 \widetilde{G}_P$,	$-q_3^2 \widetilde{G}_P + 2M(M+E)G_A$,	$q_3 G_P$



Controlling Excited State Contribution to $\langle N|A_\mu|N\rangle$



$\mathrm{Im}\,\mathcal{R}_{53}, q_3 = 0 \to G_A(Q^2)$ $\mathrm{Im}\,\mathcal{R}_{51} \to \widetilde{G}_P(Q^2)$ $\mathrm{Re}\,\mathcal{R}_5 \to G_P(Q^2)$

Fitting Q^2 dependence of the Axial Form Factor

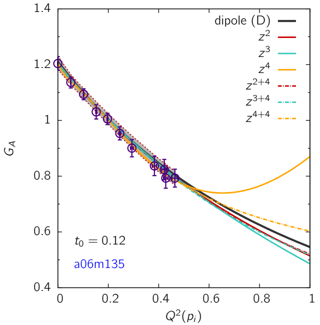
- dipole

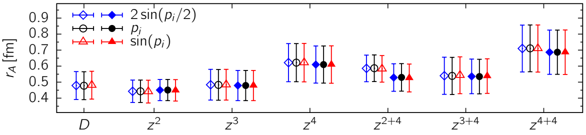
$$G_A(Q^2) = \frac{G_A(0)}{(1+Q^2/\mathcal{M}_A^2)^2} \quad \Longrightarrow \quad \langle r_A^2 \rangle = \frac{12}{\mathcal{M}_A^2}$$

- z-expansion w/o sumrule constraints $Q^n G_A(Q^2) \to 0$

$$\frac{G_A(Q^2)}{G_A(0)} = \sum_{k=0}^{\infty} a_k z(Q^2)^k \,, \; z = \frac{\sqrt{t_{\rm cut}+Q^2} - \sqrt{t_{\rm cut}+t_0}}{\sqrt{t_{\rm cut}+Q^2} + \sqrt{t_{\rm cut}+t_0}} \,,$$

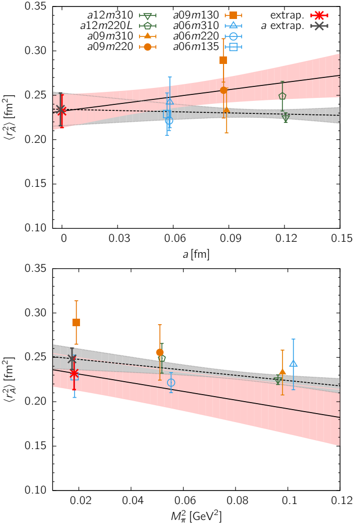
$$\sum_{k=n}^{k_{\rm max}} k(k-1)\ldots(k-n+1)a_k = 0 \qquad n = 0,1,2,3 \qquad \text{(sumrule)}$$



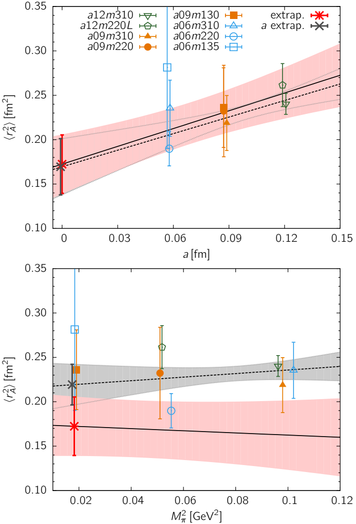


Axial Charge Radius $\langle r_A^2 \rangle$: dipole versus z^{2+4}

$$\langle r_A^2 \rangle = d_0 + d_1 a + d_2 M_\pi^2 + d_3 M_\pi^2 e^{-M_\pi L}$$



dipole ($d_3 = 0$)



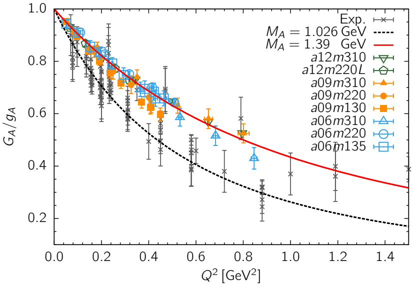
z^{2+4} ($d_3 = 0$)

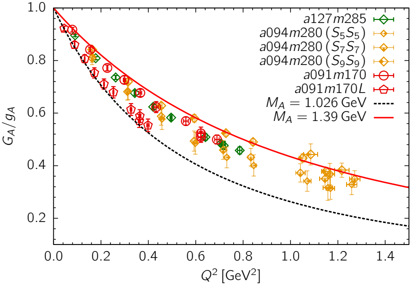
G_A: Clover-on-HISQ versus Clover-on-Clover

$a \to 0\,,\ M_\pi \to 135\ \mathrm{MeV}$ extrapolation

$\langle r_A^2 \rangle$	with FV	without FV
dipole	0.24(3)	0.23(2)
z^2	0.22(4)	0.20(3)
z^{2+4}	0.19(4)	0.17(3)
z^3	0.23(6)	0.18(4)
z^{3+4}	0.24(7)	0.18(5)

No $a \to 0\,,\ M_\pi \to 135\ \mathrm{MeV}$ extrapolation yet

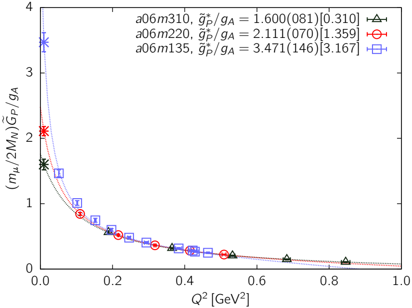


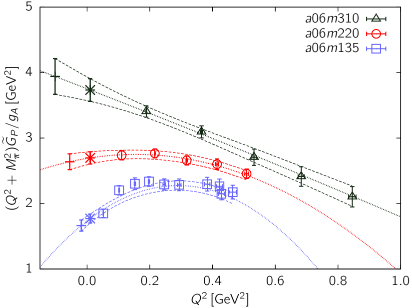


Induced Pseudoscalar Form Factor $\tilde{G}_P$

$$\frac{m_\mu}{2M_N}\frac{\tilde{G}_P(Q^2)}{g_A} = \frac{m_\mu}{2M_N}\left[\frac{4M_N^2}{Q^2+M_\pi^2}\right]\frac{G_A(Q^2)}{g_A} \quad : \text{ pion pole dominance}$$

$$\approx \frac{c_0}{Q^2+M_\pi^2} + c_1 + c_2 Q^2 \quad : \text{ fit}$$





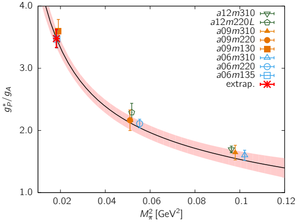
	$c_0 \times (2M_N/m_\mu)$ [GeV^2]	$c_1 \times (2M_N/m_\mu)$	$c_2 \times (2M_N/m_\mu)$ [GeV^{-2}]
a06m310	3.94(27)	-1.85(82)	-0.10(62)
a06m220	2.64(12)	0.93(61)	-2.54(84)
a06m135	1.66(09)	3.84(63)	-6.4(1.1)

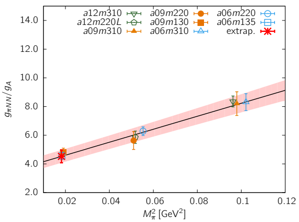
Pion pole dominance does not work as the $M_\pi \to 135$ MeV

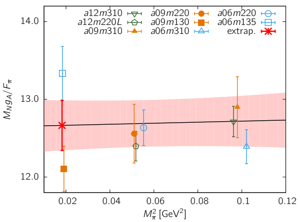
g_P^* and $g_{\pi NN}$ from fits to $\tilde{G}_P(Q^2)$

for each ensemble:

$$g_P^* \equiv \frac{m_\mu}{2M_N}\tilde{G}_P(0.88m_\mu^2) \qquad g_{\pi NN} = \lim_{Q^2\to -M_\pi^2}\frac{M_\pi^2+Q^2}{4M_N F_\pi}\tilde{G}_P \qquad g_{\pi NN} = \frac{M_N g_A}{F_\pi}\ [\text{GT}]$$







extrapolation to $a \to 0$, $M_\pi \to 135\text{MeV}$:

$$g_P^*/g_A = \frac{h_0}{0.88m_\mu^2 + M_\pi^2} + h_1 + h_2 a + h_3 M_\pi^2 \quad \to \quad 3.48(14)$$

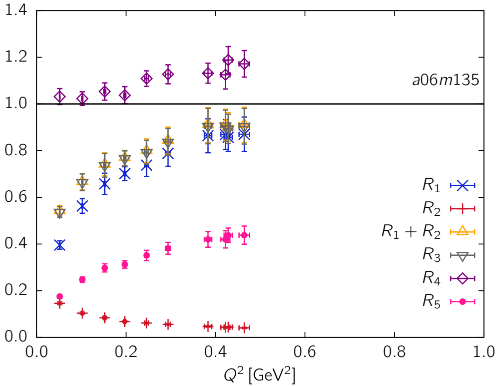
$$g_{\pi NN}/g_A = d_1 + d_2 a + d_3 M_\pi^2 \quad \to \quad 4.53(45)$$

from fit $g_P^* = 4.44(18)$ $\quad 8.06(55)$ [Mucap], $8.29^{+0.24}_{-0.13} \pm 0.52$ [χPT]

from fit $g_{\pi NN} = 5.78(57)$ $\quad 13.69 \pm 0.12 \pm 0.15$ [πNscattering lenght]

from GT $g_{\pi NN} = 12.87(34)$ $\quad 13.00$ [$g_A = 1.276$, $M_N = 939$ MeV, $F_\pi = 92.2$ MeV]

Testing PCAC: $2\widehat{m}G_P(Q^2) = 2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\widetilde{G}_P(Q^2)$



$$R_1 = \frac{Q^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)}$$

$$R_2 = \frac{2\widehat{m}}{2M_N}\frac{G_P(Q^2)}{G_A(Q^2)}$$

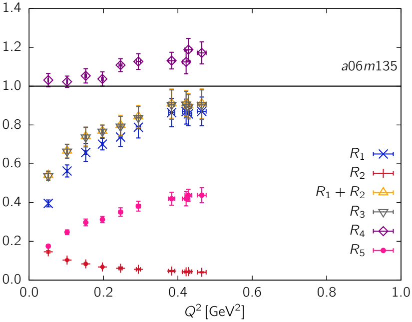
$$R_3 = \frac{Q^2 + M_\pi^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)}$$

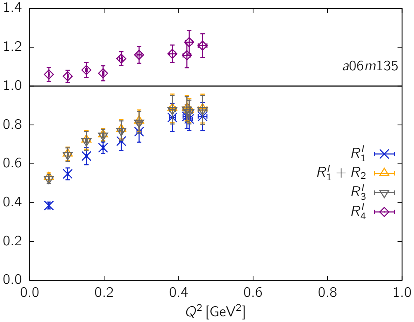
$$R_4 = \frac{4\widehat{m}M_N}{M_\pi^2}\frac{G_P(Q^2)}{\tilde{G}_P(Q^2)}$$

$$R_5 = \frac{aQ^2}{4M_N}\frac{G_P(Q^2)}{G_A(Q^2)}$$

- $R_1 + R_2 = 1$ tests PCAC relation.
  $\Rightarrow$ $Q^2 \to 0$, deviation from 1 increases.
- $R_3 = 1$ tests pion-pole dominance hypothesis.
  $\Rightarrow$ $Q^2 \to 0$, it remains close to the PCAC $R_1 + R_2$.
- $R_3 = 1$ is provided $R_4 = 1$ if PCAC is satisfied.
- $R_1 \to R_1 + 2c_A R_5$ by the $\mathcal{O}(a)$ improvement of A_μ.

$\mathcal{O}(a)$ improvement of A_μ does not fix PCAC.





$$R_1 = \frac{Q^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} \quad \longrightarrow \quad R_1^I = \frac{Q^2}{4M_N^2}\frac{\tilde{G}_P^I(Q^2)}{G_A(Q^2)}$$

$$R_3 = \frac{Q^2 + M_\pi^2}{4M_N^2}\frac{\tilde{G}_P(Q^2)}{G_A(Q^2)} \quad \longrightarrow \quad R_3^I = \frac{Q^2 + M_\pi^2}{4M_N^2}\frac{\tilde{G}_P^I(Q^2)}{G_A(Q^2)}$$

$$R_4 = \frac{4\widehat{m}M_N}{M_\pi^2}\frac{G_P(Q^2)}{\tilde{G}_P(Q^2)} \quad \longrightarrow \quad R_4^I = \frac{2\widehat{m}2M_N}{M_\pi^2}\frac{G_P(Q^2)}{\tilde{G}_P^I(Q^2)}$$

$$R_2 = \frac{2\widehat{m}}{2M_N}\frac{G_P(Q^2)}{G_A(Q^2)}\,,\quad R_5 = \frac{aQ^2}{4M_N}\frac{G_P(Q^2)}{G_A(Q^2)}$$

Vector Form Factors $G_E(Q^2)$, $G_M(Q^2)$

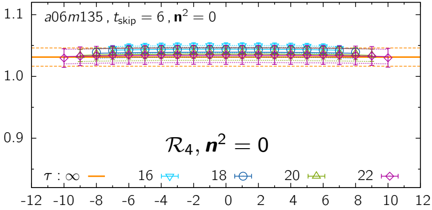
- Results obtained by using following fit.

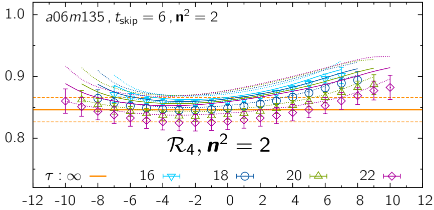
$$C_\Gamma^{(3\mathrm{pt})}(t;\tau;\boldsymbol{p}',\boldsymbol{p}) = |\mathcal{A}_0'||\mathcal{A}_0|\langle 0'|\mathcal{O}_\Gamma|0\rangle e^{-E_0 t - M_0(\tau-t)} + |\mathcal{A}_1'||\mathcal{A}_1|\langle 1'|\mathcal{O}_\Gamma|1\rangle e^{-E_1 t - M_1(\tau-t)} + |\mathcal{A}_0'||\mathcal{A}_1|\langle 0'|\mathcal{O}_\Gamma|1\rangle e^{-E_0 t - M_1(\tau-t)} + |\mathcal{A}_1'||\mathcal{A}_0|\langle 1'|\mathcal{O}_\Gamma|0\rangle e^{-E_1 t - M_0(\tau-t)}$$

- Data is displayed using the following ratio.

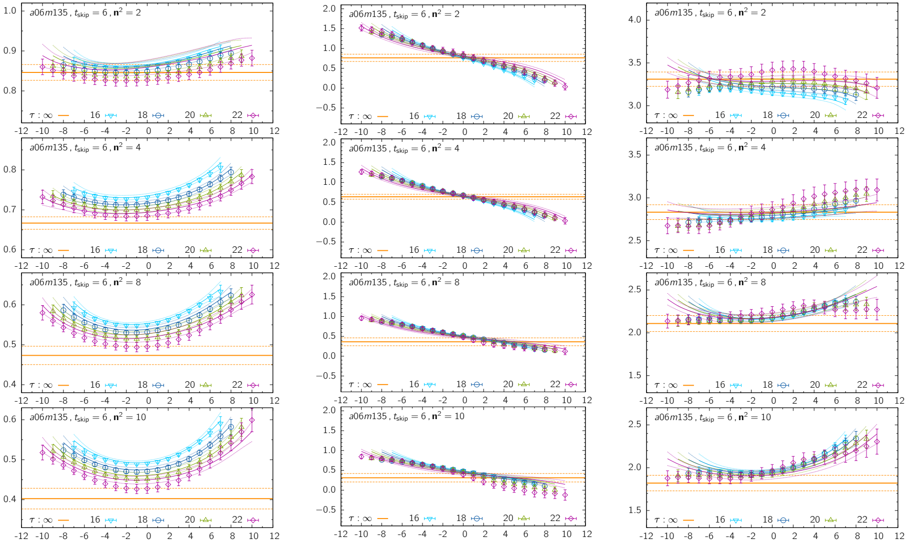
$$\mathcal{R}_\Gamma(t,\tau,\boldsymbol{p}',\boldsymbol{p}) = \frac{C_\Gamma^{(3\mathrm{pt})}(t,\tau;\boldsymbol{p}',\boldsymbol{p})}{C^{(2\mathrm{pt})}(\tau,\boldsymbol{p}')} \times \left[\frac{C^{(2\mathrm{pt})}(t,\boldsymbol{p}')C^{(2\mathrm{pt})}(\tau,\boldsymbol{p}')C^{(2\mathrm{pt})}(\tau-t,\boldsymbol{p})}{C^{(2\mathrm{pt})}(t,\boldsymbol{p})C^{(2\mathrm{pt})}(\tau,\boldsymbol{p})C^{(2\mathrm{pt})}(\tau-t,\boldsymbol{p}')}\right]^{1/2}$$

$$\begin{array}{lcccccc} \Gamma: & \gamma_1(\mathrm{Re}) & \gamma_2(\mathrm{Re}) & \gamma_1(\mathrm{Im}) & \gamma_2(\mathrm{Im}) & \gamma_3(\mathrm{Im}) & \gamma_4(\mathrm{Re}) \\ \to & -q_2 G_M, & q_1 G_M, & q_1 G_E, & q_2 G_E, & q_3 G_E, & (M+E)G_E \end{array}$$





Controlling Excited State Contribution to $\langle N|V_\mu|N\rangle$



$\mathrm{Re}\,\mathcal{R}_4 \to G_E(Q^2)$ $\quad$ $\mathrm{Im}\,\mathcal{R}_1 \to G_E(Q^2)$ $\quad$ $\mathrm{Re}\,\mathcal{R}_1 \to G_M(Q^2)$

Q^2 Fits to Electromagnetic Form Factors

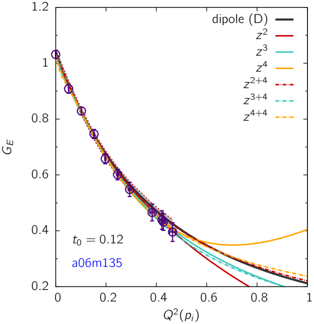
- dipole

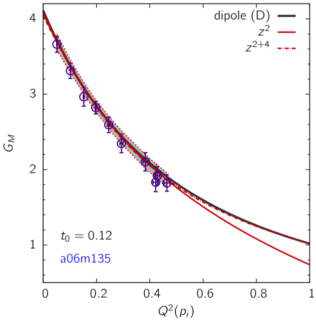
$$G_A(Q^2) = \frac{G_A(0)}{(1 + Q^2/\mathcal{M}_A^2)^2} \quad \Longrightarrow \quad \langle r_A^2 \rangle = \frac{12}{\mathcal{M}_A^2}$$

- z-expansion w/o sumrule constraints

$$\frac{G_A(Q^2)}{G_A(0)} = \sum_{k=0}^{\infty} a_k z(Q^2)^k \,, \; z = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} + t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} + t_0}} \,,$$

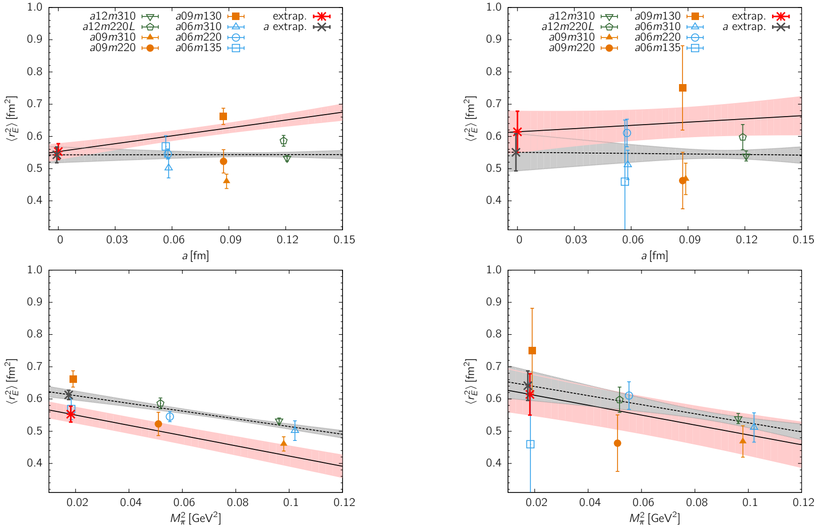
$$\sum_{k=n}^{k_{\rm max}} k(k-1)\dots(k-n+1)a_k = 0 \qquad n = 0, 1, 2, 3 \,.$$





Electric Charge Radius $\langle r_E^2 \rangle$: dipole versus z^{2+4}

$$\langle r_E^2 \rangle = d_0 + d_1 a + d_2 M_\pi^2 + d_3 M_\pi^2 e^{-M_\pi L}$$



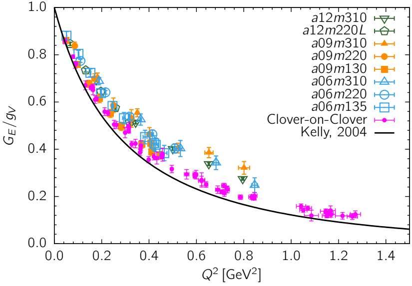
dipole ($d_3 = 0$) $\qquad$ z^{2+4} ($d_3 = 0$)

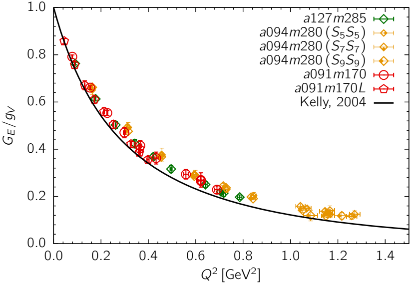
G_E: Clover-on-HISQ versus Clover-on-Clover

$a \to 0\,,\ M_\pi \to 135$ MeV extrapolation

$\langle r_E^2 \rangle$	with FV	without FV
dipole	0.53(03)	0.55(02)
z^2	0.61(05)	0.63(04)
z^{2+4}	0.60(07)	0.61(06)
z^3	0.72(09)	0.71(07)
z^{3+4}	0.88(13)	0.89(11)

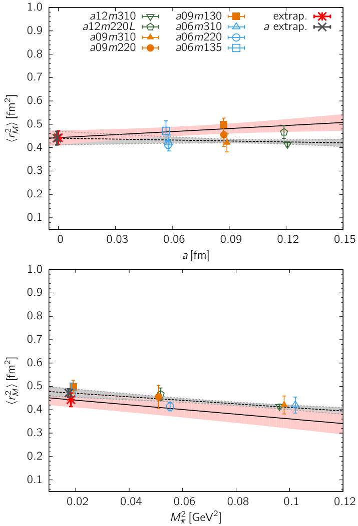
No $a \to 0\,,\ M_\pi \to 135$ MeV extrapolation yet



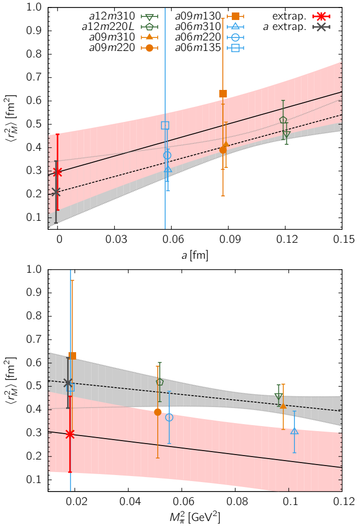


Magnetic Charge Radius $\langle r_M^2 \rangle$: dipole versus z^{2+4}

$$\langle r_M^2 \rangle = d_0 + d_1 a + d_2 M_\pi^2 + d_3 M_\pi^2 e^{-M_\pi L}$$



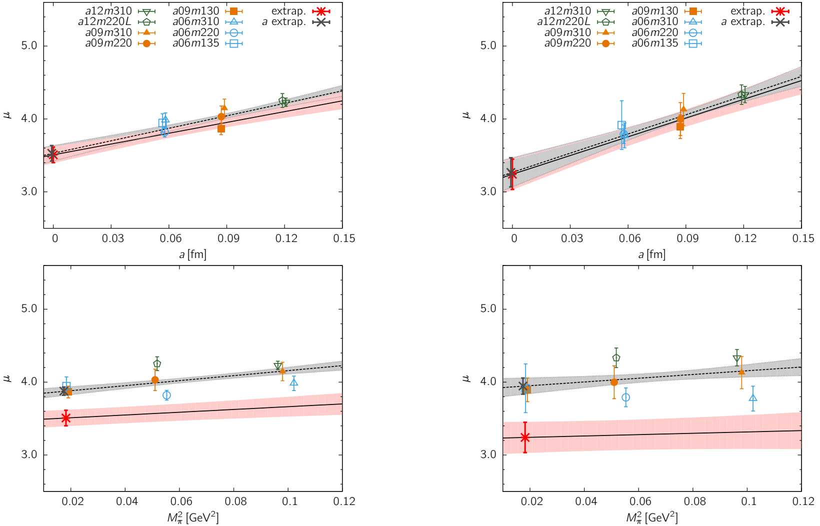
dipole ($d_3 = 0$)



z^{2+4} ($d_3 = 0$)

Magnetic Moment $\mu_p - \mu_n$: dipole versus z^{2+4}

$$\langle \mu \rangle = d_0 + d_1 a + d_2 M_\pi^2 + d_3 M_\pi^2 e^{-M_\pi L}$$



dipole ($d_3 = 0$) z^{2+4} ($d_3 = 0$)

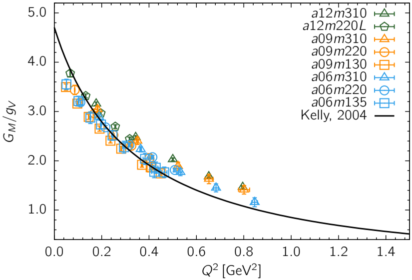
G_M: Clover-on-HISQ versus Clover-on-Clover

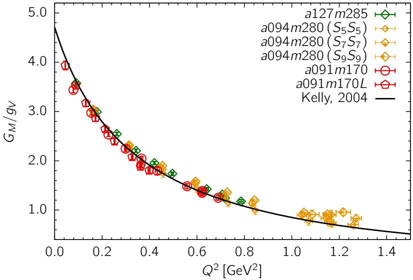
$a \to 0\,,\ M_\pi \to 135\ \mathrm{MeV}$ extrapolation

$\langle r_M^2 \rangle$	with FV	without FV
dipole	0.45(04)	0.44(03)
z^2	0.52(08)	0.51(06)
z^{2+4}	0.28(17)	0.30(16)

$\mu_p - \mu_n$	with FV	without FV
dipole	3.63(17)	3.51(11)
z^2	3.56(21)	3.45(14)
z^{2+4}	3.32(30)	3.24(21)

No $a \to 0\,,\ M_\pi \to 135\ \mathrm{MeV}$ extrapolation yet





Summary

- $r_A = 0.47(7)(5)\,\mathrm{fm}\,,\ M_A = 1.46(27)\,\mathrm{GeV}$ from $z-$expansion fit
- $r_A = 0.49(3)\,\mathrm{fm}\,,\qquad M_A = 1.39(8)\,\mathrm{GeV}$ from dipole fit
- M_A is larger than phenomenology 1.026(21) ($\nu, \bar{\nu}$ scattering), 1.069(16) (Electroproduction), and 1.00(24) (Deuterium) but consistent with the miniBooNE result $1.35(17)\,\mathrm{GeV}$.
- g_P^* and $g_{\pi NN}$ from the extrapolation of $\widetilde{G}_P(Q^2)$ are significantly smaller than the phenomenology.
- Three FFs G_A, $\widetilde{G}_P$, G_P do not satisfy PCAC relation.
  $\rightarrow$ need to understand it
- r_E, r_M, μ are smaller than the phenomenology.
- Dipole ansatz is surprisingly good.
- Estimates from dipole and z-expansion are consistent.
- Fits to G_M are the least stable.

Thank you for your attention.

Thanks for computing reasource allocations at NERSC, USQCD-Fermilab, and LANL and ALCC allocation on TITAN at OLCF